

Geometric Art

Content Summary

In Chapter 0, students make connections between geometric ideas and things they've seen before: common shapes (circles, hexagons, pentagons), mirror symmetry (or *reflectional* symmetry), and lines and angles. Students study mathematics through art, which may be a new and motivating context for them. This approach is designed to connect to and expand the intuitive understanding they already have about shapes as a whole. In addition to reviewing familiar mathematical ideas, Chapter 0 introduces rotational symmetry, compass and straightedge constructions, and tessellations.

Your student's teacher may select lessons from Chapter 0 to review skills and ideas that this class needs to understand better. The teacher can also use Chapter 0 as a gentle introduction to *Discovering Geometry's* methods for bringing about deep understanding through investigating, working in groups, and using geometry tools and perhaps software. Meanwhile, you can use this time to help establish patterns in the way you'll work with your student.

Rotational Symmetry

Most people say that a figure is *symmetric* if it has mirror symmetry—one side is a reflection of the other. There are other kinds of symmetry as well. Chapter 0 introduces *rotational symmetry*. A figure has 2-fold rotational symmetry if it looks the same after you turn it through half of a full turn—that is, through 180 degrees. It has 3-fold rotational symmetry if it looks the same after being turned one-third of a full turn—that is, through 120 degrees. And so on. The figure on the right, for example, has 3-fold rotational symmetry. All figures look the same after you turn them 360 degrees, so a figure that only satisfies this requirement is *not* considered to have rotational symmetry.



Students will revisit reflectional and rotational symmetry in the context of transformations in Chapter 7.

Tessellations

If you can make a bunch of tiles all the same shape and use those tiles to cover a flat surface, with no gaps, then you have a *tessellation*, or tiling pattern. In Chapter 0, students see a few tessellations. In Chapter 7, they'll study properties of shapes that can be used in tessellations, as well as tessellations made with more than one shape.

Constructions

Mathematicians in ancient Greece believed that the most perfect shapes were circles and straight lines, so they set out to see what they could do with a compass (to make circles) and a straightedge (to make lines). For a compass, they needed a rope with a stake tied to one end. The straightedge was unmarked, so it couldn't be used for measuring. Nevertheless, they found that with these tools they could construct many shapes and angles. Whatever can be drawn with these tools is called a *geometric construction*.

Although most of us don't carry the same idea of perfect shapes as the Greeks did 2500 years ago, studying geometric constructions is valuable for students. It gives them a feeling for shapes and relationships. This feeling is very useful in studying geometric concepts and in logical reasoning.

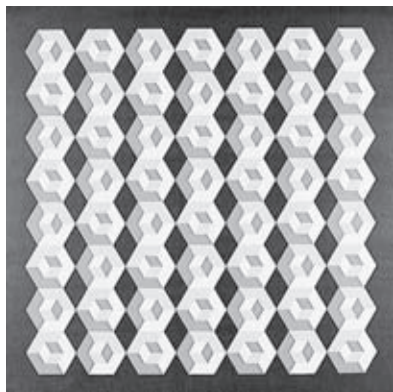
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Chapter 0 • Geometric Art (continued)

Summary Problem

You and your student might discuss this summary problem from Chapter 0. It's a good problem to revisit several times while working through the chapter.

What ideas from this chapter do you see in *Hot Blocks*, the fine art pictured on page 24, also shown below?



Hot Blocks, (1966–67), Edna Andrade,
© Edna Andrade, Philadelphia Museum of Art,
purchased by Philadelphia Foundation Fund.

Discuss these questions with your student from the point of view of *a student to your student*:

- What kinds of symmetry does the picture have?
- What kinds of symmetry does each block have?
- Could you construct one of the blocks with straightedge and compass?
- What did the artist do to make each block appear three-dimensional?
- Could you construct a different but similar drawing with straightedge and compass?
- Is your own drawing more or less elegant than this one?
- Is it okay to discuss elegance in math?

Some of these questions have several possible valid answers. It is not important that you know all the answers. Instead, as you talk about the answers, make sure your student gives a good explanation of why an answer is reasonable. For example, an answer of “yes” or “no” is not enough. Encourage your student to ask questions too.

Sample Answers

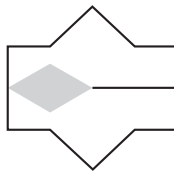
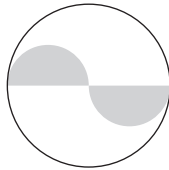
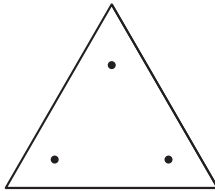
If shading is ignored, the art has 2-fold rotational symmetry, with a vertical line of symmetry through the middle column of blocks and a horizontal line of symmetry between the middle rows of blocks. Each block, if shading is ignored, has 3-fold rotational symmetry and three lines of reflection. The patterns of shading make it appear three-dimensional. The two hexagons that make up each block can be constructed: Starting with circles and using their radii to mark off six equal arcs on the circles, points can be determined to draw equilateral hexagons from which to make blocks.

Although elegance is a matter of opinion, it is acceptable to talk about elegance in any discipline. Here the drawing could be considered elegant because of its artistic value or because of the mathematics it displays.

Chapter 0 • Review Exercises

Name _____ Period _____ Date _____

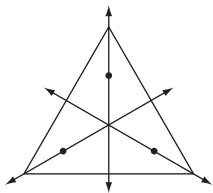
1. (*Lesson 0.1*) Which of the designs below have reflectional symmetry? Draw the lines of symmetry. Which of the designs have rotational symmetry?



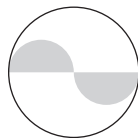
2. (*Lesson 0.2*) Name the basic tools of geometry. What are these tools used for?
3. (*Lesson 0.3*) Use your compass to create a 6-petal daisy design. Color it so it has reflectional symmetry but no rotational symmetry.
4. (*Lesson 0.4*) Use your straightedge and a copy of your daisy design from Exercise 3 to construct a regular hexagon. Use this to create one block from the Amish quilt tumbling block design pictured on page 14.

SOLUTIONS TO CHAPTER 0 REVIEW EXERCISES

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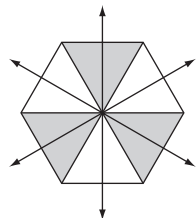
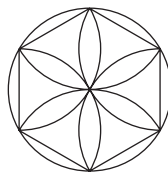


Rotational and reflectional

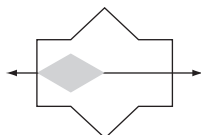


Rotational

4.



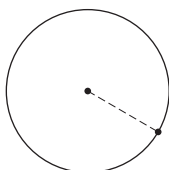
Rotational and reflectional



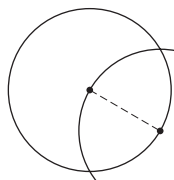
Reflectional

2. The compass is used for constructing circles and marking off equal distances, and the straightedge is used for drawing straight lines.

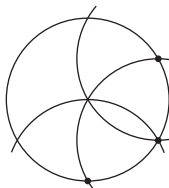
3.



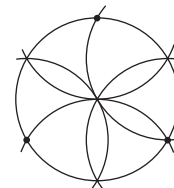
Step 1



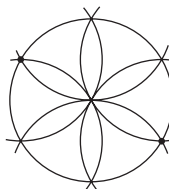
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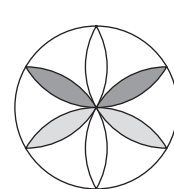
Step 3



Step 4



Step 5



Step 6

Introducing Geometry

Content Summary

Chapter 1 introduces the building blocks and vocabulary of geometry. Though it refers to many geometric shapes that students have seen before, it emphasizes what's required to make a good definition. Many of the investigations engage students in the process of writing definitions by presenting visual examples of shapes that belong to a group and shapes that do not belong to the group, called non-examples. This process begins to move students from thinking of shapes as their whole appearance toward thinking about their parts, and thinking about classes of shapes—for instance, thinking about what all rectangles have in common and what makes them rectangles. This lays the groundwork for understanding the properties of shapes, a higher level of thinking that becomes increasingly important in later chapters.

Definitions

A good definition can usually be put into the form “A [*term being defined*] is a [*general group*] that [*has some characteristic*].” For example, take the definition of a triangle.

A **triangle** is a polygon that has three sides.

For this definition to be clear, we must know what the terms used in the definition mean. What is a polygon? What is a side of a polygon? All terms used in a good definition should already be previously defined. But this leads to a problem: Where should we begin? We must agree on some terms to be understood without being defined. They can then be used to define the first definitions, which in turn can be used in other definitions. In geometry, these *undefined terms* are **point**, **line**, and **plane**. From these terms, we can define other terms, which we can then use to define the terms triangle and angle in the original example above.

Points are **collinear** if they lie on the same line.

A **line segment** is two points (called endpoints) and all the points between them that are collinear with the two points.

A **polygon** is a closed figure in a plane, formed by connecting line segments endpoint to endpoint (called vertices) with each segment intersecting exactly two others.

A **side of a polygon** is a line segment connecting consecutive vertices of the polygon.

While these definitions may not always be in the exact form described at the beginning of this section, most are based on *classifying* the term within a general group, then *differentiating* it from the group according to some characteristics.

Angles

Besides defining *angle*, the book discusses measures of angles and has students write definitions of different kinds of angles (right, acute, obtuse), as well as special pairs of angles (complementary, supplementary, vertical, linear pair).

Polygons

The book defines *polygon* and has students write definitions of special kinds of polygons (equilateral, equiangular, regular). Students also define different kinds of

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Chapter 1 • Introducing Geometry (continued)

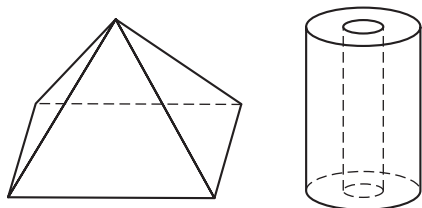
triangles (right, acute, obtuse, scalene, equilateral, isosceles) and quadrilaterals (trapezoid, kite, parallelogram, rhombus, rectangle, square).

Circles

A *circle* is the set of all points in a plane at a given distance from a given point. Students review some parts of a circle (center, radius, arc) and define other parts (chord, diameter, tangent).

Summary Problem

What geometric shapes does each of these solids contain?



Questions you might ask in your role as student to your student:

- What are the solids themselves called?
- Are there parts of the solids that are hidden in these drawings?
- What do the hidden parts look like?
- What shapes are mentioned in this chapter?
- Which of those shapes are contained in the solids? Can you color them in?
- Are any of those shapes not contained in the solids?
- Can you change these solids to some that do contain those shapes?
- What geometric relationships are mentioned in the chapter?
- Which of those relationships do shapes in the solids have?
- What measurements are mentioned in the chapter?
- Could you make any of those measurements on these solids?

Sample Answers

Together, the shapes—a rectangular pyramid and a cylinder with a cylindrical hole through it—contain line segments, collinear and coplanar points (vertices), acute and right angles, triangles, a rectangle, and concentric circles. The bottom of the cylinder, the interior of the cylindrical hole, and the back triangle in the pyramid are hidden from view. Other terms in the chapter include ray; vertical angles; polygons of several types, including several quadrilaterals; many lines related to circles, including tangent; and other solids, including cylinder, cones, and spheres. It's possible to make some of those polygons by taking a “section,” or slicing through the pyramid.

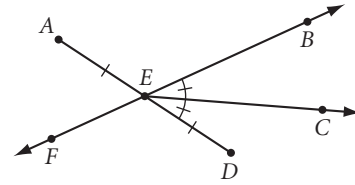
Relationships mentioned in the chapter include bisect, congruent, perpendicular, parallel, skew lines, and supplementary or complementary angles. In the figures, it appears that: The base of the pyramid has edges that are perpendicular to each other and opposite edges congruent to each other; the bases of the cylinder are parallel to each other; and if the triangles are equilateral or isosceles, they have congruent angles and congruent edges. The drawing may not show the true angles or side lengths, but otherwise you could find the measure of each angle with a protractor and the length of each side with a ruler.

Chapter 1 • Review Exercises

Name _____ Period _____ Date _____

(Lesson 1.1, 1.2) Identify the following:

1. Midpoint: _____
2. Segment: _____
3. Angle Bisector: _____
4. Ray: _____
5. Line: _____
6. Angle: _____

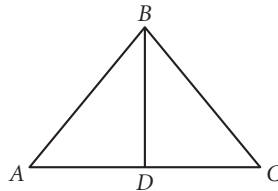


7. (Lesson 1.2) Mark the figure with the following information:

$$\angle A \cong \angle C$$

$$\overline{AB} \cong \overline{BC}$$

$$BD \perp DC$$

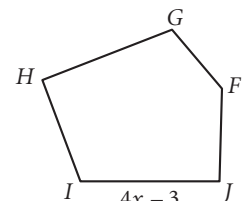
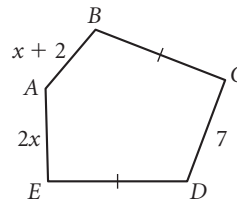


(Lessons 1.3, 1.5, 1.6, 1.7) Draw and carefully label each figure.

8. Supplementary angles $\angle ABD$ and $\angle DBC$ with $m\angle ABD = 90^\circ$
9. Isosceles right triangle
10. Circle O with diameter \overline{AB} and tangent \overleftrightarrow{CD}
11. Parallelogram ABCD

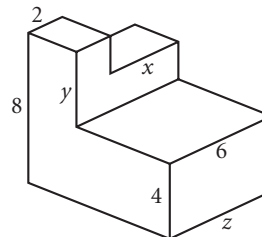
12. (Lesson 1.4) $ABCDE \cong FGHIJ$. The perimeter of ABCDE is 36 cm. Find these distances.

- a. $AB =$ _____
- b. $HI =$ _____
- c. $FJ =$ _____



13. (Lesson 1.8) Find the missing lengths. Assume all edges are perpendicular to each other.

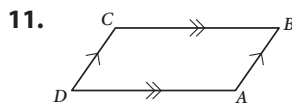
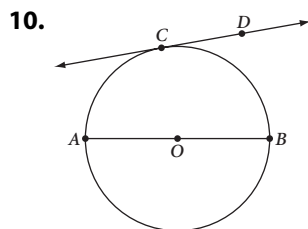
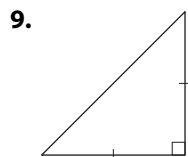
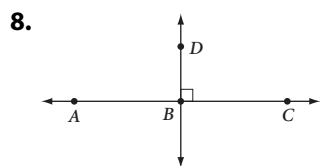
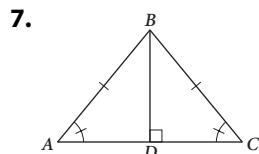
- a. $x =$ _____
- b. $y =$ _____
- c. $z =$ _____



14. (Lesson 1.9) Create a Venn Diagram to show the relationships among triangles, isosceles triangles, and right triangles.

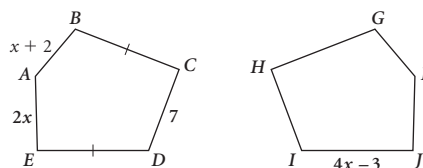
SOLUTIONS TO CHAPTER 1 REVIEW EXERCISES

1. E is the midpoint of \overline{AD} because $\overline{AE} \cong \overline{ED}$.
2. There are several segments, for example \overline{AE} or \overline{AD} .
3. \overrightarrow{EC} is the angle bisector because $\angle BEC \cong \angle CED$.
4. There are several rays, for example \overrightarrow{BF} or \overrightarrow{EC}
5. \overline{FB}
6. There are several angles, for example $\angle BEC$ or $\angle BED$.

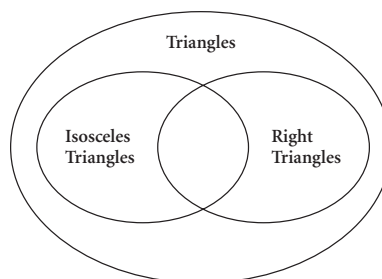


12. Because the two pentagons are congruent, $ED = JI = 4x - 3$. The marks on the pentagon show that $BC = ED$. Therefore, $BC = ED = 4x - 3$. The perimeter of $ABCDE = (x + 2) + (4x - 3) + 7 + (4x - 3) + 2x = 36$ cm. Solving for x , you get $x = 3$ cm. Substitute 3 in for x and use the fact that corresponding sides are congruent to find these side lengths.

- a. $AB = 5$ cm
- b. $HI = 7$ cm
- c. $FJ = 6$ cm



13. $x = 4$; $y = 4$; $z = 6$
14. Although it is possible for a triangle to be both isosceles and right, it is also possible for it to be one without being the other, so they are represented by two overlapping circles.



Reasoning in Geometry

Content Summary

One major purpose of any geometry course is to improve the ability of students to reason logically. Chapter 2 focuses on two basic kinds of reasoning: inductive and deductive. Students use *inductive reasoning* to identify visual and numerical patterns and to make predictions based on these patterns. Then students are introduced to the use of *deductive reasoning* to explain why these patterns are true. Students explore the relationships between the measures of angles formed by intersecting and parallel lines, make conjectures about these relationships, and learn how to use logical arguments to explain why these conjectures are true.

Inductive Reasoning

Every time you freeze your water bottle, the water expands. So you learn quickly not to put too much water in the bottle, to avoid having the top pop off or the bottle break. Reasoning in which you draw conclusions from experience is inductive.

Mathematicians use inductive reasoning to guess at what might be true. For example, if you add up positive odd numbers starting at 1, you get a pattern.

$$1 + 3 = 4$$

$$1 + 3 + 5 = 9$$

$$1 + 3 + 5 + 7 = 16$$

The sums seem to be square numbers: $4 = 2 \cdot 2$, $9 = 3 \cdot 3$, $16 = 4 \cdot 4$. Even if you “add up” only the first odd number, 1, the sum is a square: $1 \cdot 1$. You might conclude that the sum of *any* number of positive consecutive odd numbers, beginning with 1, is a square number.

But inductive reasoning is not foolproof. It may be that one time you’ll put the water bottle in the freezer and the water won’t expand. (Perhaps the freezer isn’t working, or the water is flavored in some way.) So mathematicians are not satisfied with inductive reasoning. Inductive reasoning leads only to guesses, or *conjectures*. A conjecture becomes a mathematical fact, or theorem, only if someone shows that it’s the conclusion of deductive reasoning.

Deductive Reasoning

Deductive reasoning, also called *proof*, is reasoning from proven facts using logically valid steps to arrive at a conclusion. A proof can serve many purposes. Mathematicians often use proof to verify that a conjecture is true for all cases, not just for those they examined, or to convince others. Proofs often help answer the question: *Why?* The use of proof to explain why is a natural extension for students at this point in the course and helps to deepen their understanding. This chapter stresses this illumination purpose of proof.

If you took a geometry course, you may have encountered proofs displayed in two columns: a column of statements and a column of reasons, with each statement justified by a reason. However, most students are overwhelmed by this approach. They find the format difficult to follow and miss the big picture. In *Discovering Geometry*, two-column proofs come in Chapter 13 with the study of geometry as a mathematical system, at which point students are at the appropriate developmental

(continued)

Chapter 2 • Reasoning in Geometry (continued)

level. For now, students are encouraged to use informal deductive arguments written in a paragraph form. In Chapter 4, they will be introduced to other formats for presenting proofs.

Reasoning Strategies

The most difficult part of the process for writing a deductive argument is to determine the underlying logic of the argument and what information to include. Starting in Chapter 2 and continuing throughout the book, students are taught reasoning strategies, ways of thinking that help to construct a deductive argument. You may wish to discuss these ways of thinking with your student. The first three of these reasoning strategies are presented in this chapter, and one more strategy is introduced in each subsequent chapter as indicated.

- Draw a labeled diagram and mark what you know.
- Represent a situation algebraically.
- Apply previous conjectures and definitions.
- Break a problem into parts (Chapter 3)
- Add an auxiliary line (Chapter 4)
- Think backward (Chapter 5)

Summary Problem

Draw two intersecting lines. What do you notice about the vertical angles? Can you explain any patterns you see?

Questions you might ask in your role as student to your student:

- What does the term *vertical angles* mean?
- Which pairs of angles are vertical angles?
- What conjecture might you state about the measures of vertical angles?
- Does the conjecture you're making apply to all pairs of intersecting lines that you've tried, or can you find a counterexample?
- Do you think your conjecture holds for all pairs of intersecting lines?
- How would you show that it's true for all pairs?

Sample Answers

When two lines intersect, they form four distinct angles. Two nonadjacent angles formed by the intersecting lines are vertical angles. Two intersecting lines form two pairs of vertical angles. The angles in each pair have equal measures. Explaining why could involve talking about straight lines and the various pairs of adjacent angles, or about a rotation around the vertex. You can write a deductive argument to show that the Vertical Angles Conjecture follows logically from the Linear Pair Conjecture, as shown on page 124.

Chapter 2 • Review Exercises

Name _____ Period _____ Date _____

1. (Lesson 2.1) Use the rule provided to generate the next five terms of the sequence.

$$2, 5, 9, 14, \dots, \frac{n(n+3)}{2}, \dots$$

2. (Lesson 2.2, 2.3, 2.4) Find the rule for the number of circles in the n th figure, and use that to find the number of circles in the 20th figure. Are you using inductive or deductive reasoning to answer this question?

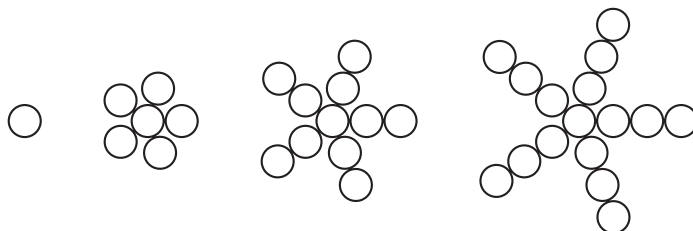
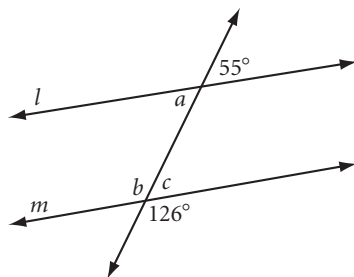
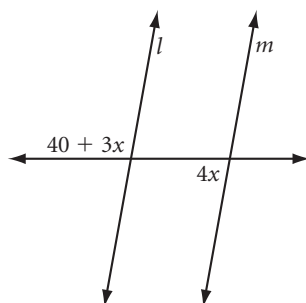


Figure number	1	2	3	4	...	n	...	20
Number of circles	1	6	11	16	

3. (Lessons 2.5, 2.6) Find the measure of each lettered angle. Is line l parallel to m ? Explain your reasoning.



4. (Lesson 2.6) Lines l and m are parallel to each other. Find x .



SOLUTIONS TO CHAPTER 2 REVIEW EXERCISES

1. 20, 27, 35, 44, 54

The first four terms come from substituting 1, 2, 3, and 4 for n in the given rule. Substitute 5, 6, 7, 8, and 9 for n to find the next five terms. See bottom of the page.

2. $5n - 4$ (or $5(n - 1) + 1$, which is equivalent); 96; inductive reasoning

First, look for the difference between the terms. In this case we add five to each term to get the next term.

Figure number	1	2	3	4	...
Number of circles	1	6	11	16	...
		+5	+5	+5	

Because the difference between consecutive terms is always five, the rule is $5n + \text{"something."}$ Let c stand for the unknown "something," so the rule is $5n + c$. To find c , replace n in the rule with one term number. For example, try $n = 3$ and set the expression equal to 11, the number of circles in the 3rd figure.

$$5(3) + c = 11$$

$$15 + c = 11$$

$$c = -4$$

Therefore, the rule is $5n - 4$. To find out how many circles there are in the 20th figure, substitute 20 for n in the rule.

$$5(20) - 4 = 96$$

3. $a = 55^\circ$ by the Vertical Angles Conjecture.

$b = 126^\circ$ by the Vertical Angles Conjecture.

$c = 54^\circ$ by the Linear Pair Conjecture.

If lines l and m were parallel, then c would equal 55° by the Corresponding Angles Conjecture. However, $c = 54^\circ$ as seen above, so l and m are not parallel.

4. $40 + 3x + 4x = 180$ Corresponding Angles Conjecture and Linear Pair Conjecture.

$$40 + 7x = 180 \quad \text{Combine like terms.}$$

$$7x = 140 \quad \text{Subtract 40 from both sides.}$$

$$x = 20 \quad \text{Divide both sides by 7.}$$

- 1.

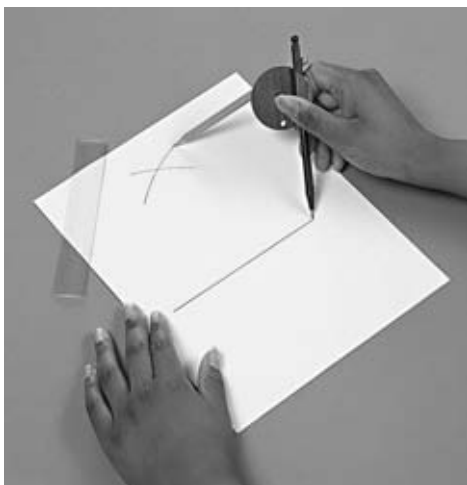
n	5	6	7	8	9
$\frac{n(n+3)}{2}$	$\frac{5(5+3)}{2} = 20$	$\frac{6(6+3)}{2} = 27$	$\frac{7(7+3)}{2} = 35$	$\frac{8(8+3)}{2} = 44$	$\frac{9(9+3)}{2} = 54$

Using Tools of Geometry

At this point in the course, you might think about how you and your student are interacting. For example, are you being a student to your student? Are you asking questions and letting your student explain? Do you explain little enough so that your student is becoming an independent learner and thinker? Do you answer questions that your student hasn't asked? Telling your student too much can waste time, because he or she might not understand; this can lead to your student feeling overwhelmed. Deeper understanding *can* result when you allow your student to teach the concept or skill to you and others.

Content Summary

In Chapter 3, students use hands-on constructions to develop an intuitive sense of the properties of shapes. This allows for a different mode of understanding how parts of a figure are related to the whole. Students learn to duplicate segments and angles, then work with perpendicular and parallel lines, angle and segment bisectors, and concurrent lines. An underlying concept is determination: What properties determine a figure's shape? That is, what properties are needed so that those properties determine one and only one shape? The idea of determination is addressed through *geometric constructions*.



Geometric Constructions

Students learn how to duplicate segments and angles, bisect segments and angles, and construct perpendicular and parallel lines. In addition to learning the classical compass and straightedge constructions, students learn how to use patty paper, small squares of waxed paper usually used between burger patties, as a unique geometry tool for constructions. You can think of geometric constructions as a game: Try to draw a figure, such as a square, using only a straightedge and compass and no measurements. The solutions can be applied to real-life problems, such as laying out the foundation for a building, but modern technology offers simpler methods. Try to help your student enjoy the game. For over 2500 years, this game has been giving students a hands-on understanding of properties that determine the shape of a figure. An understanding of determination will be especially helpful in the study of triangle congruence in Chapter 4.

(continued)

Chapter 3 • Using Tools of Geometry (continued)

Summary Problem

Draw three line segments on a sheet of paper. Using only the construction tools (unmarked straightedge and compass), duplicate them to make a triangle, if possible. If you succeed, make a few triangles. Then construct whatever you know how to construct on the triangle's sides and angles. What patterns do you see? What conjectures can you make? Can you give reasons those conjectures might be true?

Questions you might ask in your role as student to your student:

- Is it really possible to make a triangle from copies of those line segments without doing any measuring?
- Is it ever impossible to make a triangle from three given segments?
- What happens when you construct several triangles using the same three segments? Do three segments determine a triangle?
- What happens if you construct the perpendicular bisector of each side of the triangle?
- What happens if you construct the bisector of each angle of the triangle?
- What happens if you construct the midpoint of each side of the triangle and join each midpoint to other points?
- What else can you construct on this triangle, and what other patterns do you see?

Sample Answers for the Chapter 3 Summary Problem

As long as the sum of any two lines is longer than the third, a triangle can be constructed. Constructing several triangles from the same three segments always yields the same triangle (although it may be a mirror image). So, provided that a triangle can be constructed, the triangle is determined. Once you have three segments that can be duplicated to form a triangle, your student can use the copies of the triangle as they explore. The perpendicular bisector of each side can be constructed on one copy of the triangle and the angle bisectors on another. Students will find points of concurrency, points at which three lines intersect. Students will also find a relationship between the two parts of a segment determined by the point of concurrency for the medians, and relationships among some of these points of concurrency if they do the Euler Line Exploration at the end of the chapter. Your student may explore other relationships.

Chapter 3 • Review Exercises

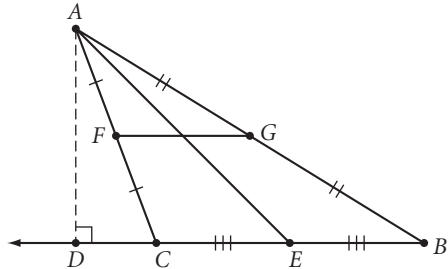
Name _____ Period _____ Date _____

1. (Lesson 3.1, 3.2) Duplicate \overline{AB} and find its perpendicular bisector.

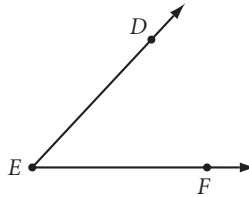


2. (Lesson 3.2, 3.3) Identify the parts of $\triangle ABC$:

- a. Median: _____
- b. Altitude: _____
- c. Midsegment: _____

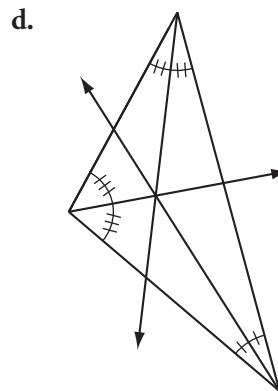
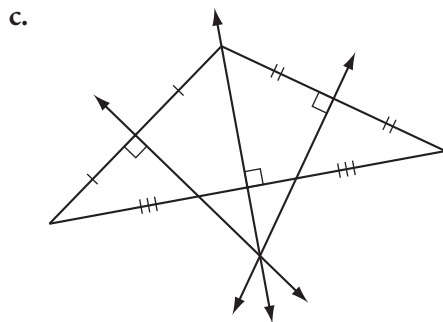
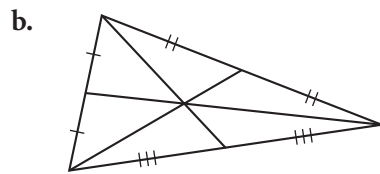
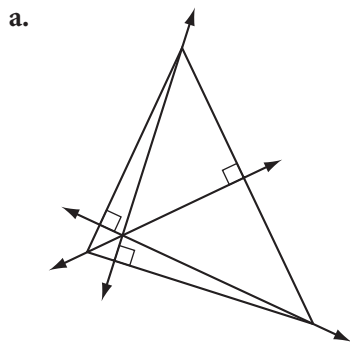


3. (Lesson 3.1, 3.4) Duplicate $\angle DEF$ and construct its angle bisector.



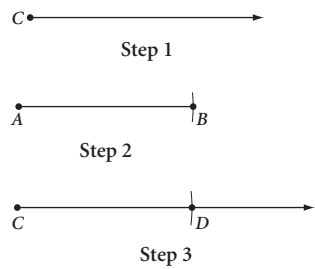
4. (Lesson 3.5, 3.6) Construct an isosceles trapezoid.

5. (Lesson 3.7) Name each center.

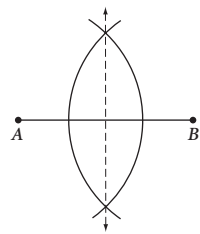


SOLUTIONS TO CHAPTER 3 REVIEW EXERCISES

1. Duplication:



Perpendicular Bisector

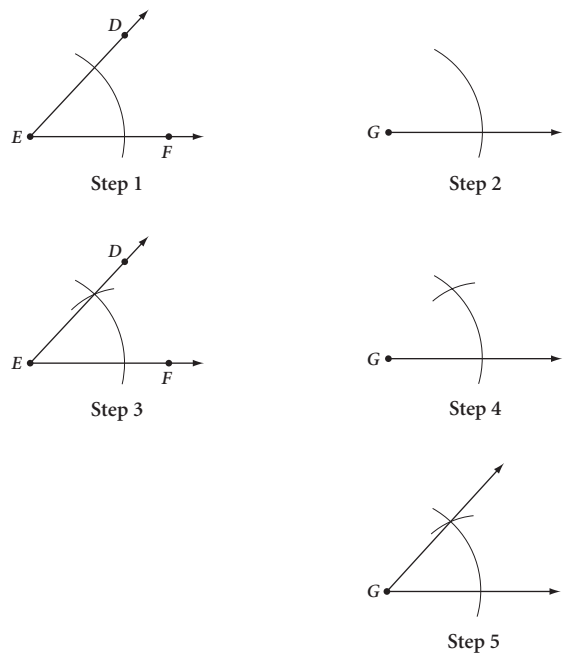


2. Median = \overline{AE} because it connects the vertex to the midpoint

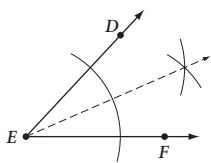
Altitude = \overline{AD} because it is the perpendicular segment from a vertex to the line containing the opposite side.

Midsegment = \overline{FG} because it connects the midpoints of two sides.

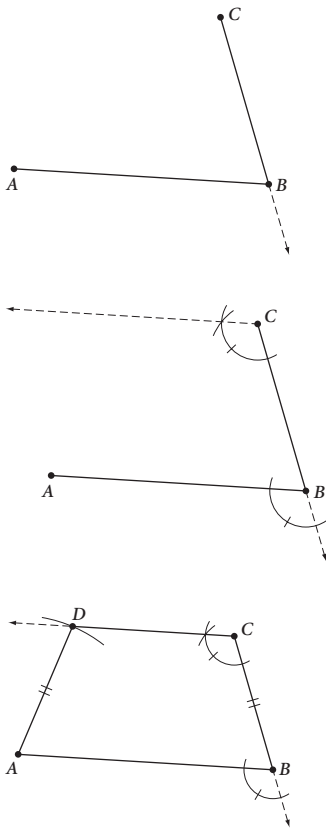
3. Duplication:



Angle bisector:



4.



- 5. a.** Orthocenter (intersection of altitudes)
- b.** Centroid (intersection of medians)
- c.** Circumcenter (intersection of perpendicular bisectors)
- d.** Incenter (intersection of angle bisectors)

Discovering and Proving Triangle Properties

Content Summary

In Chapter 4, students explore properties of triangles and the conditions that guarantee that two triangles are congruent. At first students make conjectures about the sum of interior and exterior angles, properties of isosceles triangles, and inequality relationships among sides and angles of triangles. Then they explore the characteristics needed to determine the congruence of two triangles and, finally, learn how to use this to prove their conjectures.

Angle Relationships in Triangles

Students experiment, look for patterns, and make conjectures about parts of triangles. These key conjectures result from their investigations:

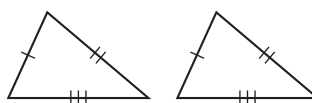
- The sum of the angles of any triangle is 180° .
- Two angles of a triangle are congruent if and only if two sides of the triangle are congruent.
- The measure of an exterior angle of a triangle equals the sum of measures of the two interior angles that are not adjacent to that exterior angle.

Triangle Congruence

The idea of congruence serves as a bridge between properties of a single triangle and properties shared by two or more triangles. In one sense, congruence is about determination. Knowing all three angles and all three sides certainly determines a triangle. In other words, if you draw a second triangle with all sides and angles congruent to those in the first triangle, the second triangle will be congruent to the first. Essentially, it will be the same triangle. So, knowing three angles and three sides guarantees the size and shape of the triangle, and all triangles that share that set of measurements are guaranteed to be congruent to each other. But is a triangle determined by fewer than six pieces of information? For example, is knowing three angles enough to determine a triangle? Is knowing two sides and an angle sufficient?

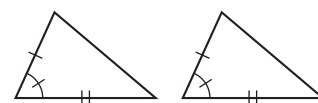
Likewise, how can you tell if two triangles are congruent? Are they congruent if their three angles have the same measure? Or, if two sides and an angle are the same? The book calls these conjectures *congruence shortcuts*. Those shortcuts that are sufficient to guarantee congruence are listed at right.

Side-Side-Side (SSS)



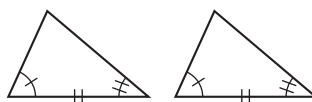
Three pairs of congruent sides

Side-Angle-Side (SAS)



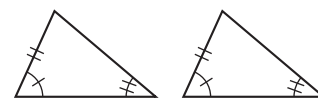
Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

Angle-Side-Angle (ASA)



Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

Side-Angle-Angle (SAA)



Two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles)

(continued)

Chapter 4 • Discovering and Proving Triangle Properties (continued)

Proof

An important reason for developing congruence shortcuts for triangles is to prove other properties of geometric figures. Chapter 4 introduces two formats for presenting proofs that are used throughout the remainder of the course. The *paragraph proof*, introduced at the beginning of the chapter, is a deductive argument that uses written sentences to support its claims with reasons. The *flowchart proof*, introduced near the end of the chapter, places statements in boxes connected by arrows to show the flow of logic, with the logical reasons presented below each box. In the last three lessons of the chapter, students apply triangle congruence shortcuts using these proof formats to prove triangle properties they discovered throughout the chapter.

Summary Problem

Suppose you know the length of the altitude to the base of an isosceles triangle and the measure of an angle between the base and another side. Is this enough information to determine a triangle, or are different triangles possible?

Questions you might ask in your role as student to your student:

- Does it help to draw particular altitudes and angles and try to make more than one triangle with the given properties?
- Do you think more than one triangle is possible? Why?
- Can you use the Triangle Sum Conjecture to help explain why?
- Can you use the Isosceles Triangle Conjecture to help explain why?
- Can you use congruence shortcuts to help explain why?
- Can you use the Vertex Angle Bisector Conjecture to help explain why?
- What if the triangle isn't isosceles?

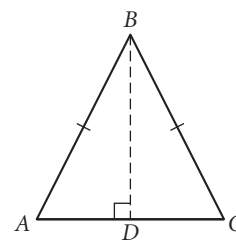
Sample Answers

Making and labeling a diagram is a good technique to help you think about a problem. In this case, your drawing will show you that there is only one possible triangle with an altitude of the specific length you drew and the angle you drew between the triangle's base and another side. To explain why, your student will use several of the conjectures. Here is one explanation, but encourage your student to give other explanations.

Because you know one of the base angles, you also know the other by the Isosceles Triangle Conjecture. The altitude of the isosceles triangle splits it into two right triangles because an altitude is defined to be perpendicular to the base. Both right triangles have the same two angles and one side (actually, two sides if you consider the shared altitude). By the Side-Angle-Angle Congruence Conjecture, all such triangles are congruent. Thus, if you construct a new isosceles triangle with the same given altitude and base angle, it will be composed of two of the same congruent right triangles, so it is determined.

Other explanations might use the Vertex Angle Bisector Conjecture along with any of the triangle congruence shortcuts to help explain why the two halves of the isosceles triangle are congruent.

Note that when these arguments are applied to a triangle that is not isosceles, they fail. The second “base angle” of the triangle need not be congruent to the first. To see this, encourage your student to draw a few noncongruent triangles that have a given altitude and a given angle between the base and one of the legs. In the diagram at right, if you are given $\angle A$, \overline{AB} , and the altitude \overline{AD} , you can locate point C anywhere along \overleftrightarrow{AD} if the triangle is not isosceles.

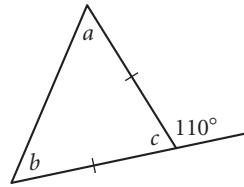


Chapter 4 • Review Exercises

Name _____ Period _____ Date _____

(Lesson 4.1, 4.2) For Exercises 1 and 2, find the missing measures.

1. Calculate the measure of each lettered angle and explain how you found it.

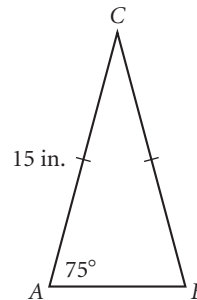


2. The perimeter of $\triangle ABC$ is 36 in.

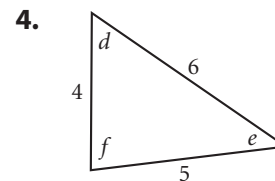
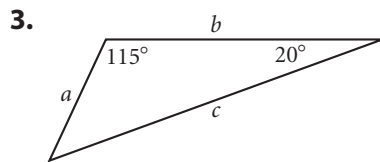
$$BC = \underline{\quad? \quad}$$

$$AB = \underline{\quad? \quad}$$

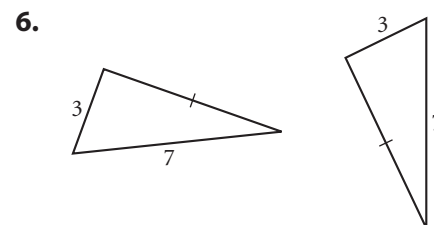
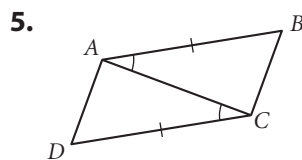
$$m\angle C = \underline{\quad? \quad}$$



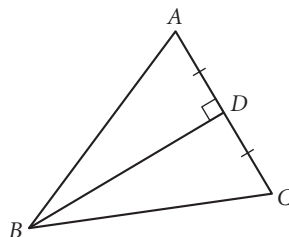
(Lesson 4.3) For Exercises 3 and 4, arrange the three unknown measures in order from greatest to least.



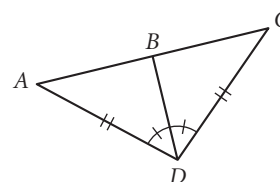
(Lesson 4.4, 4.5) For Exercises 5 and 6, decide whether the triangles are congruent. If they are, name the congruence shortcut you used.



7. (Lesson 4.6, 4.7) Create a flowchart proof to show that $\overline{AB} \cong \overline{CB}$.

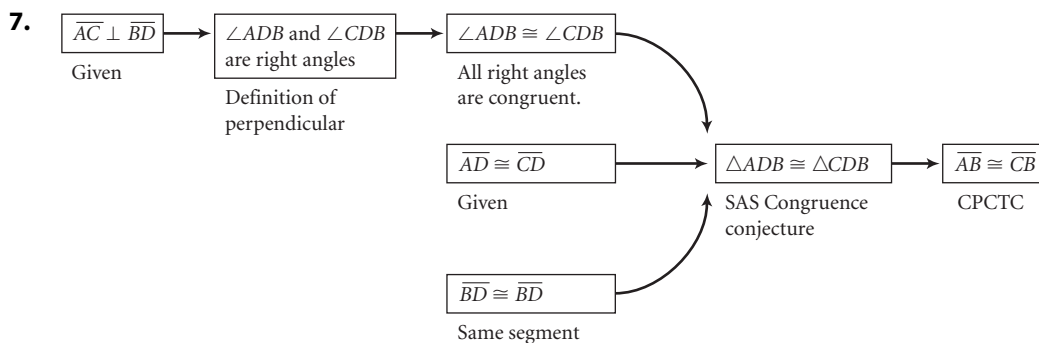


8. (Lesson 4.8) Write a paragraph proof to show that $\overline{AB} \cong \overline{CB}$.



SOLUTIONS TO CHAPTER 4 REVIEW EXERCISES

1. $c = 70^\circ$ Supplement of 110° .
 $a = b$ Isosceles triangle \cong .
 $a + b + 70^\circ = 180^\circ$ Triangle Sum Conjecture.
 $a + a + 70^\circ = 180^\circ$ Substitution.
 $2a + 70^\circ = 180^\circ$ Combine like terms.
 $2a = 110^\circ$ Subtraction.
 $a = 55^\circ$ Division.
 $b = 55^\circ$ Substitution.
2. $BC = 15$ in. Definition of an isosceles triangle.
 $AB + 15 + 15 = 36$ Perimeter.
 $30 + AB = 36$ Addition.
 $AB = 6$ in. Subtraction.
 $m\angle B = 75^\circ$ Base angles of an isosceles triangle are \cong .
 $75^\circ + 75^\circ + m\angle C = 180^\circ$ Triangle Sum Conjecture.
 $150^\circ + m\angle C = 180^\circ$ Addition.
 $m\angle C = 30^\circ$ Subtraction.
3. $c > b > a$ c is opposite the largest angle and a is opposite the smallest angle.
4. $f > d > e$ f is opposite the largest side and e is opposite the smallest side.
5. Yes, $\triangle ABC \cong \triangle CDA$ by SAS.
6. Yes, the triangles are congruent by SSS.
7. See bottom of the page.
8. We are given that $\overline{AD} \cong \overline{CD}$ and $\angle ADB \cong \angle CDB$.
 $\overline{BD} \cong \overline{BD}$ because it is the same segment,
so $\triangle ABD \cong \triangle CBD$ by SAS. Therefore, $\overline{AB} \cong \overline{CB}$
by CPCTC (Corresponding Parts of Congruent Triangles are Congruent).



Discovering and Proving Polygon Properties

Content Summary

Chapter 5 extends the explorations of triangle properties from the previous chapter to examine properties shared by all polygons. Students begin by investigating the sums of interior and exterior angles of any polygon. The chapter then focuses on quadrilaterals, which are polygons with four sides. Students explore relationships among the sides, angles, and diagonals of different special quadrilaterals, including the parallelogram family.

Polygons

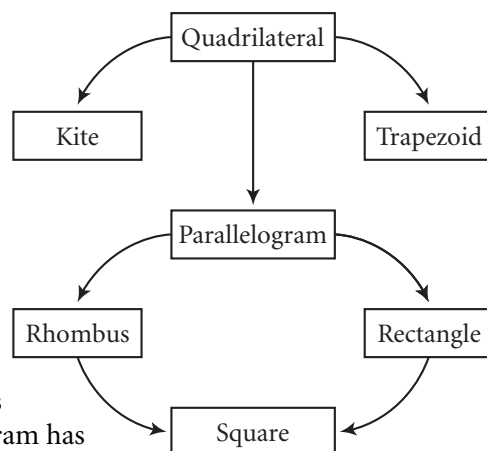
The chapter begins with conjectures about polygons in general. Students experiment to form conjectures about the sum of the angles of any polygon, and the sum of the exterior angles of any polygon. They write a paragraph proof of the first conjecture, relying on the Triangle Sum Conjecture from Chapter 4.

Quadrilaterals

The book considers properties of three categories of quadrilaterals, as in the diagram: kites, trapezoids, and parallelograms.

Students explore two kinds of parallelograms, rhombuses and rectangles, as well as squares, which are both rhombuses and rectangles. Students discover properties of all types of quadrilaterals, including how their diagonals are related. In the case of trapezoids, students investigate midsegments, which they relate to midsegments of triangles.

Properties of various quadrilaterals can be seen from their symmetry. A kite has reflectional symmetry across the diagonal through its vertex angles; an isosceles trapezoid has reflectional symmetry across the line through the midpoints of the parallel sides; and a parallelogram has 2-fold rotational symmetry about the point at which its diagonals intersect. These symmetries can help explain why certain pairs of segments or angles are congruent or perpendicular.



Summary Problem

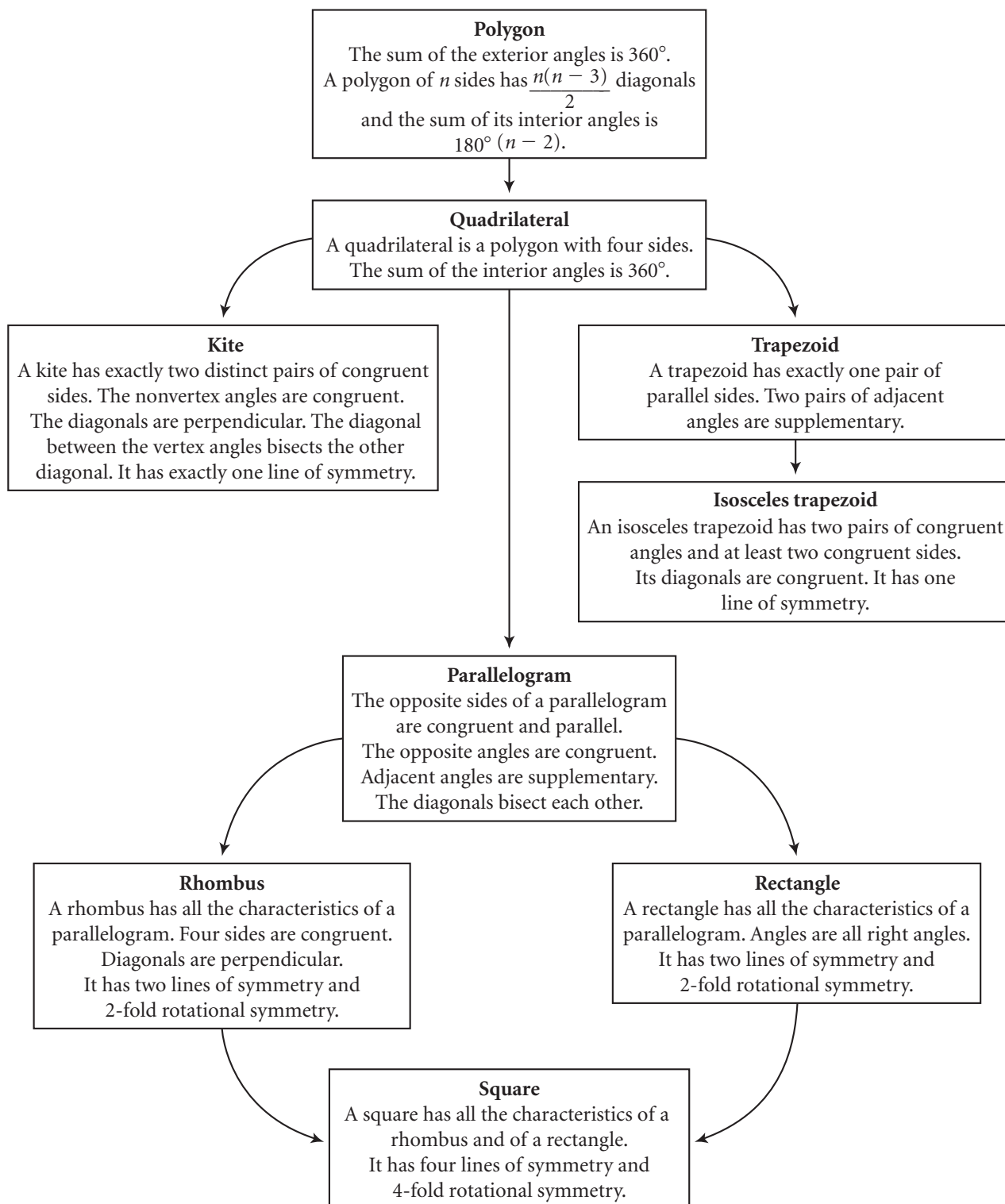
Make a copy of the quadrilaterals diagram above, but with large boxes. Write in each box the properties of that kind of figure as you encounter them in the book.

Questions you might ask in your role as student to your student:

- If you add a box to the top of the diagram for polygons in general, what properties can you put into that box?
- What other kinds of polygons might go into an expanded diagram?
- Where might isosceles trapezoids be added to your diagram?
- Where might darts be added to your diagram?
- What properties can you think of that aren't already in your diagram?
- Do you see any patterns in what properties are shared by various kinds of polygons?

(continued)

Sample Answers



Making each box in the shape of the polygon whose properties it contains might make the chart more interesting.

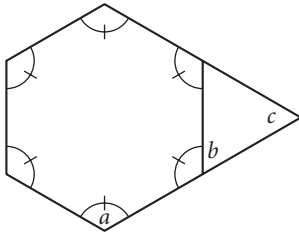
Chapter 5 • Review Exercises

Name _____ Period _____ Date _____

1. (Lessons 5.1, 5.2) Find the sum of the measures of the interior angles of a regular 14-gon. Then find the sum of the exterior angles.

(Lessons 5.1, 5.2, 5.4) For Exercises 2 and 3, find the lettered measures in each figure.

2.

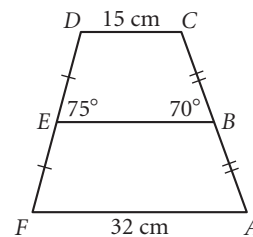


3. Given that $CD \parallel AF$,

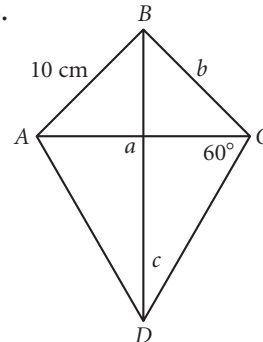
$$BE = \underline{\quad? \quad}.$$

$$m\angle ABE = \underline{\quad? \quad}.$$

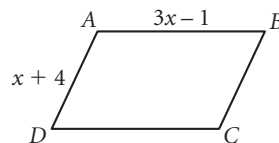
$$m\angle CDF = \underline{\quad? \quad}.$$



4. (Lesson 5.3) Given kite $ABCD$, find the missing measures.



5. (Lesson 5.5) The perimeter of parallelogram $ABCD$ is 46 in. Find the lengths of the sides.



6. (Lesson 5.6, 5.7) Draw a diagram and write a paragraph proof to show that the diagonals of a rectangle are congruent.

SOLUTIONS TO CHAPTER 5 REVIEW EXERCISES

1. Interior Angles:

$$\text{Interior angle sum} = 180(n - 2) = 180(14 - 2) = 2160^\circ$$

Exterior Angles = 360° for all polygons

2. For the hexagon:

$$\text{Interior angle sum} = 180(n - 2) = 180(6 - 2) = 720^\circ$$

$$\text{Each angle} = \frac{720^\circ}{6} = 120^\circ$$

$$a = 120^\circ$$

$$b = 60^\circ \quad \text{Linear pair.}$$

$$c = 60^\circ \quad \text{Triangle sum.}$$

3. $BE = \frac{15 + 32}{2} = 23.5 \text{ cm}$ Midsegment.

$$m\angle ABE = 110^\circ \quad \text{Linear pair.}$$

$$m\angle CDF = 105^\circ \quad \text{Supplementary angles.}$$

4. $a = 90^\circ$ Diagonals of a kite are perpendicular.

$$b = 10 \text{ cm} \quad \text{Definition of a kite.}$$

$$c = 30^\circ \quad \text{Triangle sum.}$$

5. $2(3x - 1) + 2(x + 4) = 46$

Opposite sides of a parallelogram are congruent.

$$6x - 2 + 2x + 8 = 46$$

Distributive property.

$$8x + 6 = 46$$

Combine like terms.

$$8x = 40$$

Subtraction.

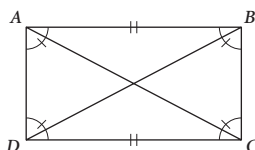
$$x = 5$$

Division.

$$AB = 3(5) - 1 = 14 \text{ in.} \quad \text{Substitution.}$$

$$AD = 5 + 4 = 9 \text{ in.} \quad \text{Substitution.}$$

6. Sample answer:



By definition, all angles of a rectangle are congruent, so $\angle ABC \cong \angle DCB$. A rectangle, like any parallelogram, has opposite sides congruent, so $\overline{AB} \cong \overline{DC}$. Because it is the same segment, $\overline{BC} \cong \overline{BC}$. Thus $\triangle ABC \cong \triangle DCB$ by SAS, and $\overline{AC} \cong \overline{DB}$ by CPCTC. Therefore, the diagonals of a rectangle are congruent.

Discovering and Proving Circle Properties

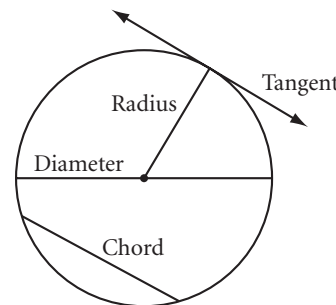
Content Summary

In Chapter 6, students continue to build their understanding of geometry as they explore properties of circles. Some of these properties are associated with line segments related to circles; other properties are associated with arcs and angles. A circle is defined as a set of points equidistant from a fixed point, its *center*.

Line Segments Related to Circles

The best-known line segments related to a circle are its radius and diameter. Actually, the word *radius* can refer either to a line segment between a point on the circle and the center, or to the length of such a line segment. Similarly, *diameter* means either a line segment that has endpoints on the circle and passes through the center, or the length of such a line segment.

The diameter is a special case because it is the longest *chord* of a circle; a chord is a line segment whose endpoints are on the circle. Another line segment associated with circles is a *tangent* segment, which touches the circle at just one point and lies on a *tangent line*, which also touches the circle at just one point and is perpendicular to the radius at this point. Students learned about these segments in Chapter 1, and Lesson 6.1 provides a quick review.



Arcs and Angles

A piece of the circle itself is an *arc*. If you join each endpoint of an arc to the center of the circle, you form the *central angle* that intercepts the arc. The size of the arc can be expressed in degrees—the number of degrees in the arc's central angle. This chapter explores several such relationships among arcs, angles, and segments. Students also write paragraph and flowchart proofs to confirm the universality of these relationships.

The size of the arc can also be expressed in length. The arc length is calculated by using the total circumference, or the distance around the circle. The number π is defined to be the circumference of any circle divided by that circle's diameter; or, the circumference is π times the diameter. For example, if an arc is $\frac{1}{4}$ of the complete circle, then its central angle measures $\frac{1}{4}$ of 360° , and its length is $\frac{1}{4}$ of the circle's circumference.

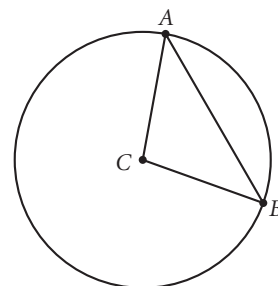
Summary Problem

Draw a diagram of a central angle intercepting a chord of a circle and its arc, as shown in the picture.

Move points A, B, and C to different locations to illustrate the ideas of the chapter.

Questions you might ask in your role as student to your student:

- What concepts are illustrated in the original drawing?
- How could you move each of the points A, B, and C to show an inscribed angle?
- How could you move each of the points A, B, and C to show tangent segments?



(continued)

Chapter 6 • Discovering and Proving Circle Properties (continued)

- How could you move points A , B , and C to show an angle inscribed in a semicircle?
- How could you move points A , B , and C to show parallel lines intercepting congruent arcs?

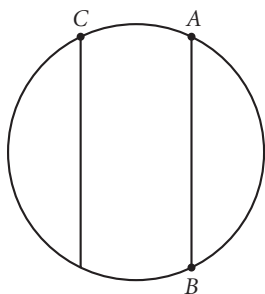
Sample Answers

The original drawing shows a central angle, a sector, a chord, and an arc. If the center, C , is moved to lie on the circle, an inscribed angle is formed.

If C is moved to be outside the circle, and A and B are moved to make \overline{AC} and \overline{BC} tangents, then those segments are congruent. Or, the chord in the original diagram could be rotated at one of its ends until it becomes a tangent segment.

If C is moved until \overline{AC} is a diameter and B remains on the circle, $\angle ABC$ is a right angle inscribed in a circle.

You would need to add another point and put C on the circle to show two chords. Parallel chords intercept equal arcs if they are equidistant from the center.

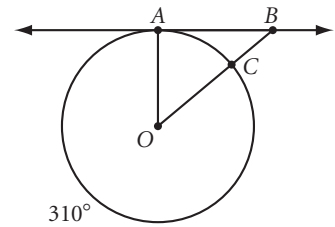


Of course many other answers are possible. Encourage your student to think of multiple ways that A , B , and C could be moved to illustrate these same concepts.

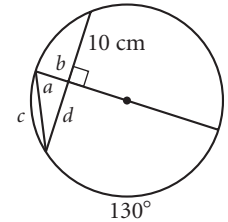
Chapter 6 • Review Exercises

Name _____ Period _____ Date _____

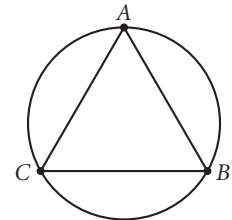
1. (Lesson 6.1) Given tangent \overleftrightarrow{AB} , find $m\angle OAB$, $m\angle AOB$, and $m\angle ABO$.



2. (Lessons 6.2, 6.3) Find the unknown measures or lengths.



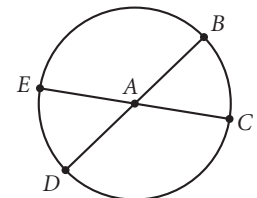
3. (Lesson 6.3) $\triangle ABC$ is an equilateral triangle. Find $m\widehat{AB}$.



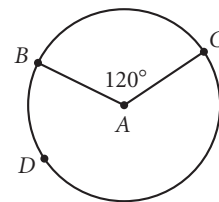
4. (Lesson 6.4) Write a paragraph proof to prove the following:

Given: Circle A with diameters \overline{EC} and \overline{BD} .

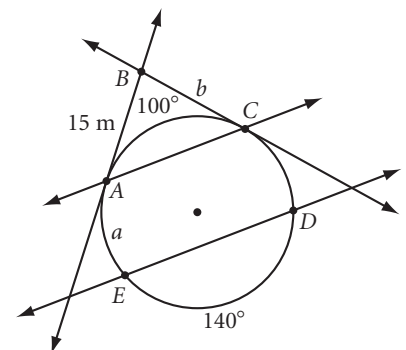
Prove: $\widehat{ED} \cong \widehat{BC}$



5. (Lessons 6.5, 6.7) Given that the circumference of circle A is 24π in., find the radius of the circle and the length of \widehat{BDC} .



6. (Lessons 6.1, 6.3) \overleftrightarrow{AB} and \overleftrightarrow{BC} are tangents to the circle as shown. $\overleftrightarrow{AC} \parallel \overleftrightarrow{ED}$. Find a and b .



SOLUTIONS TO CHAPTER 6 REVIEW EXERCISES

1. $m\angle AOB = 90^\circ$

$m\widehat{AC} = 360^\circ - 310^\circ = 50^\circ$

$m\angle AOC = m\widehat{AC} = 50^\circ$

$50^\circ + 90^\circ + m\angle ABO = 180^\circ$

$m\angle ABO = 40^\circ$

Tangent is perpendicular to radius.

360° in a circle.

Central angle is equal to arc measure.

Triangle sum.

Subtraction.
2. $b = 90^\circ$

$d = 10$ cm

$c = 180^\circ - 130^\circ = 50^\circ$

$a = \frac{1}{2}(130^\circ)$

$m\angle a = 65^\circ$

Diameter is perpendicular to the chord.

Diameter perpendicular to a chord bisects the chord.

180° in a semicircle.

Inscribed angles.

Division.
3. $AB = BC = AC$

$m\widehat{AB} = m\widehat{BC} = m\widehat{AC}$

$m\widehat{AB} + m\widehat{BC} + m\widehat{AC} = 360^\circ$

$m\widehat{AB} + m\widehat{AB} + m\widehat{AB} = 360^\circ$

$3m\widehat{AB} = 360^\circ$

$m\widehat{AB} = 120^\circ$

Equilateral triangle.

Congruent arcs of congruent chords.

360° in a circle.

Substitution.

Combine like terms.

Division.
4. Diameters \overline{CE} and \overline{BD} on circle A intersect to form congruent vertical angles, so $m\angle BAC = m\angle DAE$. The measure of an arc equals the measure of its central angle. Therefore, $m\widehat{BC} = m\widehat{ED}$ because the measures of their central angles are equal.

5. $C = 2\pi r = 24\pi$ in.; therefore, $r = 12$ in.

$m\widehat{BDC} = 360^\circ - 120^\circ = 240^\circ$

length of $\widehat{BDC} = \frac{240}{360}(24\pi) = 16\pi$ in.
6. $m\widehat{CD} = m\widehat{AE} = a$

$100^\circ + a + a + 140^\circ = 360^\circ$

$a = 60^\circ$

$b = 15$ m

Parallel secants intercept congruent arcs.

360° in a circle.

Solve.

Tangent segments from the same point are congruent.

Transformations and Tessellations

Content Summary

Thinking about ideas from different perspectives can lead to deeper understanding. For example, geometric transformations can help students deepen their understanding of congruence and symmetry.

You can think of a geometric transformation as a regular change of a figure in the plane. For example, a figure may be shifted to the right 5. Or, a figure may be enlarged to twice its original size.

Chapter 7 focuses on transformations that don't change the size or shape of figures. These transformations are called *isometries*. Expansions and contractions, called *dilations*, are studied in Chapter 11.

Isometries

There are three major kinds of isometries: translations, reflections, and rotations.

Translations are simply shifts. Students use translations when discussing tessellations, where a single shape is translated (shifted) repeatedly in different directions to cover the plane without any gaps or overlaps.

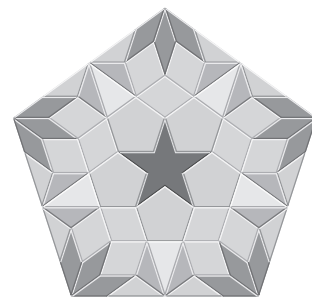
Reflections flip a shape across a line to make a mirror image. If there's a line through which a shape can be reflected to lay the image exactly on top of the original, then the figure has *reflectional symmetry*, as students saw in Chapter 0. Reflections can be used in designing figures that will tessellate the plane. They can also be used to help find the shortest path from one object to a line and then to another object.

Rotations rotate an object around a point. If there's a point around which a shape can be rotated through some angle (less than 360°) to get back to exactly the same shape, then the figure has *rotational symmetry* (also previewed in Chapter 0). Rotations can also be used in designing tessellations.

Isometries give students a new way of thinking about congruence. Two figures are congruent if one can be transformed into the other using an isometry.

Compositions of Isometries

One transformation followed by another is the *composition* of those transformations. This chapter considers compositions of two reflections: reflections across parallel lines (resulting in a translation) and reflections across nonparallel lines (resulting in a rotation). In the context of tessellations, this chapter also examines glide reflections, which are compositions of a translation and a reflection.



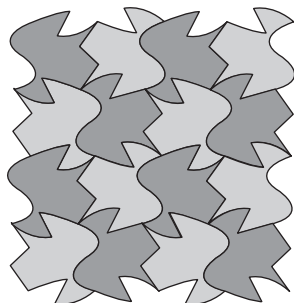
This tile pattern has both 5-fold rotational symmetry and 5-fold reflectional symmetry.

(continued)

Chapter 7 • Transformations and Tessellations (continued)

Summary Problem

Have your student trace this tessellation onto tracing paper or wax paper. Ask how it illustrates the concepts under consideration.



Questions you might ask in your role as student to your student:

- What isometries might be used to change one part of the figure to another?
- What is the underlying grid of this tessellation?
- Could this tessellation be made with just translations?
- What kinds of symmetry does the entire figure have?

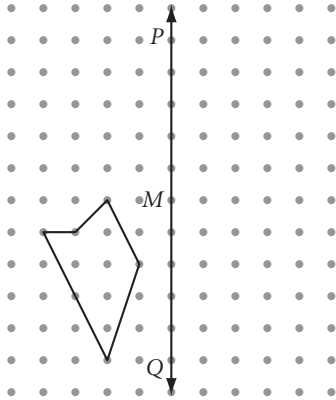
Sample Answers

The tessellation, based on a four-by-four grid of quadrilaterals, illustrates many transformations. Each of the dark figures can be translated to fit onto a dark figure two rows or columns away. Or, it can be rotated 180° to fit onto a light figure in the same row. To fit onto a light figure in an adjacent row, the dark figure must be reflected vertically, then translated. (All three transformations just described are also true for each of the light figures.) Any block of four figures that meet at one point can be translated to fill the entire plane.

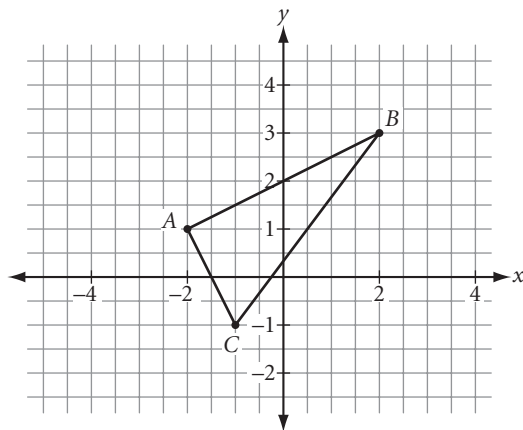
Chapter 7 • Review Exercises

Name _____ Period _____ Date _____

1. (Lesson 7.1) Reflect the figure across \overleftrightarrow{PQ} and rotate it 180° around point M . Does your answer change if you rotate it first and then reflect it?



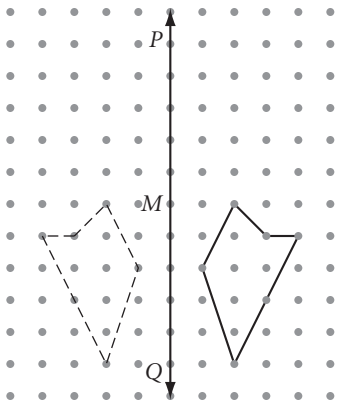
2. (Lesson 7.2) Translate $\triangle ABC$ using the rule $(x, y) \rightarrow (x - 3, y + 1)$.



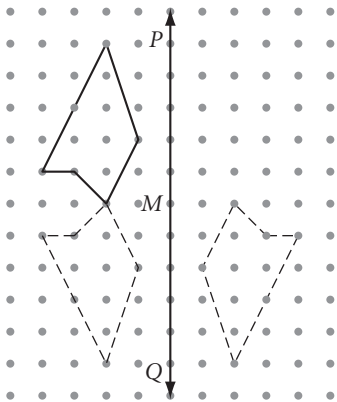
3. (Lesson 7.3) Name the single rotation that can replace the composition of these two rotations around the same center of rotation: 36° and 120° .
4. (Lesson 7.1) What capital letters have horizontal but not vertical symmetry?
5. (Lesson 7.1) What capital letters have horizontal, vertical, and rotational symmetry?

SOLUTIONS TO CHAPTER 7 REVIEW EXERCISES

1. Reflection:

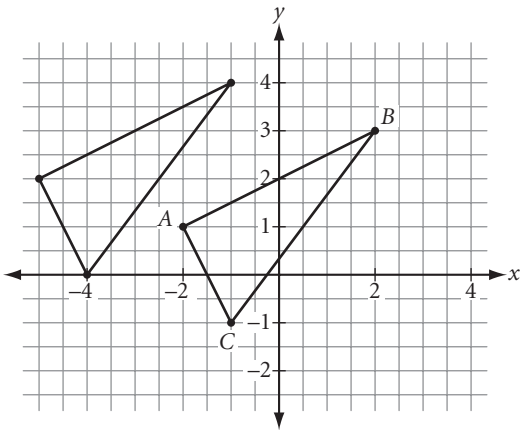


Rotation:



No, the final answer is the same.

2.



3. $36^\circ + 120^\circ = 156^\circ$

4. B, C, D, E, K

5. H, I, O, X

Content Summary

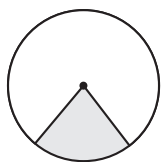
Your student may have already seen formulas for the areas of standard geometric figures. If so, ask for the meaning of the formulas. Some appropriate questions might be, “What does area mean?” and “Does this formula still apply if the figure is rotated to have a different base?” Focus on the fact that area is measured by the number of square units that could fill a region.

Areas of Polygons

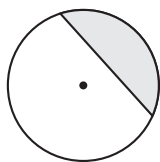
Some students lack an important concept regarding area: When a figure is cut up and its pieces are rearranged, the resulting figure has the same area as the original. With this concept, students can understand why the area formulas for parallelograms, triangles, and trapezoids can be found using the formula for the area of a rectangle, and how the areas of other plane figures can be found using areas of triangles.

Areas of Circles and Parts of Circles

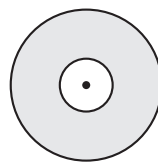
Mathematicians in ancient times realized that the ratio of circumference to diameter is the same for every circle, a number we now call π . What wasn't realized until later is that this same number appears in a formula for the area of the region enclosed by a circle. *Discovering Geometry* shows ways of connecting the ideas of circumference and area to make the formulas more understandable. Students also learn how to find the areas of the parts of a circle shown below by relating these shapes to circles and triangles.



Sector of a circle



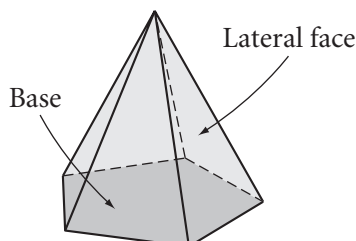
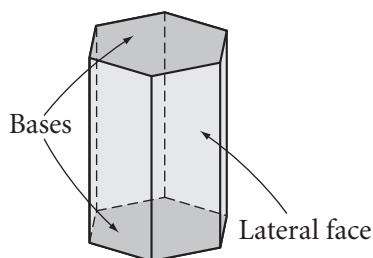
Segment of a circle



Annulus

Surface Area

The surface area of a solid refers to the area of the surface, which is actually two-dimensional. A solid's surface is often composed of standard shapes; to find the surface, your students will find the sum of the areas of these shapes.



Some solids have a base or two whose areas must be included in the sum. Also included are the areas of the sides (or lateral faces), which in this book are primarily rectangles or triangles. The lateral face of a cylinder, when flattened out, is a rectangle. The lateral face of a cone, when flattened out, is a sector of a circle. The surface area of a sphere is considered in Chapter 10.

(continued)

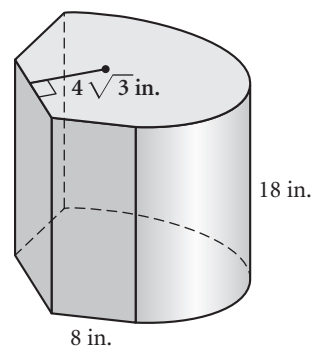
Chapter 8 • Area (continued)

Summary Problem

What is the surface area of this stone sculpture? Each of the parallel bases began as a circle. Half of each circle was left untouched. The other half was cut three times, leaving three sides of a regular hexagon.

Questions you might ask in your role as student to your student:

- Can you find the areas of parts of the figure?
- How does a regular hexagon relate to the circle in which it's inscribed?
- How does the radius of the circle relate to the length of one of the straight sides?
- For an extra challenge, consider what would happen if more stone were cut from the sculpture until it tapered evenly from the lower base to a point at the center of where the top base is now. What would the surface area be?



Sample Answers

Calculating surface area involves identifying all the surfaces, then systematically calculating and totaling their areas. Encourage your student to be systematic and organized. When doing the calculations, students should check their work. It also helps to estimate the answer first to be sure no large errors were made.

Each base consists of a semicircle and half of a regular hexagon. As shown in diagram at right, the half-hexagon can be divided into three congruent equilateral triangles, so the radius of the semicircle is 8 in. The area of the semicircle is $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(8)^2 = 32\pi \text{ in}^2$. The area of each equilateral triangle is $\frac{1}{2}(8)(4\sqrt{3}) = 16\sqrt{3} \text{ in}^2$, so the area of the half-hexagon is $48\sqrt{3} \text{ in}^2$. The area of each base, then, is $48\sqrt{3} + 32\pi \text{ in}^2$.

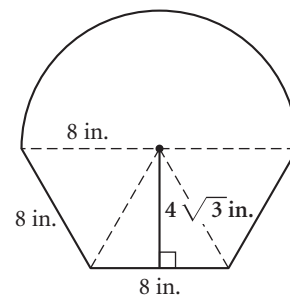
Each of the three rectangular lateral faces has area $8 \cdot 18 = 144 \text{ in}^2$. If you were to cut off the curved lateral face and lay it flat, it would form a rectangle of height 18 in. The width of that rectangle would be half the circle's circumference, or $\frac{1}{2} \cdot 2\pi r = \frac{1}{2} \cdot 2\pi(8) = 8\pi \text{ in}$. The area of that lateral face, then, is $18 \cdot 8\pi = 144\pi \text{ in}^2$.

So, the sculpture's faces have these areas in square inches:

- 2 bases: $48\sqrt{3} + 32\pi \text{ in}^2$
- 3 rectangular lateral faces: 144 in^2
- 1 curved lateral face: $144\pi \text{ in}^2$

The total surface area is $2(48\sqrt{3} + 32\pi) + 3(144) + 144\pi = 96\sqrt{3} + 208\pi + 432$, or about 1252 in^2 .

If the top base of the sculpture were shaved to a point, the curved face would become half of a cone and the rest would become half of a hexagonal pyramid. You can use this question to motivate the study of the Pythagorean Theorem in Chapter 9. A student who remembers this theorem from a previous course can calculate the slant height of the cone to be $\sqrt{8^2 + 18^2} = \sqrt{388}$ and the slant height of the lateral faces of the half-pyramid to be $\sqrt{(4\sqrt{3})^2 + 18^2} = \sqrt{372}$. The surface area of a cone is $\pi r l + \pi r^2 = \pi(8)(\sqrt{388}) + \pi(8)^2$. Half of the cone would have half of this surface area, or $4\pi\sqrt{388} + 32\pi$. The lateral faces of the half-pyramid would have area $\frac{1}{2}bh = \frac{1}{2}(8)(\sqrt{372})$, or $4\sqrt{372}$. The bottom base would still have area $48\sqrt{3} + 32\pi$. Therefore, the total area of the figure would be $4\pi\sqrt{388} + 32\pi + 3(4\sqrt{372}) + 48\sqrt{3} + 32\pi$, which is about 763 in^2 .



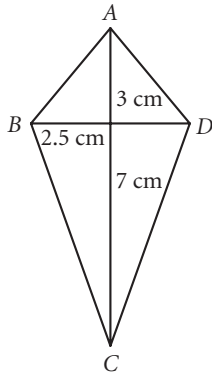
Chapter 8 • Review Exercises

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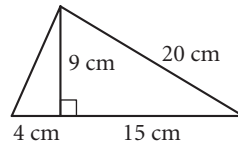
1. (Lesson 8.1) Find the area of quadrilateral $ABCD$ with vertices $(1, 2)$, $(4, 2)$, $(-1, -2)$, $(2, -2)$.

2. (Lessons 8.2, 8.4) Find the area of each figure.

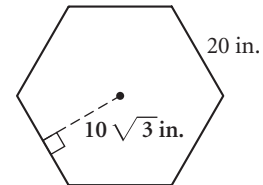
a. Kite $ABCD$



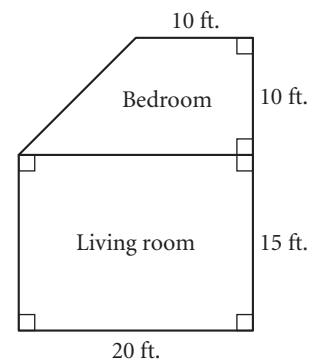
b. Triangle



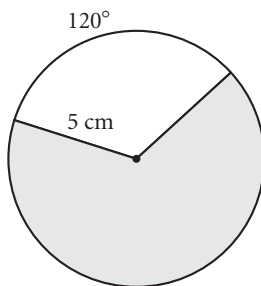
c. Regular hexagon



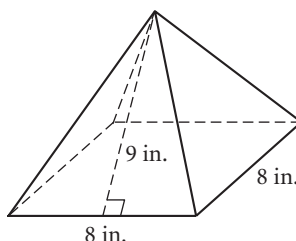
3. (Lessons 8.1, 8.2, 8.3) Shahin wants to carpet the floors in his living room and bedroom. If the carpet, padding, and installation cost \$30 per square yard, how much will it cost?



4. (Lessons 8.5, 8.6) Find the area of the shaded region. Leave answers in terms of π .

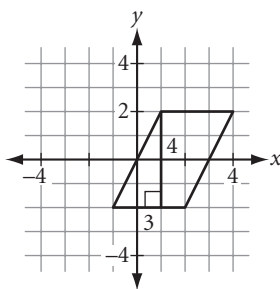


5. (Lesson 8.7) Find the surface area of this square pyramid.



SOLUTIONS TO CHAPTER 8 REVIEW EXERCISES

1. After plotting the points, we find the quadrilateral is a parallelogram with base equal to 3 units and height equal to 4 units. Find the area:



$$A = bh$$

$$A = (3)(4)$$

$$A = 12 \text{ square units}$$

2. a. $A = \frac{1}{2}d_1d_2$

$$A = \frac{1}{2}(10)(5) = 25 \text{ cm}^2$$

b. $A = \frac{1}{2}bh$

$$A = \frac{1}{2}(19)(9) = 85.5 \text{ cm}^2$$

c. $A = \frac{1}{2}aP$

$$P = 6 \cdot 20 = 120 \text{ in.}$$

$$A = \frac{1}{2}(10\sqrt{3})(120) \approx 1039 \text{ in}^2$$

3. Bedroom (trapezoid): $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2} \cdot 10(10 + 20) = 150 \text{ ft}^2$

Living room (rectangle): $A = bh = (15)(20) = 300 \text{ ft}^2$

Total area = $150 + 300 = 450 \text{ ft}^2$

To find the area in square yards:

$$450 \text{ ft}^2 \left(\frac{1 \text{ yd}}{3 \text{ ft}} \right)^2 = 450 \text{ ft}^2 \cdot \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = 50 \text{ yd}^2$$

$$\text{Total cost} = 50 \text{ yd}^2 \cdot \frac{\$30}{\text{yd}^2} = \$1500.00$$

4. $r = 5 \text{ cm}$

Angle measure of sector = $360^\circ - 120^\circ = 240^\circ$

$$\text{Area of a sector} = \frac{240}{360} \cdot \pi r^2 = \frac{240}{360} \cdot \pi(5)^2 = \frac{50\pi}{3} \text{ cm}^2$$

5. $A = 4\left(\frac{1}{2}bh\right) + b^2$

$$A = 4\left(\frac{1}{2} \cdot 8 \cdot 9\right) + 8^2 = 208 \text{ in}^2$$

The Pythagorean Theorem

Content Summary

One of the best-remembered theorems of geometry is the Pythagorean Theorem. This chapter begins by reviewing the Pythagorean Theorem and then considering its converse. It then examines shortcuts in the case of special triangles. Finally, the Pythagorean Theorem is used to calculate distances on the coordinate plane, making it possible to write an equation for a circle.

The Pythagorean Theorem and Its Converse

Many people identify the Pythagorean Theorem as $a^2 + b^2 = c^2$, without recalling what a , b , and c stand for. The theorem actually says that if a , b , and c are the lengths of sides of a right triangle whose hypotenuse has length c , then $a^2 + b^2 = c^2$.

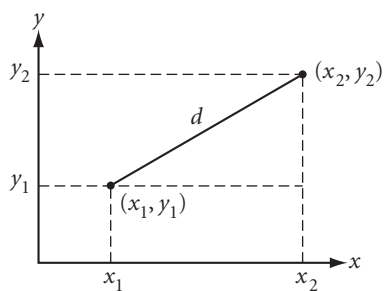
The converse of the Pythagorean Theorem is also true: If a , b , and c are the lengths of sides of a triangle, and if $a^2 + b^2 = c^2$, then the triangle is a right triangle whose hypotenuse has length c .

Applications to Triangles

Discovering Geometry illustrates how to apply the Pythagorean Theorem to solve real-world problems. It also applies the theorem to find the relative lengths of sides of two special right triangles: the 45° - 45° - 90° triangle, which is half of a square; and the 30° - 60° - 90° triangle, which is half of an equilateral triangle.

Applications to Circles

From algebra, students know how to write equations to represent lines and parabolas. Coupled with the conjectures from Chapter 6, the Pythagorean Theorem can be used to develop equations for circles. The restatement of the Pythagorean Theorem in coordinate geometry is the distance formula, which tells how to find the distance between two points whose coordinates are known. From this formula, the book derives the equation of a circle.



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

(continued)

Chapter 9 • The Pythagorean Theorem (continued)

Summary Problem

Why is the general equation of a circle $(x - h)^2 + (y - k)^2 = r^2$?

Questions you might ask in your role as student to your student:

- What numbers do h , k , and r represent?
- What points satisfy the equation if h and k are both 0?
- What can you say about the distance from any point on a circle to the center?
- How does this equation compare with the distance formula?
- Why does the sum of the squares on the left equal the square on the right?

Sample Answers

Encourage your student to relate this equation to the distance formula shown above. Looking at the diagram, imagine that the point (x_1, y_1) is the center of a circle, and all the points on the circle are a distance d away from it. If you rename the center (h, k) and rename the distance r , you can see that the circle is all the points (x, y) that are this distance away from (h, k) .

If the center is at the point $(0, 0)$, the points satisfying the equation are all a distance r from the origin—that is, they all lie on a circle centered at the origin and having radius r .

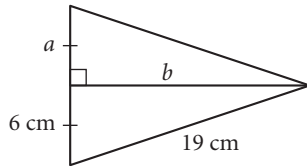
The equation of that circle is $x^2 + y^2 = r^2$, which is the Pythagorean Theorem. Try having your student use a compass to draw a circle centered at the origin with a certain radius (for example, 5 or 10), and then ask him or her to identify a few points on that circle and substitute them into the equation.

Chapter 9 • Review Exercises

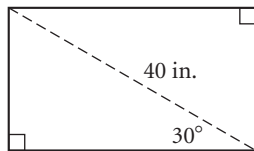
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Round solutions to the nearest tenth of a unit.

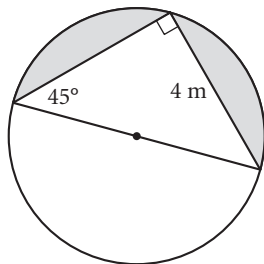
1. (Lesson 9.1) Find each missing length.



2. (Lesson 9.2) Is a triangle with sides measuring 3 ft., 4 ft., and 5 ft. a right triangle? Justify your answer.
3. (Lessons 9.3, 9.4) A 40-in. TV has a diagonal equal to 40 in., as shown here. Find the dimensions of the TV.



4. (Lesson 9.5) Find the equation of the circle with diameter endpoints $(-2, 2)$ and $(4, 2)$.
5. (Lesson 9.6) Find the area of the shaded region.



SOLUTIONS TO CHAPTER 9 REVIEW EXERCISES

1. $a = 6$ cm Congruent segments.
 $6^2 + b^2 = 19^2$ Pythagorean Theorem.

$$36 + b^2 = 361$$

$$b^2 = 325$$

$$b \approx 18.0 \text{ cm}$$

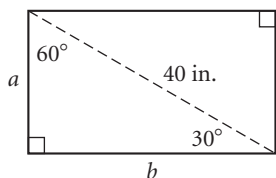
2. If sides with lengths 3, 4, and 5 form a right triangle, the Pythagorean Theorem should make a true statement:

$$3^2 + 4^2 = 5^2$$

$$25 = 25$$

Therefore, it is a right triangle.

3. Use the Triangle Sum Conjecture to find the other angle of the right triangle. The triangle is a 30° - 60° - 90° triangle, so you can use the shortcut.



$$a = \frac{1}{2}(40) = 20 \text{ in.}$$

$$b = a\sqrt{3} = 20\sqrt{3} \approx 34.6 \text{ in.}$$

4. Using the given diameter, the center is at $(1, 2)$, and the radius is 3. Enter these values into the circle equation.

$$(x - 1)^2 + (y - 2)^2 = 9$$

5. The triangle is a 45° - 45° - 90° triangle, therefore the diameter is $4\sqrt{2}$ m.

$$r = \frac{1}{2}(4\sqrt{2}) = 2\sqrt{2} \text{ m}$$

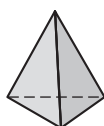
$$\text{Area of semicircle} = \frac{1}{2} \text{ area of circle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2\sqrt{2})^2 = \frac{1}{2}\pi(4 \cdot 2) = 4\pi \text{ m}^2$$

$$\text{Area of triangle} = \frac{1}{2}bh = \frac{1}{2}(4)(4) = 8 \text{ m}^2$$

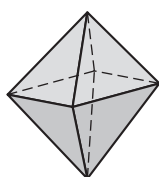
$$\text{Area of shaded region} = \text{area of semicircle} - \text{area of triangle} = 4\pi - 8 \approx 4.6 \text{ m}^2$$

Content Summary

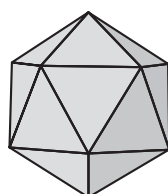
In geometry, the word *solid* refers to the shell of a three-dimensional figure, not including its interior. Solids can be classified in many ways, based on their shape. The five *regular* solids have faces that are congruent regular polygons and angles that are all congruent.



Regular tetrahedron
(4 faces)



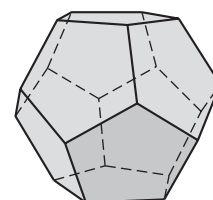
Regular octahedron
(8 faces)



Regular icosahedron
(20 faces)



Regular hexahedron
(6 faces)



Regular dodecahedron
(12 faces)

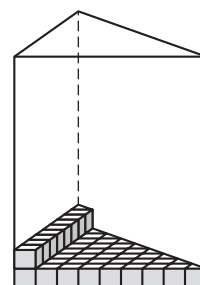
Just as area is measured as the number of unit squares that completely fill a region, volume is measured as the number of unit cubes that completely fill a space. They may be cubic inches (in^3), cubic centimeters (cm^3), or other units of volume.

This chapter contains formulas for the volume of quite a few solids. Your student should be able to recognize those formulas through visualization and a good understanding of volume, and use them to solve practical problems.

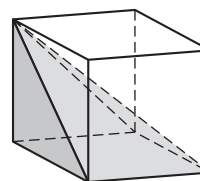
Prisms and Cylinders

Many common solids, such as cardboard boxes, have at least one pair of faces parallel and, usually, congruent. If these faces are polygons and are connected by sides that are parallelograms, the solid is called a *prism*. If the faces are circles joined by a single side, the solid is a *cylinder*. In either case, the parallel faces are called the *bases*.

To help your student think about the volume of a prism or cylinder, you might suggest imagining the prism on a table with one base downward. Then cover the bottom base with one layer of cubes. Some cubes may have to be cut to fit inside the solid. In any event, each of these cubes is providing a unit square on the base, so the number of these cubes is the area of the base. If you put another layer of cubes on top of the first, and then another layer on top of this, and so on, you can fill the prism. The volume—the number of cubes in the whole space—is equal to the number of cubes on the base times the number of layers. That is, the volume is the area of the base times the height of the solid.

**Pyramids and Cones**

Suppose you have a pyramid or a cone. The solid has one base and tapers to a point. If that base is a polygon, the pointed solid is a *pyramid*. If the base is a circle, the pointed solid is a *cone*. If you lay it on its base, each layer will be smaller than the layer below. It is interesting to compare the volume of a pointed solid with the volume of a prism or cylinder with the same base. Somewhat surprisingly, the ratio of their volumes is always 1 to 3. So, the volume of the pointed solid is $\frac{1}{3}$ the volume of the prism or cylinder with the same base and height.

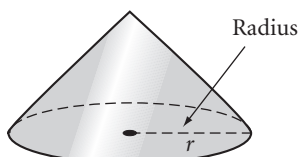


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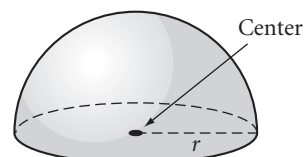
Chapter 10 • Volume (continued)

Spheres

Discovering Geometry asks students to find the volume of a sphere (actually, a hemisphere) by measuring the amount of water it will hold compared to a cylinder with the same radius and height. To help your student remember the formula, you might use a different comparison: Ask how the volume of a hemisphere relates to the volume of a cone with the same base and height. Clearly, the hemisphere has a larger volume. It turns out that the hemisphere has twice as much volume. If the sphere has radius r , then the area of the base (of the cone and hemisphere) is πr^2 . The height of the hemisphere and the cone is also r , so the volume of the cone is $\frac{1}{3}(\pi r^2)(r)$, or $\frac{1}{3}\pi r^3$, therefore the volume of the hemisphere is $\frac{2}{3}\pi r^3$. The volume of the sphere, then, is twice that amount, or $\frac{4}{3}\pi r^3$.



Cone

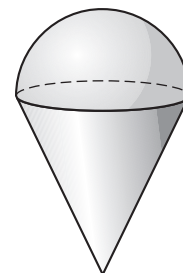


Hemisphere

To help remember the formula for the surface area of a sphere, you might suggest that your student imagine a piece of paper cut in a circle to match the base of the hemisphere. That paper would not be enough to cover the hemisphere itself. Interestingly, however, it would cover exactly half of it. So, the surface area of the sphere is 4 times the area of that circle. That is, the surface area is $4\pi r^2$.

Summary Problem

One treat at an ice-cream store has the shape of a cone with a hemisphere on top. The cone has height 22 cm. The diameter of the hemisphere is 13 cm. The treat comes packed tightly in a box shaped like a prism with square bases. What are the volumes of the box and the treat?



Questions you might ask in your role as student to your student:

- What are the dimensions (length, width, and height) of the packing box?
- What is the volume of the packing box?
- What would be the volume of the packing box if it were changed to a cylinder?
- What is the volume of the cone?
- What is the volume of the hemisphere?

Sample Answers

Let's assume the treat is packed perfectly upright. (If the treat is packed tilted so that the tip of the cone slips into a corner, a slightly smaller box may be possible.) The dimensions of the base of the packing box must be the same as the diameter of the hemisphere, or 13 cm. The hemisphere has radius 6.5 cm, so the height of the packing box is 28.5 cm. Therefore, the volume of the packing box is $13 \cdot 13 \cdot 28.5$, or 4816.5 cm^3 . If the packing box were a cylinder instead, the bases would be circles with radii 6.5 cm, so their areas would be $\pi(6.5)^2$, and the volume of the box would be $\pi(6.5)^2(28.5)$, or about 3783 cm^3 .

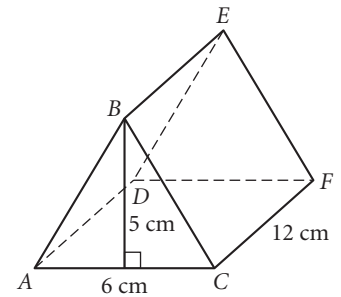
The cone has a base of area $\pi(6.5)^2$ and it has height 22, so its volume is $\frac{1}{3}\pi(6.5)^2(22)$, or about 973.4 cm^3 . The hemisphere has volume $\frac{1}{2}[\frac{4}{3}\pi(6.5)^3]$, which is about 575.2 cm^3 . So, the volume of the treat is about 1549 cm^3 .

Chapter 10 • Review Exercises

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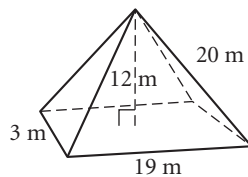
1. (Lessons 10.1, 10.2) Use the triangular prism to answer the following:

- Name the bases of the prism.
- What is the area of one base?
- What is the height of the prism?
- Find the volume.

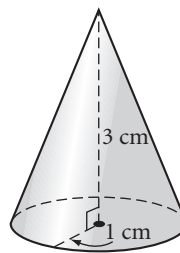


2. (Lessons 10.2, 10.3) Find the volume of each solid.

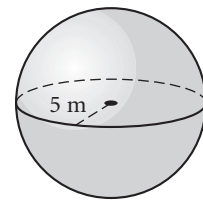
- a. Rectangular pyramid



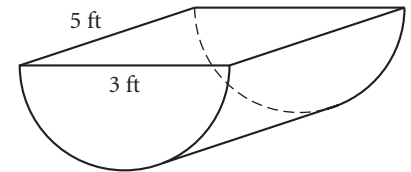
- b. Cone



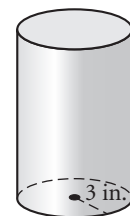
- c. Sphere



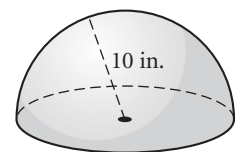
3. (Lessons 10.3, 10.4) A semicircular cylinder trough has a diameter of 3 ft and a length of 5 ft. If 1 cubic foot of water is about 7.5 gallons, how many gallons will the trough hold?



4. (Lesson 10.5) You drop a rock into a cylinder with a radius of 3 in., raising the water level by 1 in. What is the volume of the rock?



5. (Lesson 10.7) Find the volume and surface area of this hemisphere.



SOLUTIONS TO CHAPTER 10 REVIEW EXERCISES

1. a. $\triangle ABC$ and $\triangle DEF$ are the bases.

b. $A = \frac{1}{2}bh$

$$A = \frac{1}{2}(6)(5) = 15 \text{ cm}^2$$

c. $H = 12 \text{ cm}$

- d. $V = BH$ (B = area of base; H = height of prism)

$$V = (15)(12) = 180 \text{ cm}^3$$

2. a. $V = \frac{1}{3}BH$

$$B = (3)(19) = 57 \text{ m}^2$$

$$H = 12 \text{ m}$$

$$V = \frac{1}{3}(57)(12) = 228 \text{ m}^3$$

- b. $V = \frac{1}{3}BH$

$$V = \frac{1}{3}(\pi r^2)H$$

$$V = \frac{1}{3}\pi(1)^2(3) = \pi \approx 3.1 \text{ cm}^3$$

- c. $V = \frac{4}{3}\pi r^3$

$$V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3} \approx 523.6 \text{ m}^3$$

3. $r = \frac{1}{2}(3) = 1.5 \text{ ft.}$

$V = BH$, where the base is a semicircle and H is the length of the trough

$$V = \left(\frac{1}{2}\pi r^2\right)H$$

$$V = \frac{1}{2}\pi(1.5)^2(5) \approx 17.7 \text{ ft}^3$$

$$17.7 \text{ ft}^3 \cdot \frac{7.5 \text{ gal}}{1 \text{ ft}^3} \approx 133 \text{ gal}$$

4. This new "slice" of water is shaped like a cylinder with radius 3 and height 1. The volume of this cylinder is $= \pi r^2 H = \pi(3)^2(1) = 9\pi \approx 28.3 \text{ in}^3$.

5. Volume of hemisphere $= \frac{1}{2}$ volume of sphere

$$V = \frac{1}{2} \left[\frac{4}{3}\pi r^3 \right]$$

$$V = \frac{1}{2} \left[\frac{4}{3}\pi(10)^3 \right] \approx 2094.4 \text{ in}^3$$

Surface area of hemisphere $= \frac{1}{2}$ surface area of sphere + area of base

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$SA = \frac{1}{2}[4\pi(10)^2] + \pi(10)^2 = 300\pi \approx 942.5 \text{ in}^2$$

Content Summary

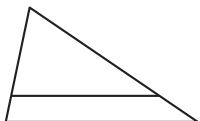
One of the most important kinds of thinking in mathematics is proportional reasoning—applying what we know about a figure to larger or smaller figures. In Chapter 11, students have the opportunity to practice this kind of thinking by applying it to similar figures—figures that are enlargements (stretches) or reductions (shrinks) of each other. A transformation that stretches or shrinks a figure by the same factor in all directions is a *dilation*, and the factor is called a *scale factor*.

Similar Polygons

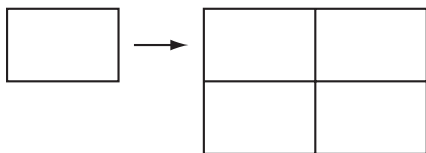
In everyday usage, “similar” means alike in some ways. But in geometry, *similar* means exactly the same shape (but not necessarily the same size). Two polygons are similar if their corresponding angles are congruent and the lengths of corresponding sides all have the same ratio. This ratio is called the scale factor. Ratios of lengths of other corresponding parts of similar triangles are equal to the scale factor. The book concentrates on similar triangles and—as with congruence—finds shortcuts for determining similarity. These shortcuts can be applied to calculate lengths that can’t be measured directly, such as the height of a flagpole.

Parallel Lines

If lines are drawn between two sides of a triangle and they are parallel to the third side, then each of the lines creates a new triangle that is similar to the original. Therefore, these lines divide proportionately the two sides that they intersect.

**Area and Volume**

Some of the most important and surprising results involve relationships among the areas or volumes of similar figures. Suppose you double the length and width of a rectangle to form a larger rectangle. You might expect the area to double, but in fact it quadruples, or increases by a factor of 4.



If any two-dimensional figure is dilated by a scale factor of r , then its area is multiplied by r^2 . Similarly, if a three-dimensional figure is dilated by a factor of r , then its volume is multiplied by r^3 . (Surface areas of its faces, which are two-dimensional, are multiplied by r^2 .) For example, if a sphere is enlarged by a factor of 1.5 (its radius is multiplied by 1.5), then its volume is multiplied by $1.5^3 = 3.375$, and its surface area is multiplied by $1.5^2 = 2.25$.

(continued)

Chapter 11 • Similarity (continued)

Summary Problem

Draw a pentagon on graph paper, as in Lesson 11.1, Investigation 2. Choose any constant as your scale factor. Multiply the coordinates of each vertex by that constant to locate the vertices of a new pentagon. How do the two pentagons compare?

Questions you might ask in your role as student to your student:

- Are the pentagons similar?
- How many ways can you prove that they're similar?
- How does the area of the new pentagon compare to the area of the original?
- What scale factors would you use to enlarge the original pentagon? What scale factors would you use to make a smaller pentagon?

Sample Answers

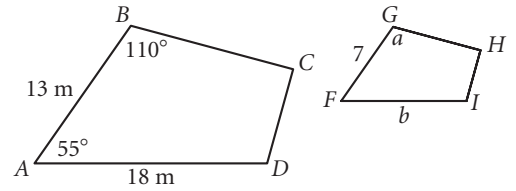
To verify similarity, we have to show that the corresponding sides are proportional and that the corresponding angles are congruent. Your student can verify that the sides are proportional either by measuring the lengths of corresponding sides and writing their ratios or by drawing segments from the origin to the vertices and applying the Extended Parallel/Proportionality Conjecture (finding five pairs of similar triangles). He or she can verify the congruence of corresponding angles by measuring or by applying the SSS similarity shortcut to triangles into which the pentagon can be divided. Once the scale factor is known, it can be squared to determine the ratio of areas. Scale factors larger than 1 will enlarge the figure, whereas scale factors smaller than 1 will reduce the size.

Chapter 11 • Review Exercises

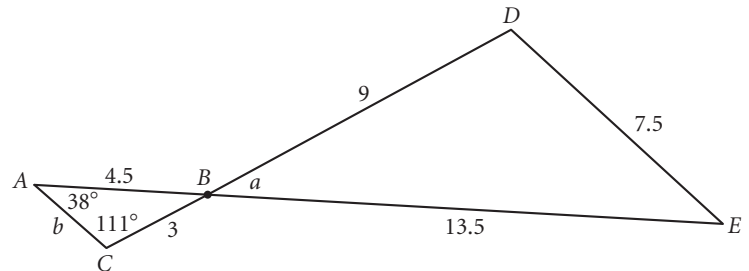
Name _____ Period _____ Date _____

Round your solutions to the nearest tenth of a unit unless otherwise stated.

1. (Lesson 11.1) $ABCD \sim FGHI$. Find a and b .

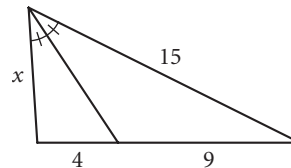


2. (Lesson 11.2) Which similarity shortcut can be used to show that $\triangle ABC \sim \triangle EBD$? Find a and b .



3. (Lesson 11.3) Rafael is 1.8 m tall and casts a 0.5 m shadow. If at the same time a nearby tree casts a 1.5 m shadow, how tall is the tree?

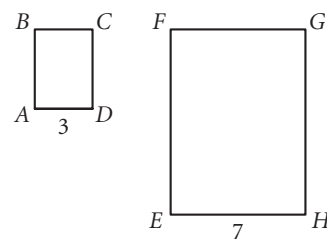
4. (Lesson 11.4) Find x .



5. (Lesson 11.5) $ABCD \sim EFGH$.

Area of $ABCD = 15 \text{ cm}^2$

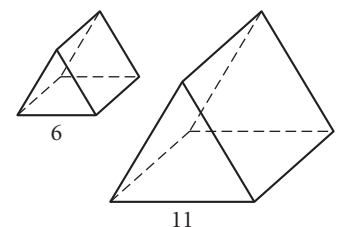
Area of $EFGH = ?$



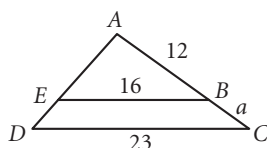
6. (Lesson 11.6) Small triangular prism \sim Large triangular prism

Volume of small prism $= 48 \text{ in}^3$

Volume of large prism $= ?$



7. (Lesson 11.7) $EB \parallel CD$. Find a .



SOLUTIONS TO CHAPTER 11 REVIEW EXERCISES

1. Because the quadrilaterals are similar, corresponding angles are congruent.

Therefore, $a = m\angle G = m\angle B = 110^\circ$.

Because the two quadrilaterals are similar, we can set up a proportion to find the missing length.

$$\frac{13}{7} = \frac{18}{b}$$

$$13b = 18 \cdot 7 \quad \text{Multiply both sides by 7; multiply both sides by } b.$$

$$b \approx 9.7 \text{ m} \quad \text{Solve for } b.$$

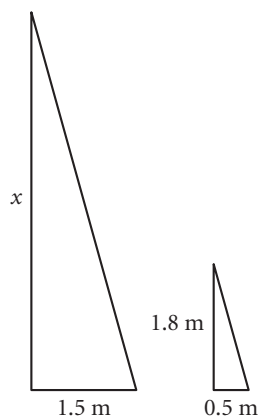
2. Two pairs of corresponding sides are proportional ($\frac{13.5}{4.5} = \frac{9}{3}$) and their included angles are congruent (vertical angles $\angle ABC$ and $\angle EBD$), so the triangles are similar because of the SAS Similarity Conjecture. To find a , use the Triangle Sum Conjecture to find $\angle ABC$: $180^\circ - 38^\circ - 111^\circ = 31^\circ$. Because $\angle ABC \cong \angle EBD$, $a = m\angle EBD = 31^\circ$. To find b , solve the proportion.

$$\frac{9}{3} = \frac{7.5}{b}$$

$$9b = 7.5 \cdot 3 \quad \text{Multiply both sides by 3; multiply both sides by } b.$$

$$b = 2.5 \text{ cm} \quad \text{Solve for } b.$$

3. Because the sun's rays all come in at the same angle at a particular time of day, all people or objects and their shadows form similar right triangles. So, Rafael's height and the length of his shadow are proportional to the tree's height and the length of its shadow.



$$\frac{1.8}{0.5} = \frac{x}{1.5}$$

$$x = 5.4 \quad \text{Multiply both sides by 1.5.}$$

The tree is 5.4 m tall.

4. The angle bisector divides the opposite side into segments that are proportional to the other two sides of the triangle. Set up a proportion and solve:

$$\frac{x}{15} = \frac{4}{9}$$

$$x \approx 6.7 \text{ cm}$$

5. When two figures are similar, the ratio of their areas is the square of the scale factor.

$$\frac{AD}{EH} = \frac{3}{7}$$

The scale factor is the ratio of corresponding sides.

$$\frac{\text{Area of } ABCD}{\text{Area of } EFGH} = \left(\frac{3}{7}\right)^2 = \frac{9}{49}$$

The ratio of areas is the square of the scale factor.

$$\frac{15}{x} = \frac{9}{49}$$

Substitute values.

$$x \approx 81.7 \text{ cm}^2 \quad \text{Solve for } x.$$

6. Scale factor = $\frac{6}{11}$

Find the scale factor.

$$\frac{V_{\text{small prism}}}{V_{\text{large prism}}} = \left(\frac{6}{11}\right)^3$$

The ratio of the volumes is equal to the cube of the scale factor.

$$\frac{48}{x} = \frac{216}{1331}$$

Substitute values.

$$x \approx 295.8 \text{ in}^3$$

Solve for x .

7. Because of parallel proportionality, corresponding sides are proportional.

$$\frac{AB}{AC} = \frac{EB}{DC}$$

$$\frac{12}{12 + a} = \frac{16}{23}$$

Substitute values.

$$12 \cdot 23 = 16(12 + a)$$

Multiply both sides by $23(12 + a)$.

$$276 = 192 + 16a$$

Distribute.

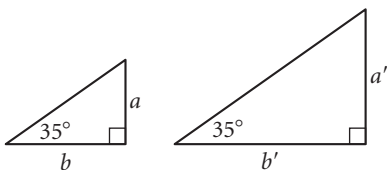
$$a = 5.25$$

Solve for a .

Trigonometry

Content Summary

If two triangles are similar, then their corresponding sides have the same ratio of lengths. Say, $\frac{a}{a'} = \frac{b}{b'}$. But the ratios of the corresponding side lengths of each triangle are also equal: $\frac{a}{b} = \frac{a'}{b'}$.



Now if two *right* triangles have a second angle congruent (in addition to their right angles), then the third angles will also be congruent (by the Triangle Sum Conjecture), so the triangles will be similar (by AAA). Therefore, ratios of the corresponding side lengths of each triangle are also equal. In other words, all right triangles with a certain acute angle measure, say 35° , are similar to one another. So, associated with any acute angle are various ratios of the side lengths, which are the same for all right triangles containing this angle.

Trigonometric Ratios

The study of the relationships between sides and angles of triangles is called *trigonometry*. The ratio of the lengths of two sides in a right triangle is called a *trigonometric ratio*. The three most commonly used ratios—*sine*, *cosine*, and *tangent*—are the focus of this chapter. The *sine* (abbreviated *sin*) of an angle is the ratio of the length of the opposite leg to the length of the hypotenuse. For example, the sine of 30° is $\frac{1}{2}$. This means that in any right triangle with a 30° angle, the side opposite the 30° angle is $\frac{1}{2}$ the length of the hypotenuse. The *cosine* (abbreviated *cos*) of an angle is the ratio of the adjacent leg to the hypotenuse. The *tangent* (abbreviated *tan*) is the ratio of the opposite leg to the adjacent leg. The trigonometric ratios for various angle measures can be looked up in a trigonometric table. They can also be found using a calculator with *sin*, *cos*, and *tan* keys. For example, *sin* 30 will give 0.5, as long as the calculator is in degree mode. Some problems require the inverse of the sine, cosine, or tangent—the ratio of lengths is known and the angle is needed. The inverse can also be found using a calculator. For example, $\sin^{-1} 0.5 = 30$.

This chapter focuses primarily on right triangles, but any triangle can always be divided into two right triangles. Therefore, trigonometry also helps us understand other kinds of triangles and provides shortcuts for working with non-right triangles as well. The Law of Sines, the Law of Cosines, and the SAS Area Conjecture apply to all triangles.

(continued)

Chapter 12 • Trigonometry (continued)

Summary Problem

How can you find the area of a triangular plot of land whose sides have lengths 5 km, 8 km, and 9 km?

Questions you might ask in your role as student to your student:

- What do you know about areas of triangles?
- What would you need to know to apply the formula involving base and altitude?
- How could you determine those quantities?

Sample Answers

Your student may or may not have seen Hero's formula in the Exploration Alternative Area Formulas in Chapter 8. That formula yields an area of about 19.9 km^2 . More likely, students will think of the standard formula for the area of a triangle, $A = \frac{1}{2}bh$. However, the height needs to be measured perpendicular to a base. Any side can serve as a base; to find the altitude to that side, students need an angle. They can use the Law of Cosines to find an angle at one end of the chosen base. Then they can apply the SAS Triangle Area Conjecture to find the area. For example, if the base is the side of length 9 km, then the angle x between that side and the side of length 8 km can be found using the Law of Cosines:

$$5^2 = 9^2 + 8^2 - 2(9)(8)\cos x \quad \text{Substitute values into the Law of Cosines.}$$

$$\frac{25 - 81 - 64}{-144} = \cos x \quad \text{Solve for } \cos x.$$

$$x = \cos^{-1}\left(\frac{5}{6}\right) \quad \text{Use the inverse cosine to find } x.$$

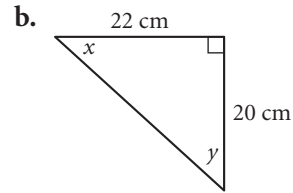
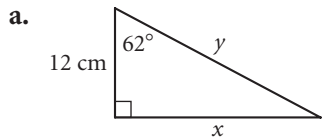
$$x \approx 33.6^\circ$$

The sine of this angle is the ratio of the altitude to the side with length 8 km. The altitude then is 8 times the sine of this angle, or about 4.42 km. The area is $0.5(4.42)(9)$, approximately 19.9 km^2 .

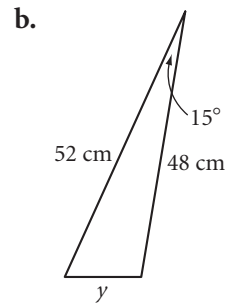
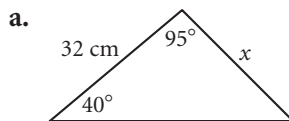
Chapter 12 • Review Exercises

Name _____ Period _____ Date _____

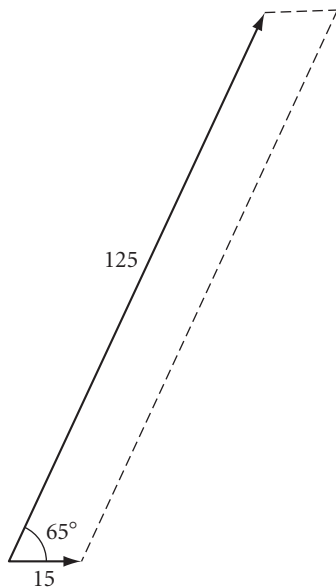
1. (Lessons 12.1, 12.2) Find the missing measures to the nearest tenth of a unit.



2. (Lessons 12.3, 12.4) Find the missing lengths to the nearest centimeter.



3. (Lesson 12.5) Radha is flying her plane on a heading, as shown. Her air speed is 125 mi/h. There is a crosswind of 15 mi/h. What is her resulting speed T ?



SOLUTIONS TO CHAPTER 12 REVIEW EXERCISES

1. a. $\tan 62^\circ = \frac{x}{12}$
 $x = 12 \tan 62^\circ$
 $x \approx 22.6 \text{ cm}$
 $\cos 62^\circ = \frac{12}{y}$
 $y \cos 62^\circ = 12$
 $y = \frac{12}{\cos 62^\circ}$
 $y \approx 25.6 \text{ cm}$

b. $\tan x = \frac{20}{22}$
 $\tan x = 0.909$
 $x = \tan^{-1}(0.909)$
 $x \approx 42.3^\circ$
 $y = 180^\circ - 90^\circ - x$
 $y \approx 47.7^\circ$

2. a. By the Triangle Sum Conjecture, the missing angle is 45° .

Law of Sines:

$$\frac{\sin 45^\circ}{32} = \frac{\sin 40^\circ}{x}$$

$$x \sin 45^\circ = 32 \sin 40^\circ$$

$$x = \frac{32 \sin 40^\circ}{\sin 45^\circ}$$

$$x \approx 29.1 \text{ cm}$$

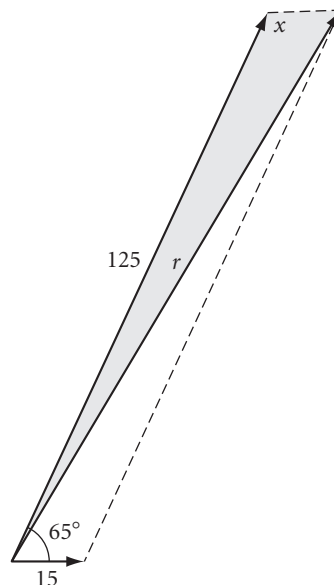
- b. Law of Cosines:

$$y^2 = 52^2 + 48^2 - 2(52)(48) \cos 15^\circ$$

$$y^2 \approx 186.1$$

$$y \approx 13.6 \text{ cm}$$

3. The diagonal of the parallelogram is the resulting speed. You can use the Law of Cosines, but you need to know at least one angle measure in the shaded triangle. Because the diagram is a parallelogram, the adjacent angles are supplementary.



Therefore, $65^\circ + x = 180$, so $x = 115^\circ$. Now you can use the Law of Cosines to find r .

$$r^2 = 15^2 + 125^2 - 2(15)(125) \cos 115^\circ$$

$$r^2 \approx 17,434.8$$

$$r \approx 132 \text{ mi/h}$$

Geometry as a Mathematical System

Content Summary

Having experienced all the concepts of a standard geometry course, students are ready to examine the framework of the geometry knowledge they have built. Students now review and deepen their understanding of those concepts by proving some of the most important conjectures in the context of a logical system, starting with the premises of geometry.

Premises and Theorems

A complete deductive system must begin with some assumptions that are clearly stated and, ideally, so obvious that they need no defense. Chapter 13 begins by laying out its assumptions: properties of arithmetic and equality, postulates of geometry, and a definition of congruence for angles and line segments. These basic assumptions are called *premises*. Everything else builds on these premises.

Next, students develop proofs of their conjectures concerning triangles, quadrilaterals, circles, similarity, and coordinate geometry. Once a conjecture has been proved, it is called a *theorem*. Each step of a proof must be supported by a premise or a previously proved theorem.

Developing a Proof

Developing a proof is more art than science. Mathematicians don't sit down and write a proof from beginning to end, so encourage your student not to expect to do so. Proofs require thought and creativity. Generally, the student will start by writing down what's given and what's to be shown—the beginning and end of the proof. Then, perhaps using diagrams, the student will restate these first and last statements in several ways, looking for an idea of how to get from one to the other logically. You might remind your student about the reasoning strategies that can help in planning a proof:

- Draw a labeled diagram and mark what you know
- Represent a situation algebraically
- Apply previous conjectures and definitions
- Break a problem into parts
- Add an auxiliary line
- Think backward

Questions you may ask your student include “What can you conclude from the given statements?” and “What's needed to prove the last statement?” Seeing connections, the student can develop a plan, perhaps expressed using flowcharts. There are several ways to express the plan, and your student may use more than one. Then he or she can write the proof, being careful to check the reasoning. A good way to be careful about details is to write a two-column proof, with statements in the first column and reasons in the second. Your student might find gaps in his or her reasoning and have to go back to the planning stage.

If there doesn't seem to be any way to prove the statement, you might suggest indirect reasoning, in which the student proves that the *negation* of the theorem is *false*. It then follows that the theorem must be true.

(continued)

Chapter 13 • Geometry as a Mathematical System (continued)

Summary Problem

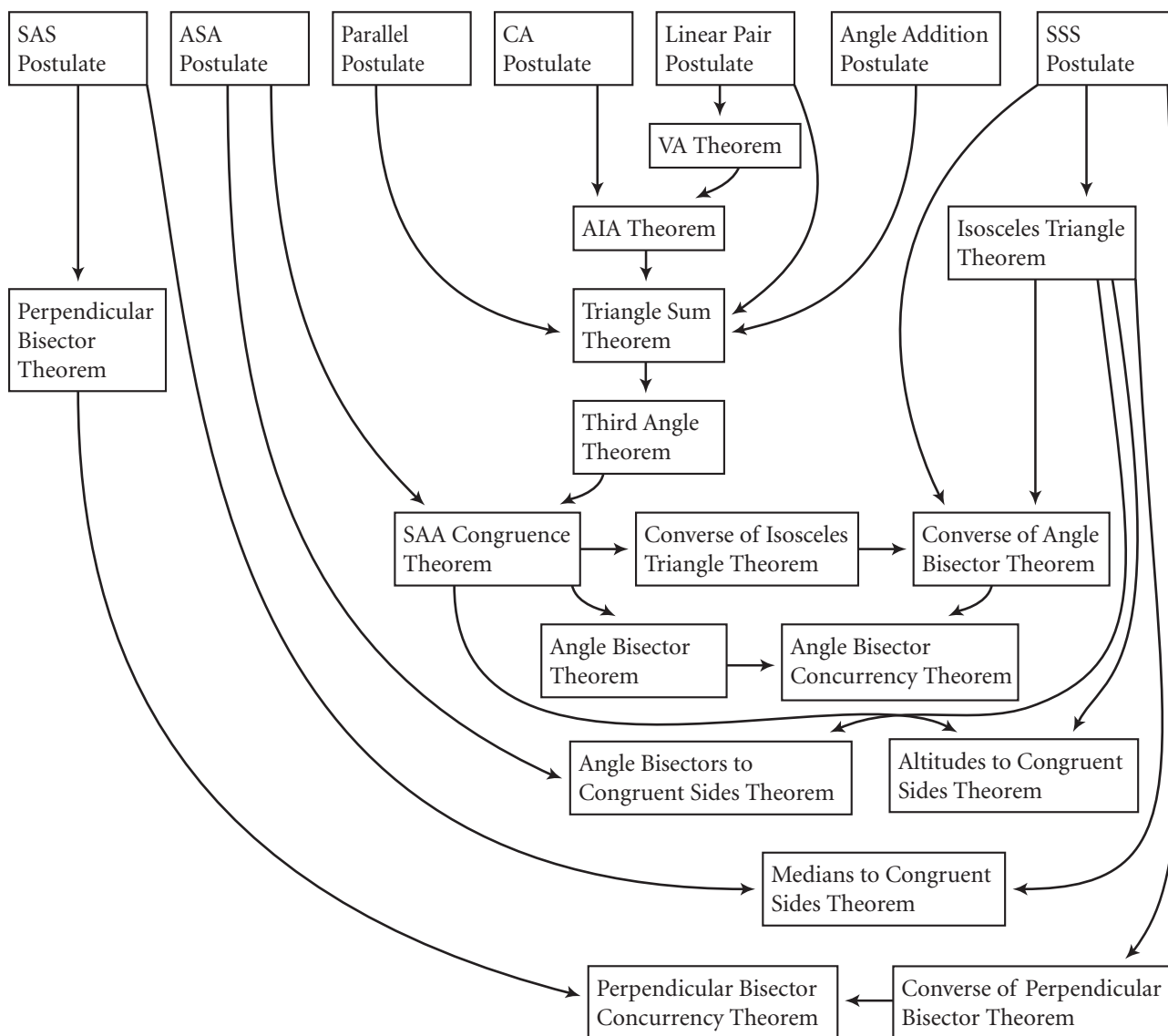
Draw a diagram that gives a family tree for all triangle theorems that appear in the exercises for Lesson 13.3. Include postulates and theorems, but not definitions or properties.

Questions you might ask in your role as student to your student:

- Can you build on the tree on page 707?
- What do the arrows represent in words?

Sample Answers

A family tree shows how theorems support each other. A theorem may rely on several theorems, each of which relies on other theorems, and so on, all the way up to the postulates of geometry. The top of the family tree should be all postulates, and all the arrows should flow down from there. The diagrams can get quite complex. Don't worry too much about neatness or completeness; the goal is to see how the structure can be built while reviewing the theorems. Here is the complete tree.



Chapter 13 • Review Exercises

Name _____ Period _____ Date _____

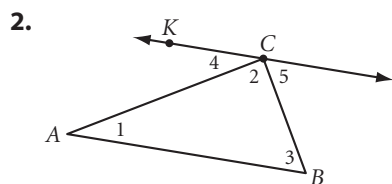
1. (*Lesson 13.1*) Name the property that supports each statement:
 - a. If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
 - b. If $\overline{AB} \cong \overline{CD}$, then $AB = CD$.
2. (*Lessons 13.2, 13.3*) In Lesson 13.2, Example B, the Triangle Sum Theorem is proved with a flowchart proof. Rewrite this proof using a two-column proof.

Given: $\angle 1$, $\angle 2$, and $\angle 3$ are the three angles of $\triangle ABC$

Show: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$
3. (*Lessons 13.2, 13.4*) Answer the following questions for the statement, “The diagonals of an isosceles trapezoid are congruent.”
 - a. **Task 1:** Identify what is given and what you must show.
 - b. **Task 2:** Draw and label a diagram to illustrate the given information.
 - c. **Task 3:** Restate what is given and what you must show in terms of your diagram.
4. (*Lesson 13.6*) Write a proof for the Parallel Secants Congruent Arcs Theorem: Parallel lines intercept congruent arcs on a circle.
5. (*Lesson 13.7*) Write a proof for the Corresponding Altitudes Theorem: If two triangles are similar, then corresponding altitudes are proportional to the corresponding sides.

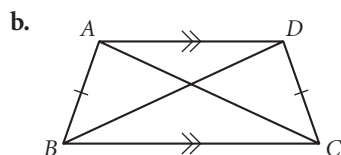
SOLUTIONS TO CHAPTER 13 REVIEW EXERCISES

1. a. Transitive Property
b. Definition of Congruence

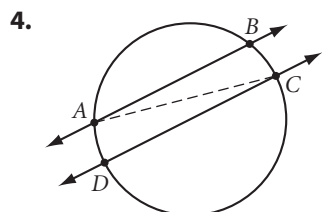


Statement	Reason
$\angle 1, \angle 2,$ and $\angle 3$ of $\triangle ABC$	Given
Construct $\overrightarrow{KC} \parallel \overrightarrow{AB}$	Parallel Postulate
$\angle 1 \cong \angle 4; \angle 3 \cong \angle 5$	Alternate Interior Angles Theorem
$m\angle 1 = m\angle 4; m\angle 3 = m\angle 5$	Definition of Congruence
$m\angle 4 + m\angle 2 = m\angle KCB$	Angle Addition Postulate
$\angle KCB$ and $\angle 5$ are supplementary	Linear Pair Postulate
$m\angle KCB + m\angle 5 = 180^\circ$	Definition of Supplementary
$m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$	Substitution Property of Equality
$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$	Substitution Property of Equality

3. a. **Given:** Isosceles trapezoid
Show: Diagonals are congruent



- c. **Given:** $\overline{AB} \cong \overline{DC}; \overline{AD} \parallel \overline{BC}$
Show: $\overline{AC} \cong \overline{DB}$

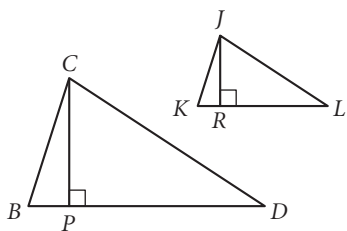


Given: $\overline{AB} \parallel \overline{DC}$

Show: $\overline{AD} \cong \overline{BC}$

Statement	Reason
$\overline{AB} \parallel \overline{DC}$	Given
Construct \overline{AC}	Line Postulate
$\angle DCA \cong \angle BAC$	Alternate Interior Angles Theorem
$m\angle DCA = m\angle BAC$	Definition of Congruence
$\frac{1}{2}m\angle BAC = \frac{1}{2}m\angle DCA$	Multiplication Property of Equality
$m\widehat{AD} = \frac{1}{2}m\angle DCA$	Inscribed Angle Theorem
$m\widehat{BC} = \frac{1}{2}m\angle BAC$	Inscribed Angle Theorem
$m\widehat{AD} = m\widehat{BC}$	Substitution Property of Equality
$\widehat{AD} \cong \widehat{BC}$	Definition of Congruence

5.



Given: $\triangle CBD \sim \triangle JKL$; Altitudes \overline{CP} and \overline{JR}

Show: $\frac{CP}{JR} = \frac{CB}{JK}$

Statement	Reason
$\triangle CBD \sim \triangle JKL$; Altitudes \overline{CP} and \overline{JR}	Given
$\overline{CP} \perp \overline{BD}$; $\overline{JR} \perp \overline{KL}$	Definition of Altitude
$\angle CPB$ and $\angle JRK$ are right angles	Definition of Perpendicular
$\angle CPB \cong \angle JRK$	Right Angles are Congruent Theorem
$\angle CBD \cong \angle JKL$	Corresponding Angles of Similar Triangles are Congruent
$\triangle CBP \sim \triangle JKR$	AA Similarity Postulate
$\frac{CP}{JR} = \frac{CB}{JK}$	Corresponding Sides of Similar Triangles are Proportional

1.1

Building Blocks of Geometry

In this lesson you will

- Learn about **points**, **lines**, and **planes** and how to represent them
- Learn definitions of **collinear**, **coplanar**, **line segment**, **congruent segments**, **midpoint**, and **ray**
- Learn **geometric notation** for lines, line segments, rays, and congruence

Points, **lines**, and **planes** are the building blocks of geometry. Read about these three concepts on page 28 of your book.

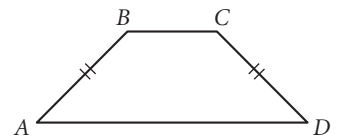
A **definition** is a statement that explains the meaning of a word or phrase. It is impossible to define *point*, *line*, and *plane* without using other words that need to be defined. For this reason, these terms remain undefined. Using these three undefined terms, you can define all other geometric figures and terms.

Keep a list of definitions in your notebook. Each time you encounter a new geometric definition, add it to your list. Illustrate each definition with a sketch. Begin your list with the definitions of **collinear**, **coplanar**, and **line segment** given on pages 29–30 of your book.

Make sure you understand the two ways to express the length of a segment. For example, to express the fact that the length of segment FG is 12 units, you can write either $FG = 12$ or $m\overline{FG} = 12$.

Two segments with equal measures, or lengths, are said to be **congruent**. The symbol for congruence is \cong . It is important to remember that the equals symbol, $=$, is used between equal numbers or measures, while the congruence symbol, \cong , is used between congruent figures.

In geometric drawings, congruent segments are marked with identical symbols. In the figure at right, \overline{AB} is congruent to \overline{DC} . You can indicate that these segments have the same length in any of the following ways:
 $\overline{AB} \cong \overline{DC}$, $AB = DC$, $m\overline{AB} = m\overline{DC}$.

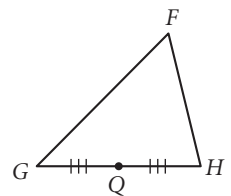
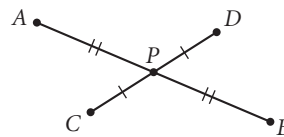


The **midpoint** of a segment is a point that divides the segment into two congruent segments. Work through the example in your book, which gives you practice identifying midpoints and congruent segments and using geometric notation. Below is another example.

EXAMPLE

Study the diagrams at right.

- Name each midpoint and the segment it bisects.
- Name all the congruent segments.
Use the congruence symbol to write your answer.

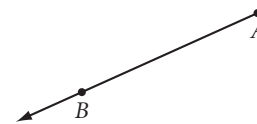
**► Solution**

- P is the midpoint of both \overline{AB} and \overline{CD} . Q is the midpoint of \overline{GH} .
- $\overline{AP} \cong \overline{PB}$, $\overline{CP} \cong \overline{PD}$, $\overline{GQ} \cong \overline{QH}$

(continued)

Lesson 1.1 • Building Blocks of Geometry (continued)

A **ray** is a part of a line that begins at a point and extends infinitely in one direction. A ray is named with two letters. The first letter is the endpoint, and the second letter is any other point on the ray. So ray AB , abbreviated \overrightarrow{AB} , is the part of line AB that contains point A and all the points on \overleftrightarrow{AB} that are on the same side of point A as point B .



Now look back through Lesson 1.1 in your book and make sure you have recorded all the new definitions in your notebook.

Investigation: Mathematical Models

In your book, use the photograph and the figure at the beginning of the investigation to identify examples of real-world and mathematical models for each of the following terms: point, line, plane, line segment, congruent segments, midpoint of a segment, and ray. For example, a line segment can be modeled by the top edge of a window in the photograph, and a point is modeled by the dot labeled A in the figure.

Now explain in your own words what each of these terms means.

1.2

Poolroom Math

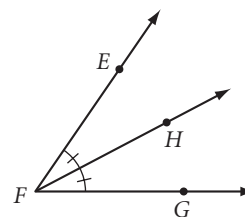
In this lesson you will

- Learn about **angles** and how to measure them
- Identify **congruent angles** and **angle bisectors**
- Use your knowledge of angles to solve problems involving pool

An **angle** is two rays with a common endpoint, provided the two rays do not lie on the same line. The two rays are the **sides** of the angle, and the common endpoint is the **vertex**. In your book, read the text before Example A, which explains how to name angles. Then work through Example A.

The **measure of an angle** is the smallest amount of rotation about the vertex from one ray to the other. The **reflex measure of an angle** is the largest amount of rotation less than 360° between the two rays. In this course, angles are measured in **degrees**. Read the text about angle measures in your book, paying close attention to the instructions for using a **protractor**. Then use your protractor to measure the angles in Example B.

Two angles are **congruent** if and only if they have equal measures. A ray is the **angle bisector** if it contains the vertex and divides the angle into two congruent angles. In the diagram at right, \overrightarrow{FH} bisects $\angle EFG$, so $\angle EFH \cong \angle GFH$. Identical markings are used to show that two angles are congruent.

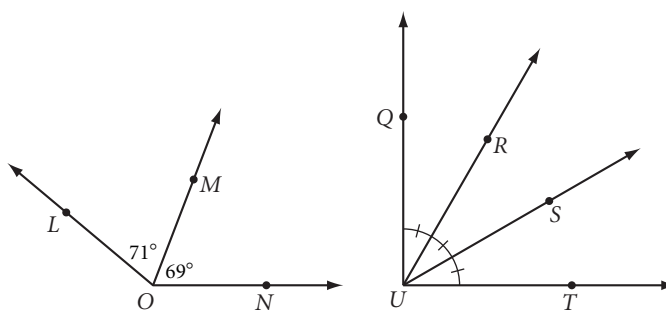


Work through Example C in your book. Here is another example.

EXAMPLE

Look for angle bisectors and congruent angles in the diagrams below.

- Name each angle bisector and the angle it bisects.
- Name all the congruent angles.

**► Solution**

- \overrightarrow{US} bisects $\angle RUT$. \overrightarrow{UR} bisects $\angle QUS$.
- $\angle SUT \cong \angle RUS \cong \angle QUR$ and $\angle QUS \cong \angle RUT$

(continued)

Lesson 1.2 • Poolroom Math (continued)

Investigation: Virtual Pool

Billiards, or pool, is a game of angles. Read about **incoming angles** and **outgoing angles** in your book.

Look at the diagram of the pool table on page 42 of your book. Imagine that the ball is shot toward point C . The incoming angle is $\angle 1$. Use your protractor to find the measure of $\angle 1$.

The measure of the outgoing angle must equal the measure of the incoming angle. Measure $\angle BCP$ and $\angle ACP$. Which of these angles is the outgoing angle? Which point will the ball hit, point A or point B ?

Now imagine you want to hit the ball against cushion \overleftrightarrow{WA} so that the ball bounces off and hits the 8-ball. Which point— W , X , or Y —should you hit? One way to find the answer is to measure each possible incoming angle and then check whether the ray for the congruent outgoing angle passes through the 8-ball.

Now think about how you would have to hit the ball against cushion \overleftrightarrow{CP} so that it would bounce back and pass over its starting point. If you don't know, try experimenting with several different incoming angles. Each time, think about how you can adjust the angle to make the ball pass closer to its starting point.

Suppose you want to hit the ball so that it bounces off the cushions at least three times but never touches cushion \overleftrightarrow{CP} . Again, if you don't know, experiment. Try different incoming angles and different cushions until you start to see a pattern.

1.3

What's a Widget?

In this lesson you will

- Learn how to write a good definition
- Write definitions for geometric terms
- Test definitions by looking for **counterexamples**

In geometry, it is very important to be able to write clear, precise definitions. The text on page 47 of your book discusses how to write a good definition. Read this text carefully. Then work through Example A. Here is another example.

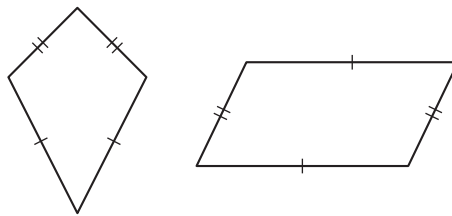
EXAMPLE A

Consider this “definition” of *rectangle*: “A rectangle is a figure with two pairs of congruent sides.”

- Sketch a **counterexample**. That is, sketch a figure with two pairs of congruent sides that is not a rectangle.
- Write a better definition for *rectangle*.

► **Solution**

- Here are two counterexamples.



- One better definition is “A rectangle is a 4-sided figure in which opposite sides are congruent and all angles measure 90° .”

Read the “Beginning Steps to Creating a Good Definition” in your book, and make sure you understand them. Look at the symbols for parallel, perpendicular, and 90° . Now work through Example B, which asks you to write definitions for *parallel lines* and *perpendicular lines*.

Investigation: Defining Angles

In this investigation you will write definitions for some important terms related to angles.

On page 49 of your book, look at the examples of right angles and of angles that are not right angles. What do the right angles have in common? What characteristics do the right angles have that the other angles do not have? You should notice that all the right angles measure 90° . The angles in the other group have measures less than or greater than 90° . Based on this information, you could write the following definition for *right angle*.

A **right angle** is an angle that measures 90° .

(continued)

Lesson 1.3 • What's a Widget? (continued)

Now look at the acute angles and the angles that are not acute. Try to use the examples to write a definition for **acute angle**. When you have written your definition, test it by trying to come up with a counterexample. When you are satisfied with your definition, look at the next set of examples, and write a definition for **obtuse angle**.

The remaining sets of examples show angle pairs. Look at the pairs of vertical angles and the pairs of angles that are not vertical angles. What do you notice? You should see that each pair of vertical angles is formed by two intersecting lines. You might start with the following definition.

Two angles are a **pair of vertical angles** if they are formed by two intersecting lines.

However, $\angle 1$ and $\angle 2$ in the “not pairs of vertical angles” group are also formed by two intersecting lines. What makes the angle pairs in the “vertical angles” group different from this pair? When you know the answer, try completing this definition.

Two angles are a **pair of vertical angles** if they are formed by two intersecting lines and _____.

Now look at the linear pairs of angles and the pairs of angles that are not linear pairs. Write a definition for **linear pair of angles**. Be sure to test your definition by looking for a counterexample. Here is one possible definition. You may have written a different definition.

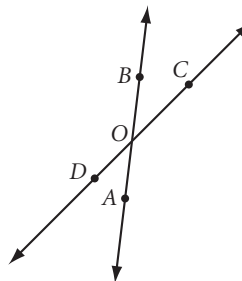
Two angles are a **linear pair of angles** if they share a side and their other sides form a straight line.

Repeat this process to define **pair of complementary angles** and **pair of supplementary angles**. Think carefully about the difference between a supplementary pair and a linear pair. Make sure your definitions account for the difference.

A labeled figure can often be helpful when writing a geometric definition. Work through Example C in your book. Here is another example.

EXAMPLE B | Use a labeled figure to define a *vertical pair of angles*.

► **Solution** | $\angle AOC$ and $\angle BOD$ are a pair of vertical angles if \overleftrightarrow{AB} and \overleftrightarrow{CD} intersect at point O , and point O is between points A and B and also between points C and D .



In the definition, is it necessary to state that point O is between the other two points on each line? Sketch a counterexample that shows why this part of the definition is necessary.

Add definitions for all the new terms in this lesson to your definition list. Be sure to include a sketch with each definition.

1.4

Polygons

In this lesson you will

- Learn the definition of **polygon**
- Learn the meaning of terms associated with polygons, such as **concave**, **convex**, **equilateral**, **equiangular**, and **regular**
- Identify **congruent polygons**

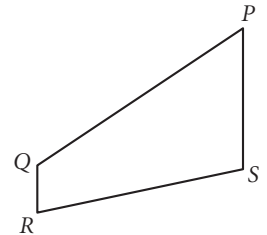
A **polygon** is a closed figure in a plane, formed by connecting line segments endpoint to endpoint with each segment intersecting exactly two others.

Look closely at the examples of “polygons” and “not polygons” on page 54 of your book. Check that each figure in the “polygons” group fits the definition. Then try to explain why each figure in the “not polygons” group is not a polygon.

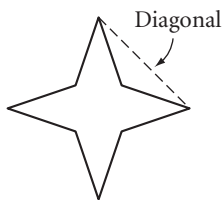
Each line segment in a polygon is a **side** of a polygon. Each endpoint where the sides meet is a **vertex** of the polygon.

Polygons are classified by the number of sides they have. The chart on page 54 of your book gives the names of polygons with different numbers of sides.

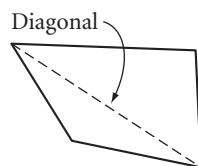
You name a polygon by listing the vertices in consecutive order. It does not matter which vertex you start with. For example, you could name this polygon quadrilateral $PQRS$ or $RQPS$, but not $PRQS$. When you name a triangle, you can use the \triangle symbol. For example, $\triangle XYZ$ means triangle XYZ .



A **diagonal** of a polygon is a line segment that connects two *nonconsecutive* vertices. A polygon is **convex** if no diagonal is outside the polygon. A polygon is **concave** if at least one diagonal is outside the polygon. See page 54 of your book for more examples of convex and concave polygons.



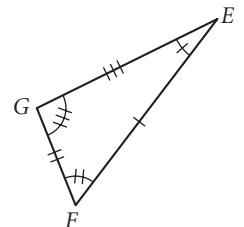
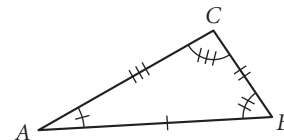
Concave polygon



Convex polygon

Two polygons are **congruent polygons** if and only if they are exactly the same size and shape. If two polygons are congruent, then their corresponding angles and sides are congruent. For example, triangle ABC is congruent to triangle EFG , so their three pairs of corresponding angles and three pairs of corresponding sides are also congruent.

$$\begin{array}{lll} \angle A \cong \angle E & \angle B \cong \angle F & \angle C \cong \angle G \\ \overline{AB} \cong \overline{EF} & \overline{BC} \cong \overline{FG} & \overline{CA} \cong \overline{GE} \end{array}$$



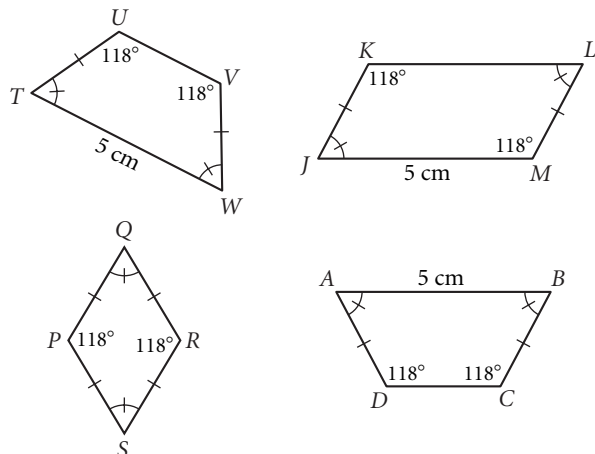
When you write a statement of congruence, always write the letters of the corresponding vertices in an order that shows the correspondence. For example, when referring to the triangles at right, the statements $\triangle ABC \cong \triangle EFG$ and $\triangle CAB \cong \triangle GEF$ are correct, but $\triangle ABC \cong \triangle FEG$ is incorrect.

(continued)

Lesson 1.4 • Polygons (continued)

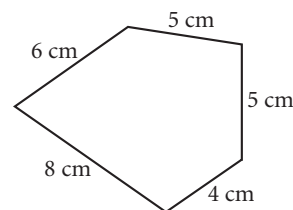
EXAMPLE

Which polygon is congruent to $TUVW$?



► **Solution** | Polygon $TUVW \cong$ polygon $BCDA$. You could also say $TUVW \cong ADCB$.

The **perimeter** of a polygon is the sum of the lengths of its sides. The perimeter of the polygon at right is 28 cm.



Investigation: Special Polygons

In this investigation you will write definitions for three special polygons.

In your book, look at the polygons that are equilateral and the polygons that are not equilateral. What characteristics do the equilateral polygons have that the other polygons don't have? You should notice that in each equilateral polygon all sides have equal length, whereas in a polygon that is not equilateral, not all sides have equal length. From this observation you could write the following definition:

An equilateral polygon is a polygon in which all sides have equal length.

Now look at the polygons that are equiangular and the polygons that are not equiangular. Use the examples to write a definition of **equiangular polygon**.

Finally, look at the polygons that are regular polygons and the ones that are not regular polygons. Decide which characteristics separate the polygons into the two groups and write a definition of **regular polygon**. Your definition might be in this form:

A regular polygon is a polygon that is both _____ and _____.

Add definitions for all the new terms in this lesson to your definition list. Be sure to include a sketch with each definition.

Triangles

In this lesson you will

- Learn how to interpret geometric diagrams
- Write definitions for types of triangles

When you look at a geometric diagram, you must be careful not to assume too much from it. For example, you should *not* assume that two segments that appear to be the same length actually are the same length, unless they are marked as congruent.

You may assume

that lines are straight and if two lines intersect they intersect at one point.

that all points on a line are collinear and all points on a diagram are coplanar unless planes are drawn to show they are noncoplanar.

You may not assume

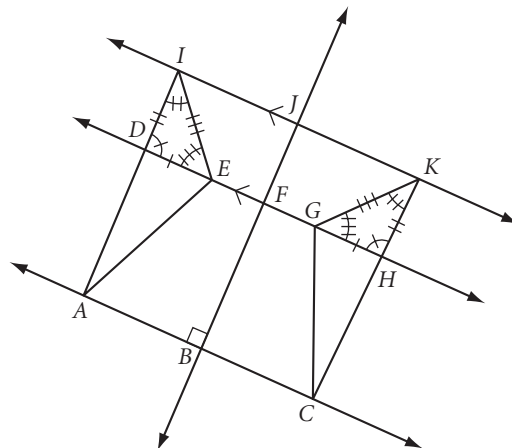
that lines are parallel unless they are marked as parallel.

that lines are perpendicular unless they are marked as perpendicular.

that pairs of angles, segments, or polygons are congruent unless they are marked with information that tells you they are congruent.

EXAMPLE

In the figure below, which pairs of lines are parallel? Which pairs of lines are perpendicular? Which pairs of triangles are congruent?



► Solution

Lines IJ and EG are parallel. Lines BF and AC are perpendicular. Triangles IDE and KHG are congruent.

(continued)

Lesson 1.5 • Triangles (continued)

Investigation: Triangles

In this investigation, you will write definitions for types of triangles.

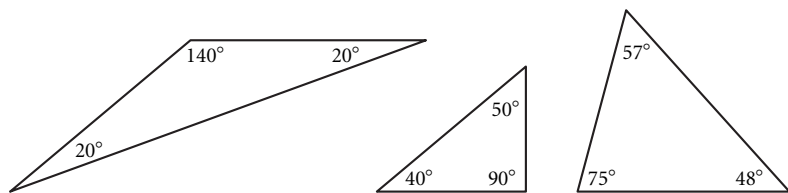
In your book, look at the right triangles and the figures that are not right triangles. What do the right triangles have in common? What characteristics do the right triangles have that the other triangles do not have? You should notice that all the right triangles have a right angle (an angle that measures 90°). None of the other triangles have a right angle. Based on this information, you could write the following definition for a right triangle.

A **right triangle** is a triangle with one right angle.

Now, look at the acute triangles and the triangles that are not acute.

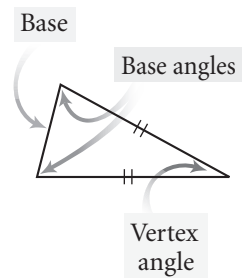
Use the examples to write a definition for **acute triangle**. When you have written your definition, test it by trying to come up with a counterexample. When you are satisfied with your definition, look at the next set of examples, and write a definition for **obtuse triangle**.

Look back at your definitions of *right triangle*, *acute triangle*, and *obtuse triangle*. If you have written correct definitions, any triangle you are given will fit *one and only one* of these definitions. Check that each triangle below fits one and only one of your definitions. If not, go back and refine your definitions.



Look at the next three sets of examples, and write definitions for **scalene triangle**, **equilateral triangle**, and **isosceles triangle**. (Hint: Focus on the side lengths of the triangles.) If your definitions are correct, any triangle will fit one and only one of these definitions. Draw several triangles and test each one to make sure this is true. If it is not, refine your definitions.

In an isosceles triangle, the angle between the two sides of equal length is called the **vertex angle**. The side opposite the vertex angle is called the **base** of the isosceles triangle. The two angles opposite the two sides of equal length are called **base angles** of the isosceles triangle.



Add the definition for each type of triangle to your definition list. Be sure to include a labeled drawing with each definition.

1.6

Special Quadrilaterals

In this lesson you will

- Write definitions for special types of quadrilaterals

All of the quadrilaterals in this lesson can be made by attaching two congruent triangles along a corresponding side. Depending on the type of triangles you join, the quadrilaterals have different properties. In the following investigation you will define different types of special quadrilaterals based on relationships of their sides and angles.

Investigation: Special Quadrilaterals

On page 64 of your book, look at the figures that are trapezoids and the figures that are not trapezoids. What differentiates the trapezoids from the nontrapezoids? Each trapezoid is a figure with a pair of parallel sides. So, you might start with this definition.

A **trapezoid** is a figure with a pair of parallel sides.

However, two of the figures in the “not trapezoids” group also have a pair of parallel sides. One of these figures has two pairs of parallel sides, while all the trapezoids have only one pair. So, you could refine the definition like this.

A **trapezoid** is a figure with exactly one pair of parallel sides.

This definition is better, but one of the nontrapezoids satisfies it. Notice, though, that this nontrapezoid has five sides, while each trapezoid has four sides. You can refine the definition once again.

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides.

This definition fits all the trapezoids and none of the nontrapezoids.

Now write definitions for the next two terms, **kite** and **parallelogram**. Note that these figures have some things in common, but there are also important differences between them. Make sure your definitions account for these differences.

Move on to write definitions for **rhombus**, **rectangle**, and **square**. There are several correct definitions for each of these terms. Once you define a term, you can use it in the definition for another term. For example, a rhombus is a special type of parallelogram, so your definition might be in this form.

A **rhombus** is a parallelogram with _____.

A square is a special type of rhombus *and* a special type of rectangle, so your definition might be in one of the following forms.

A **square** is a rhombus with _____.

A **square** is a rectangle with _____.

Here is another possible definition.

A **square** is a rhombus that is also a rectangle.

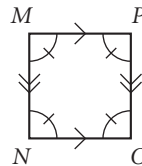
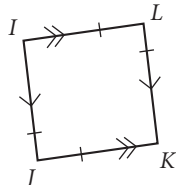
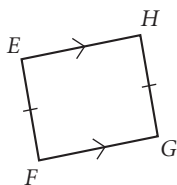
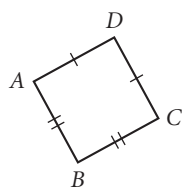
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Lesson 1.6 • Special Quadrilaterals (continued)

Add the definition for each type of quadrilateral to your definition list. Be sure to include a labeled drawing with each definition.

EXAMPLE

Look carefully at the quadrilaterals. Classify each figure as a trapezoid, kite, rhombus, rectangle, or square. Explain your thinking.



► Solution

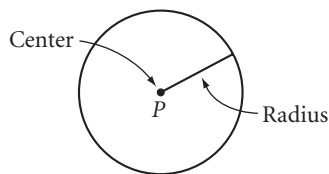
Quadrilateral $ABCD$ is a kite because two pairs of adjacent sides are congruent. Quadrilateral $EFGH$ is a trapezoid because it has only one pair of parallel sides. Quadrilateral $IJKL$ is a rhombus because it is a parallelogram and all of its sides have equal length. Quadrilateral $MNOP$ is a rectangle because it is a parallelogram and all of its angles have equal measures.

Circles

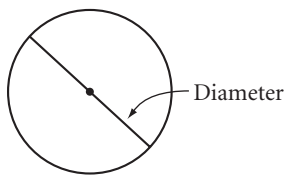
In this lesson you will

- Learn the definition of **circle**
- Write definitions for **chord**, **diameter**, and **tangent**
- Learn about three types of **arcs** and how they are measured

A **circle** is the set of all points in a plane that are a given distance from a given point. The given distance is called the **radius** and the given point is called the **center**. The word *radius* is also used to refer to a segment from the center to a point on the circle. You name a circle by its center. The circle below is circle *P*.



The **diameter** of a circle is a line segment containing the center, with its endpoints on the circle. The word *diameter* is also used to refer to the length of this segment.



Investigation: Defining Circle Terms

In this investigation you will write definitions for terms associated with circles.

In your book, look at the examples of chords and nonchords. What do the chords have in common? What characteristics do the chords have that the nonchords do not? For example, the chords are all segments, and each chord has two points on the circle. One of the nonchords, namely \overline{RS} , also has these properties. However, each chord has *both endpoints* on the circle, while \overline{RS} has only one of its endpoints on the circle. Using these observations, you could write this definition.

A **chord** of a circle is a segment with both of its endpoints on the circle.

Now study the examples of diameters and nondiameters. Use your observations to write a definition for *diameter*. Because you have already defined *chord*, you can use this term in your definition. Your definition might be in one of the following forms.

A **diameter** of a circle is a segment that _____.

A **diameter** of a circle is a chord that _____.

Finally, study the examples of tangents and nontangents, and use your observations to define **tangent**. Be sure to check your definition by looking

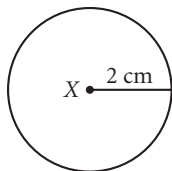
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Lesson 1.7 • Circles (continued)

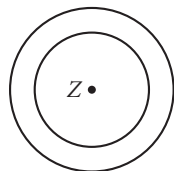
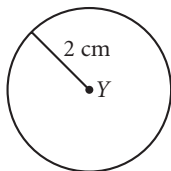
for a counterexample. Note that the point where the tangent touches the circle is called the **point of tangency**.

Look at the questions in Steps 2 and 3 in your book. Make sure you can answer these questions. Think about the definitions you wrote in Step 1 and how they are similar and different.

Congruent circles are circles with the same radius. **Concentric circles** are circles in the same plane with the same center.



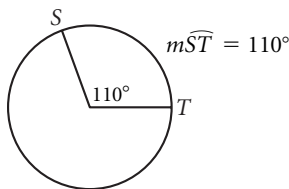
Congruent circles



Concentric circles

An **arc of a circle** is two points on the circle and the continuous (unbroken) part of the circle between the two points. Arcs can be classified into three types. A **semicircle** is an arc of a circle whose endpoints are on a diameter. A **minor arc** is an arc that is smaller than a semicircle. A **major arc** is an arc that is larger than a semicircle. You name a minor arc with the letters of its endpoints. You name semicircles and major arcs with the letters of three points—the first and last letters are the endpoints, and the middle letter is any other point on the arc. See the diagram on page 71 of your book for examples of each type of arc.

Arcs are measured in degrees. A full circle has an arc measure of 360° , a semicircle has an arc measure of 180° , and so on. The measure of a minor arc is equal to the measure of the *central angle* associated with the arc. The **central angle** is the angle with its vertex at the center of the circle and its sides passing through the endpoints of the arc.



Add definitions for all the new terms in this lesson to your definition list. Be sure to include a labeled drawing with each definition.

1.8

Space Geometry

In this lesson you will

- Learn the mathematical definition of **space**
- Learn the names of common three-dimensional objects and how to draw them
- Solve problems that require you to visualize objects in space

The work you have done so far has involved objects in a single plane. In this lesson you will need to visualize objects in three dimensions, or space. Read about space on page 75 of your book.

In geometry, it is important to be able to recognize three-dimensional objects from two-dimensional drawings, and to create drawings that represent three-dimensional objects. Pages 75–77 in your book show examples of common three-dimensional objects and give tips for drawing these objects. Read this text carefully and practice drawing the objects.

Investigation: Space Geometry

In this investigation you need to decide whether statements about geometric objects are true or false. You can make sketches or use physical objects to help you visualize each statement. For example, you might use a sheet of paper to represent a plane and a pencil to represent a line. In each case, try to find a counterexample to the statement. If you find one, the statement must be false. If a statement is false, draw a picture and explain why it is false.

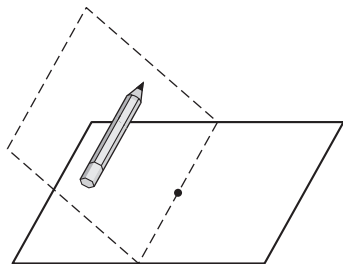
Below are some suggestions for visualizing the situations described in the statements. Try to determine whether each statement is true or false on your own before you read the suggestion.

1. For any two points, there is exactly one line that can be drawn through them.

Draw two points on a sheet of paper and draw a line through them. Is there a way to draw another straight line through the points? Remember that you are not limited to the surface of the paper.

2. For any line and a point not on the line, there is exactly one plane that contains them.

Draw a dot on a sheet of paper to represent the point, and use a pencil to represent the line. Hold the pencil above the paper and imagine a plane passing through both the point and the line.



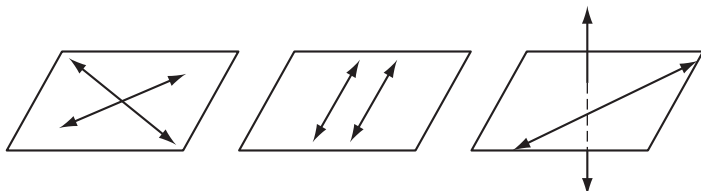
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Lesson 1.8 • Space Geometry (continued)

Without moving the point or the line, try to imagine a different plane passing through them. Can you do it? Change the position of the pencil and the paper so that they represent a different point and line. Can you imagine more than one plane passing through them? Experiment until you think you know whether the statement is true or false.

3. For any two lines, there is exactly one plane that contains them.

There are three situations that you must consider: intersecting lines, parallel lines, and skew lines.



First, look at the intersecting lines. They are drawn on a sheet of paper, which can represent a plane containing the lines. Try to imagine a different plane that also contains both lines. Can you do it?

Next, study the parallel lines contained in the plane of the sheet of paper. Can a different plane contain both parallel lines? Finally, look at the third pair of lines, which are skew lines, or lines that are not parallel and do not intersect. Can you imagine a sheet of paper that will contain these lines?

4. If two coplanar lines are both perpendicular to a third line in the same plane, then the two lines are parallel.

Notice that all the lines mentioned in this statement are in the same plane. You can use a sheet of paper to represent the plane. On the paper, draw a line and then draw two lines that are each perpendicular to the line. Are the two lines parallel? Make more sketches if you need to.

5. If two planes do not intersect, then they are parallel.

Use two sheets of paper or cardboard to represent the planes. You'll need to picture the sheets extending forever. Can you arrange the planes so that they will never intersect but so they are *not* parallel?

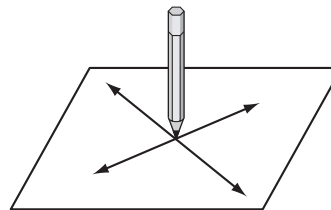
6. If two lines do not intersect, then they are parallel.

You know that if lines *in the same plane* do not intersect, then they must be parallel. But what if the lines are in different planes? You can use two pencils to represent two lines. See if you can position the lines so that they will not intersect and are *not* parallel.

7. If a line is perpendicular to two lines in a plane, but the line is not contained in the plane, then the line is perpendicular to the plane.

You can use a sheet of paper to represent the plane. Draw two lines on the paper to represent the two lines in the plane. The third line is not contained in the plane. Represent this line with a pencil. Hold the pencil so that it is perpendicular to both of the lines in the plane. (Note: In order for you to do this, the lines in the plane must intersect.)

Is the pencil perpendicular to the plane? Experiment until you are convinced you know whether the statement is true or false.



A Picture Is Worth a Thousand Words

In this lesson you will

- Solve problems that require visual thinking
- Draw diagrams to help you solve problems

When you are solving a problem that requires you to visualize something, it often helps to draw a diagram. In the examples in this lesson, you will apply your visualization skills to solve problems. Work through all the examples in your book, using diagrams to help you find the solutions. Below are some additional examples. Try to solve each problem yourself before looking at the solution.

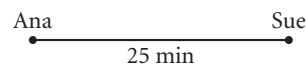
EXAMPLE A

Five friends rode in a 50-mile bike race. Sue finished 25 minutes after Ana. Ana finished 40 minutes before Mel. Mel finished 25 minutes after Jing. Rosi finished 20 minutes before Jing. If Ana finished at 1:30 P.M., what time did each of the other girls finish?

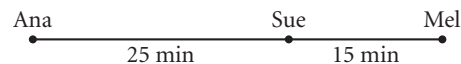
► Solution

You can plot the information, one fact at a time, on a “time line.”

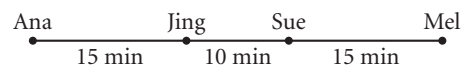
Sue finished 25 minutes after Ana.



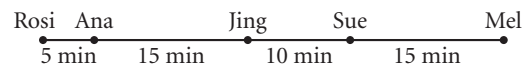
Ana finished 40 minutes before Mel.



Mel finished 25 minutes after Jing.



Rosi finished 20 minutes before Jing.



Use the fact that Ana finished at 1:30 P.M., along with the information on the time line, to figure out when each girl finished.

Rosi: 1:25 P.M. Jing: 1:45 P.M. Sue: 1:55 P.M. Mel: 2:10 P.M.

In the next example, you need to identify a **locus** of points.

EXAMPLE B

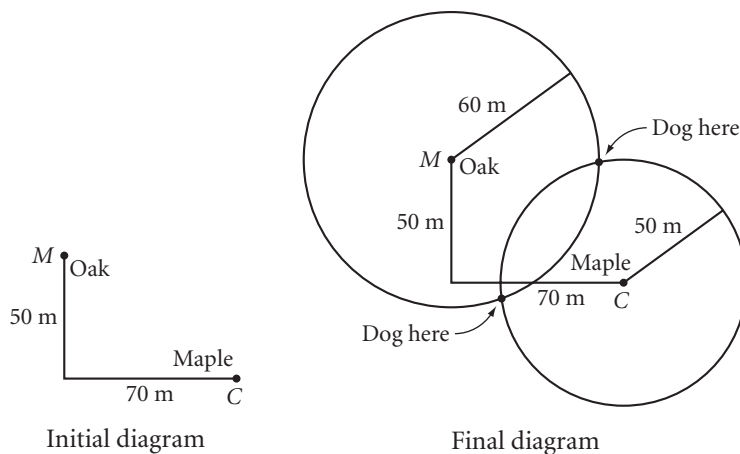
Oak Street and Maple Street are perpendicular to one another. Maya and Chris are looking for their lost dog. Maya is on Oak Street, 50 meters north of the corner of Oak and Maple. Chris is on Maple Street, 70 meters east of the corner. The dog is 60 meters from Maya and 50 meters from Chris. Make a diagram showing the places where the dog might be located.

(continued)

Lesson 1.9 • A Picture Is Worth a Thousand Words (continued)

► Solution

Start by drawing a diagram showing the two streets and the locations of Maya and Chris. Because the dog is 60 meters from Maya, draw a circle with radius 60 meters centered at point M . Because the dog is 50 meters from Chris, draw a circle with radius 50 meters centered at point C . The intersection of the circles marks the two places the dog might be.



A Venn diagram is a large circle (or oval) that contains smaller circles (or ovals). The large circle represents a whole collection of things, and the smaller circles represent special parts (or subsets) of the whole collection.

EXAMPLE C

Create a Venn diagram showing the relationships among polygons, equilateral polygons, and regular polygons.

► Solution

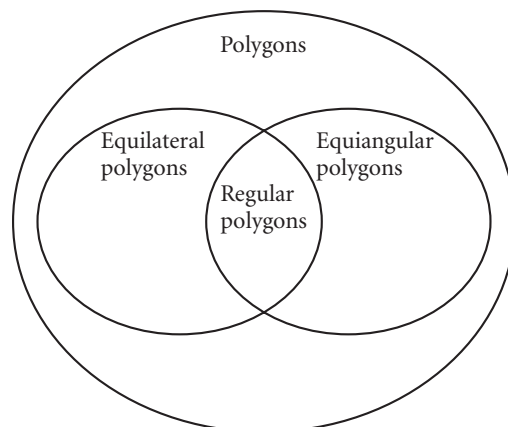
The large circle in your Venn diagram must represent what polygons, equilateral polygons, and regular polygons all have in common. All of these are polygons. So, the large outer circle represents polygons.

An equilateral polygon is a special polygon with all sides the same length. So, a small inner circle can represent equilateral polygons.

An equiangular polygon is a special polygon in which all angles have equal measures. Therefore, a second small inner circle can represent equiangular polygons.

The intersection of the two inner circles represents polygons that are both equilateral and equiangular. This is the definition of a regular polygon.

The Venn diagram at right can visually represent the relationships among these figures.



2.1

Inductive Reasoning

In this lesson you will

- Learn how **inductive reasoning** is used in science and mathematics
- Use inductive reasoning to make **conjectures** about sequences of numbers and shapes

Inductive reasoning is the process of observing data, recognizing patterns, and making generalizations based on those patterns. You probably use inductive reasoning all the time without realizing it. For example, suppose your history teacher likes to give “surprise” quizzes. You notice that, for the first four chapters of the book, she gave a quiz the day after she covered the third lesson. Based on the pattern in your observations, you might generalize that you will have a quiz after the third lesson of every chapter. A generalization based on inductive reasoning is called a **conjecture**.

Example A in your book gives an example of how inductive reasoning is used in science. Here is another example.

EXAMPLE A

In physics class, Dante’s group dropped a ball from different heights and measured the height of the first bounce. They recorded their results in this table.

Drop height (cm)	120	100	160	40	200	80
First-bounce height (cm)	90	74	122	30	152	59

Make a conjecture based on their findings. Then predict the first-bounce height for a drop height of 280 cm.

► **Solution**

If you divide each first-bounce height by the corresponding drop height, you get the following results: 0.75, 0.74, 0.7625, 0.75, 0.76, 0.7375. Based on these results, you could make this conjecture: “For this ball, the height of the first bounce will always be about 75% of the drop height.”

Based on this conjecture, the first-bounce height for a drop height of 280 cm would be about $280 \cdot 0.75$, or 210 cm.

Example B in your book illustrates how inductive reasoning can be used to make a conjecture about a number sequence. Here is another example.

EXAMPLE B

Consider the sequence

10, 7, 9, 6, 8, 5, 7, . . .

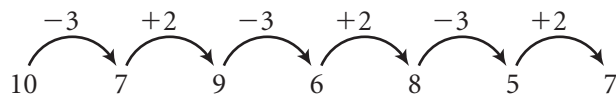
Make a conjecture about the rule for generating the sequence. Then find the next three terms.

(continued)

Lesson 2.1 • Inductive Reasoning (continued)

► **Solution**

Look at how the numbers change from term to term.



The 1st term in the sequence is 10. You subtract 3 to get the 2nd term. Then you add 2 to get the 3rd term. You continue alternating between subtracting 3 and adding 2 to generate the remaining terms. The next three terms are 4, 6, and 3.

In the investigation you look at a pattern in a sequence of shapes.

Investigation: Shape Shifters

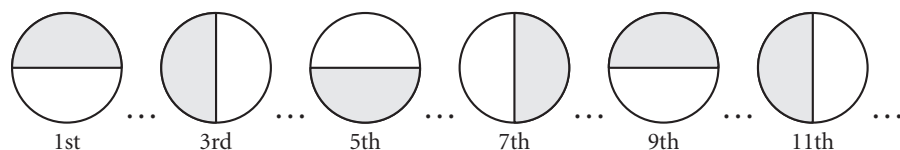
Look at the sequence of shapes in the investigation in your book. Complete each step of the investigation. Below are hints for each step if you need them.

Step 1 Are the shapes the same or different? How does the shaded portion of the shape change from one odd-numbered shape to the next?

Step 2 First, focus on the polygon shape. Does the polygon change from one even-numbered shape to the next? If so, how does it change? Second, focus on the small circles inside the shape. How do these circles change from one even-numbered shape to the next?

Step 3 The next shape is the 7th shape. Because it is an odd-numbered shape, use the patterns you described in Step 1 to figure out what it will look like. The 8th shape is an even-numbered shape, so it should follow the patterns you described in Step 2.

Step 4 Notice that the odd-numbered shapes go through a cycle that repeats every eight terms. So, for example, the 1st, 9th, and 17th shapes look the same; the 3rd, 11th, and 19th shapes look the same; and so on. Use this idea to figure out what the 25th shape looks like.



Step 5 How many sides does the 2nd shape have? The 4th shape? The 6th shape? The n th shape? How many sides will the 30th shape have? How will the dots be arranged on the 30th shape?

Read the text following the investigation in your book.

2.2

Finding the n th Term

In this lesson you will

- Learn how to write **function rules** for number sequences with a constant difference
- Write a rule to describe a geometric pattern
- Learn why a rule for a sequence with a constant difference is called a **linear function**

Consider the sequence 20, 27, 34, 41, 48, 55, 62, Notice that the difference between any two consecutive terms is 7. We say that this sequence has a *constant difference* of 7. To find the next two terms in the sequence, you could add 7 to the last term to get 69, and then add 7 to 69 to get 76. But what if you wanted to find the 200th term? It would take a long time to list all the terms. If you could find a rule for calculating the n th term of the sequence for any number n , you could find the 200th term without having to list all the terms before it. This rule is called the **function rule**. In the investigation you will learn a method for writing a rule for any sequence that has a constant difference.

Investigation: Finding the Rule

Copy and complete each table in Step 1 of the investigation. Then find the difference between consecutive values. If the difference is constant, look for a connection between the difference and the rule.

Here is the completed table for part c. Notice that the values have a constant difference of -2 , which is equal to the coefficient of n in the rule $-2n + 5$.

n	1	2	3	4	5	6	7	8
$-2n + 5$	3	1	-1	-3	-5	-7	-9	-11

For each table, you should have found a constant difference and observed that the constant difference is equal to the coefficient of n in the rule. If you didn't, go back and check your work. In general, if the difference between consecutive terms in a sequence is a constant value a , then the coefficient of n in the rule is a .

Now return to the sequence 20, 27, 34, 41, 48, 55, 62, . . . from the beginning of the lesson. You can organize the terms in a table.

Term n	1	2	3	4	5	6	7	...	200
Value $f(n)$	20	27	34	41	48	55	62	...	

The constant difference for this sequence is 7, so you know that part of the rule is $7n$. The value of the first term ($n = 1$) of the sequence is 20. Substituting 1 for n in $7n$ gives $7(1) = 7$. To get 20, you need to add 13. So, the rule is $7n + 13$.

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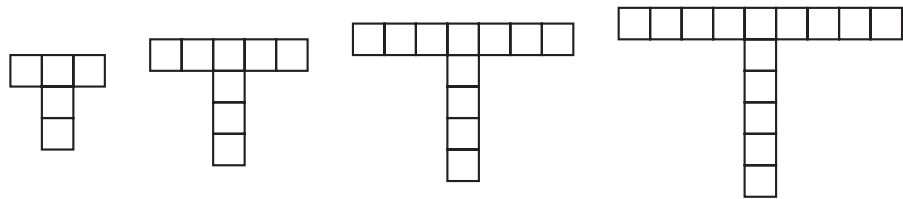
Lesson 2.2 • Finding the *n*th Term (continued)

Check this rule by trying it for other terms of the sequence. For example, when $n = 4$, $7n + 13 = 28 + 13 = 41$, which is the 4th term in the sequence.

You can use the rule to find the 200th term in the sequence. The 200th term is $7(200) + 13$, or 1413.

To get more practice writing rules for patterns, work through Examples A and B in your book. Below is another example.

EXAMPLE If the pattern of T-shapes continues, how many squares will be in the 100th T-shape?



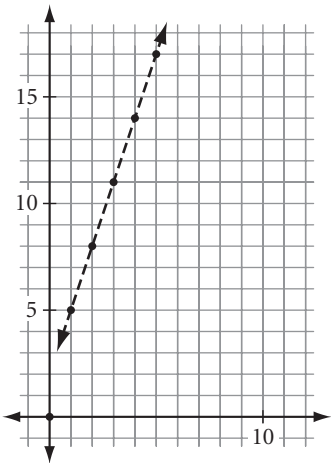
► **Solution** Make a table showing the number of squares in each of the T-shapes shown.

T-shape	1	2	3	4	...	n	...	100
Number of squares	5	8	11	14	

Now try to find a rule for the number of squares in the n th T-shape. Because the constant difference is 3, the rule is of the form $3n + c$. Because the number of squares in the first shape ($n = 1$) is 5, $c = 2$. The rule is $3n + 2$. Therefore, there are $3(100) + 2$, or 302, squares in the 100th T-shape.

In the T-shape example, the process of looking at patterns and generalizing a rule for the n th shape is inductive reasoning. The rule found in this example is called a **linear function**. A linear function is a rule that generates a sequence with a constant difference.

To see why the rule in the T-shape example is a linear function, you can graph the value for the sequence as ordered pairs of the form (*term number*, *value*) on the coordinate plane. The points on your graph will be collinear, because each time the term number increases by 1, the value goes up by the constant difference 3. The line $y = 3x + 2$ passes through the points.



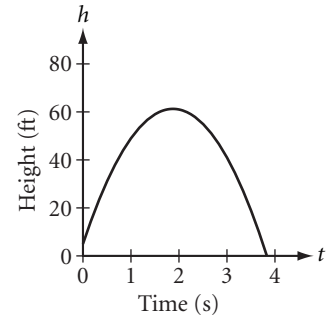
2.3

Mathematical Modeling

In this lesson you will

- Attempt to solve a problem by **acting it out**
- Create a **mathematical model** for a problem
- Learn about **triangular numbers** and the formula for generating them

When you represent a situation with a graph, diagram, or equation, you are creating a **mathematical model**. Suppose you throw a ball straight up into the air with an initial velocity of 60 ft/s. You may recall from algebra that if you release the ball from a height of 5 ft, then the height h of the ball after t seconds can be modeled with the equation and graph shown at right. Once you have created a model, you can use it to make predictions. For example, you could use the equation or graph to predict the height of the ball after 2 seconds or to predict when the ball will hit the ground.



$$h = -16t^2 + 60t + 5$$

In the investigation you will solve a problem by creating mathematical models.

Investigation: Party Handshakes

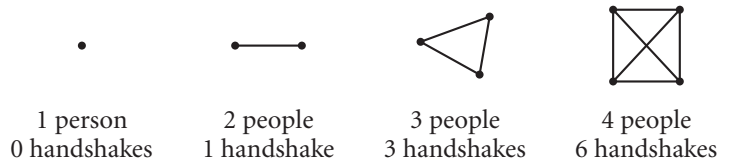
If each of the 30 people at a party shook hands with everyone else, how many handshakes were there altogether?

If you can gather a group of four people, act out the problem for “parties” of one, two, three, and four people and record your results in a table. Your table should look like this.

People	1	2	3	4	...	30
Handshakes	0	1	3	6	...	

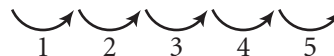
Can you generalize from your table to find the number of handshakes for 30 people? It would certainly help if you had more data. However, gathering many people to act out the problem is not very practical. You could instead try using a mathematical model.

Model the problem by using points to represent people and line segments connecting the points to represent handshakes.



Record your results in a table. This table gives the results for up to six people, but you may want to find results for larger groups of people to help you find a pattern.

Number of points (people)	1	2	3	4	5	6	...	n	...	30
Number of segments (handshakes)	0	1	3	6	10	15	



Notice that the pattern does not have a constant difference, so the rule is not a linear function.

(continued)

Lesson 2.3 • Mathematical Modeling (continued)

Read the dialogue between Erin and Stephanie in your book. According to Erin and Stephanie’s line of reasoning, the diagram with 5 points should have 4 segments per point, so the total number of segments should be $\frac{5 \cdot 4}{2}$, or 10. This matches the data in the table.

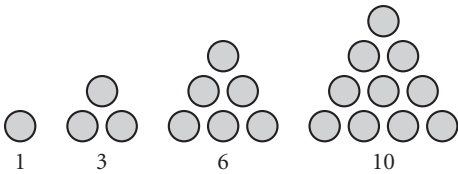
Copy and complete the table given in Step 6 in your book. Make sure you can answer these questions about the expressions for the number of handshakes.

- What does the larger of the two factors in each numerator represent?
- What does the smaller factor represent?
- Why is the product of the factors divided by 2?

You should find that the rule $\frac{n(n-1)}{2}$ models the number of handshakes for a group of n people. So, for 30 people, there would be $\frac{30 \cdot 29}{2}$, or 435, handshakes.

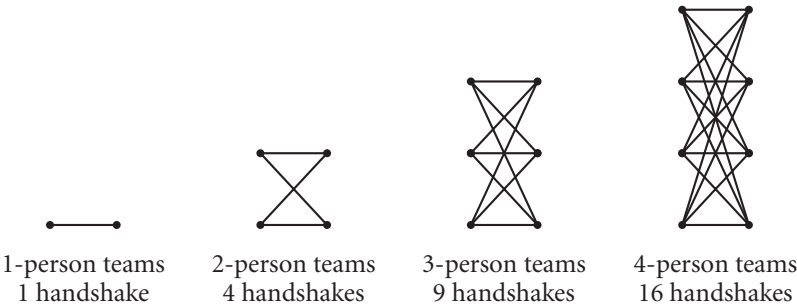
The numbers in the pattern in the investigation are called **triangular numbers** because you can represent them with a triangular pattern of points.

Read the text after the investigation in your book, which shows how you can derive a formula for the triangular numbers using **rectangular numbers**. As expected, the formula is the same one you found in the investigation. Here is another example of a handshake problem.



EXAMPLE Before a soccer game, each of the 11 players on one team shook hands with each player on the other team. How many handshakes were there?

► **Solution** Draw diagrams to represent this situation for teams of one, two, three, and four players, and record the results in a table. (Keep in mind that the players do *not* shake hands with members of their own team.)



Players per team	1	2	3	4	...	n	...	11
Number of handshakes	1	4	9	16	

Do you see a pattern? In each case, the number of handshakes is the square of the number of people on each team. The rule for the number of handshakes between two n -player teams is n^2 . So, for the 11-player soccer teams, there were 11^2 , or 121, handshakes.

2.4

Deductive Reasoning

In this lesson you will

- Learn about **deductive reasoning**
- Use deductive reasoning to justify the steps in the solution of an equation
- Use a deductive argument to explain why a geometric conjecture is true

In Lessons 2.1–2.3, you used inductive reasoning to make conjectures based on observed patterns. To explain *why* a conjecture is true, you need to use *deductive* reasoning. **Deductive reasoning** is the process of showing that certain statements follow logically from accepted facts. Read about deductive reasoning on page 114 of your book.

When you give a reason for each step in the process of solving an equation, you are using deductive reasoning. Example A in your book shows the steps involved in solving a particular algebraic equation. Here is another example.

EXAMPLE Solve the equation for x . Give a reason for each step in the solution process.

$$5x^2 + 19x - 45 = 5x(x + 2)$$

► **Solution**

$$5x^2 + 19x - 45 = 5x(x + 2) \quad \text{The original equation.}$$

$$5x^2 + 19x - 45 = 5x^2 + 10x \quad \text{Distribute.}$$

$$19x - 45 = 10x \quad \text{Subtract } 5x^2 \text{ from both sides.}$$

$$-45 = -9x \quad \text{Subtract } 19x \text{ from both sides.}$$

$$5 = x \quad \text{Divide both sides by } -9.$$

Read Example B in your book carefully. It shows three examples of a ray bisecting an obtuse angle. In each case, the two newly formed congruent angles are acute. From these examples, *inductive reasoning* is used to form the following conjecture.

If an obtuse angle is bisected, then the two newly formed congruent angles are acute.

Once the conjecture is stated, a *deductive argument* is used to show it is true. Notice that, by using a variable, m , to represent the measure of an obtuse angle, the argument shows that the conjecture is true for *any* obtuse angle. As another example of using a deductive argument to explain why a conjecture is true, consider the three obtuse angles in Example B of your book with their bisector, \overrightarrow{AC} . This time make a conjecture which states the minimum angle measure for each of the newly formed congruent angles. Looking again at the measures of the new angles— 60° , 79° , and 46° —you notice that each is larger than 45° . So, one possible conjecture is stated here.

Conjecture: If an obtuse angle is bisected, then the two newly formed congruent angles have measures greater than 45° .

(continued)

Lesson 2.4 • Deductive Reasoning (continued)

Complete the following deductive argument to show that this conjecture is true.

Deductive Argument: Let a be the measure of an obtuse angle.

$$a > \underline{\hspace{2cm}}$$

When an angle is bisected, the newly formed angles each measure half the original angle.

$$\frac{1}{2}a > \underline{\hspace{2cm}}$$

Therefore, the newly formed congruent angles have measures greater than 45° .

In the investigation you will use inductive reasoning to form a conjecture and deductive reasoning to explain why it is true.

Investigation: Overlapping Segments

Look at the two diagrams at the beginning of the investigation. In each diagram, $\overline{AB} \cong \overline{CD}$.

For each diagram, find the lengths of \overline{AC} and \overline{BD} . What do you notice? You should find that in each case $\overline{AC} \cong \overline{BD}$.

Now draw your own segment AD . Place points B and C on the segment so that $\overline{AB} \cong \overline{CD}$ and B is closer to point A than to point D . Measure \overline{AC} and \overline{BD} . You should find that, as in the diagrams in the investigation, $\overline{AC} \cong \overline{BD}$.

Use your findings to complete the conjecture that is started in the book in Step 4. Your completed conjecture should be similar to this one:

If \overline{AD} has points A , B , C , and D in that order with $\overline{AB} \cong \overline{CD}$, then the overlapping segments AC and BD are congruent (that is, $\overline{AC} \cong \overline{BD}$).

This conjecture is known as the overlapping segments property. Now write a deductive argument to explain why the conjecture is true. (Hint: Use the facts that \overline{BC} is a part of both \overline{AC} and \overline{BD} and that the other parts of \overline{AC} and \overline{BD} —namely, \overline{AB} and \overline{CD} —are congruent.) After you have written your own deductive argument, compare it to the one below.

Deductive Argument: Because $\overline{AB} \cong \overline{CD}$, you know that

$$AB = CD$$

You can add the same thing to both sides of an equation and the equation remains true.

$$AB + BC = BC + CD$$

Using segment addition, $AB + BC = AC$ and $BC + CD = BD$. By substituting AC for $AB + BC$ and BD for $BC + CD$ in the equation above, you get the following:

$$AC = BD$$

By the definition of congruent segments, $\overline{AC} \cong \overline{BD}$.

Read the text following the investigation in your book.

2.5

Angle Relationships

In this lesson you will

- Make a conjecture about angles that form a **linear pair**
- Make and prove a conjecture about pairs of **vertical angles**
- Write the **converse** of an “if-then” statement and determine whether it is true

In this lesson you will use inductive reasoning to discover some geometric relationships involving angles.

Investigation 1: The Linear Pair Conjecture

Repeat Step 1 of Investigation 1 three times, creating three different pairs of linear angles.

You should find that, for each pair of linear angles, the sum of the angle measures is 180° . This discovery leads to the following conjecture.

Linear Pair Conjecture If two angles form a linear pair, then the measures of the angles add up to 180° .

C-1

Keep a list of important conjectures in your notebook. Make a sketch for each conjecture. The Linear Pair Conjecture (C-1) should be the first entry in your list.

Investigation 2: Vertical Angles Conjecture

Follow Steps 1–3 in your book. You should find that the vertical angles are congruent. That is, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.

Now draw a different pair of intersecting lines on patty paper and repeat Steps 2 and 3. Are the vertical angles congruent?

Fill in the conjecture using your work in this investigation.

Vertical Angles Conjecture If two angles are vertical angles, then they are _____.

C-2

You used inductive reasoning to discover the Linear Pair Conjecture and the Vertical Angles Conjecture. The example in your book shows that, if you accept that the Linear Pair Conjecture is true, you can use deductive reasoning to show that the Vertical Angles Conjecture must also be true. Read the example very carefully and make sure you understand each step of the deductive argument.

(continued)

Lesson 2.5 • Angle Relationships (continued)

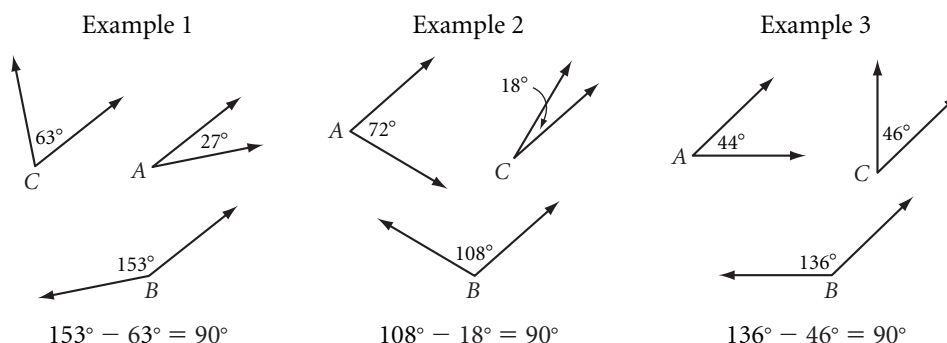
Here is another example to help you write deductive arguments.

EXAMPLE

- Use inductive reasoning to complete this conjecture.
If $\angle B$ is the supplement of an acute angle A and $\angle C$ is the complement of $\angle A$, then $m\angle B - m\angle C = \underline{\hspace{2cm}}$.
- Write a deductive argument to show why the conjecture is true.

► Solution

- The diagrams below show three examples.



In each case, $m\angle B - m\angle C = 90^\circ$. Based on these examples, the conjecture would be

If $\angle B$ is the supplement of an acute angle A and $\angle C$ is the complement of $\angle A$, then $m\angle B - m\angle C = 90^\circ$.

Deductive Argument: The measures of supplementary angles add up to 180° .

$$m\angle A + m\angle B = 180^\circ$$

The measures of complementary angles add up to 90° .

$$m\angle A + m\angle C = 90^\circ$$

Solve the second equation for $m\angle A$.

$$m\angle A = 90^\circ - m\angle C$$

Substitute $90^\circ - m\angle C$ for $m\angle A$ in the first equation.

$$90^\circ - m\angle C + m\angle B = 180^\circ$$

Subtract 90° from both sides of the equation.

$$-m\angle C + m\angle B = 90^\circ$$

Rewrite the left side of the equation to complete the deductive argument.

$$m\angle B - m\angle C = 90^\circ$$

The Vertical Angles Conjecture states that *if* two angles are vertical angles, *then* they are congruent. When you reverse the “if” and “then” parts of an “if-then” statement, you get the **converse** of the statement. Here is the converse of the Vertical Angles Conjecture.

If two angles are congruent, *then* they are vertical angles.

Is this statement true? The diagram on page 124 of your book shows a *counterexample* to this statement. If you can find even one counterexample, then the statement is false. So, the converse of the Vertical Angles Conjecture is false.

2.6

Special Angles on Parallel Lines

In this lesson you will

- Make three conjectures about the angles formed when two parallel lines are intersected by a **transversal**
- Determine whether the converse of each conjecture is true
- Prove one of the conjectures assuming one of the other conjectures is true

A line that intersects two or more coplanar lines is called a **transversal**. In your book, read about the three types of angle pairs formed when a transversal intersects two lines. In the investigation you will look at the angles formed when a transversal intersects two *parallel* lines.

Investigation 1: Which Angles Are Congruent?

Follow the instructions before Step 1 to create parallel lines k and ℓ intersected by transversal m . Number the angles as shown.

Place a piece of patty paper over the set of angles 1, 2, 3, and 4. Copy the two intersecting lines m and k and the four angles onto the patty paper.

Angles 1 and 5 are corresponding angles. Place the tracing of $\angle 1$ over $\angle 5$. How do the angles compare? Repeat this process for the other pairs of corresponding angles ($\angle 3$ and $\angle 7$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$). Use your findings to complete this conjecture.

Corresponding Angles Conjecture, or CA Conjecture If two parallel lines are cut by a transversal, then corresponding angles are _____.

C-3a

Angles 3 and 6 are alternate interior angles. Place the tracing of $\angle 3$ over $\angle 6$. How do the angles compare? Repeat this process for the other pair of alternate interior angles ($\angle 4$ and $\angle 5$), and then complete this conjecture.

Alternate Interior Angles Conjecture, or AIA Conjecture If two parallel lines are cut by a transversal, then alternate interior angles are _____.

C-3b

Angles 1 and 8 are alternate exterior angles. Place the tracing of $\angle 1$ over $\angle 8$. How do the angles compare? Repeat this process for the other pair of alternate exterior angles ($\angle 2$ and $\angle 7$), and then complete this conjecture.

Alternate Exterior Angles Conjecture, or AEA Conjecture If two parallel lines are cut by a transversal, then alternate exterior angles are _____.

C-3c

The three conjectures above can be combined to create the Parallel Lines Conjecture.

Parallel Lines Conjecture If two parallel lines are cut by a transversal, then corresponding angles are _____, alternate interior angles are _____, and alternate exterior angles are _____.

C-3

(continued)

Lesson 2.6 • Special Angles on Parallel Lines (continued)

Now draw two lines that are *not* parallel and a transversal cutting both lines. Use the same process you used above to compare corresponding angles, alternate interior angles, and alternate exterior angles. Do the conjectures work for nonparallel lines?

Investigation 2: Is the Converse True?

In this investigation you'll consider the converse of the Corresponding Angles Conjecture: If two lines are cut by a transversal to form a pair of congruent corresponding angles, then the lines are parallel. Do you think this statement is true? Investigate by following Step 1 in your book.

Now write the converse of each of the other two conjectures. Do you think the converses are true? Investigate by following Step 2 in your book. Then complete the following conjecture.

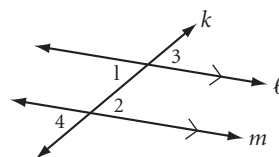
Converse of the Parallel Lines Conjecture If two lines are cut by a transversal to form a pair of congruent corresponding angles, congruent alternate interior angles, or congruent alternate exterior angles, then the lines are _____.

C-4

If you accept any of the three parallel lines conjectures as true, you can use deductive reasoning (and possibly some earlier conjectures) to show that the others are true. The example in your book shows that if you accept the Corresponding Angles and Vertical Angles Conjectures as true, then you can prove that the Alternate Interior Angles Conjecture is true. Read the example carefully. Here is another example.

EXAMPLE

Suppose you assume that the Alternate Interior Angles Conjecture is true. Write a deductive argument showing that the Alternate Exterior Angles Conjecture must be true. (You may assume that the Vertical Angles Conjecture is true.)



► Solution

Deductive Argument: In the diagram, lines ℓ and m are parallel and intersected by transversal k . Because we assume that the alternate interior angles are congruent

$$m\angle 1 = m\angle 2$$

Because we assume the Vertical Angles Conjecture is true

$$m\angle 1 = m\angle 3 \text{ and } m\angle 2 = m\angle 4$$

Substitute $m\angle 3$ for $m\angle 1$ and $m\angle 4$ for $m\angle 2$ in the first equation.

$$m\angle 3 = m\angle 4$$

So, the alternate exterior angles, $\angle 3$ and $\angle 4$, are congruent.

Therefore, if the alternate interior angles are congruent, then the alternate exterior angles are also congruent.

3.1

Duplicating Segments and Angles

In this lesson you will

- Learn what it means to create a **geometric construction**
- **Duplicate a segment** by using a straightedge and a compass and by using patty paper and a straightedge
- **Duplicate an angle** by using a straightedge and a compass and by using patty paper and a straightedge

In geometry, there are several methods for creating a figure.

- You can *sketch* a figure without using geometry tools. Make a sketch when exact measurements are not important.
- You can *draw* a figure using measuring tools, such as a protractor and a ruler. Make a drawing when it is important for lengths and angle measures to be precise.
- You can *construct* a figure using a compass and straightedge. When you make a construction, do *not* use your measuring tools. Compass-and-straightedge constructions allow you to accurately draw congruent segments and angles, segment and angle bisectors, and parallel and perpendicular lines.
- You can also *construct* a figure using patty paper and a straightedge. As with compass-and-straightedge constructions, patty-paper constructions do not use measuring tools.

In this lesson you will focus on constructions. You can read about the history of constructions in the lesson introduction in your book.

Investigation 1: Duplicating a Segment

In this investigation you will copy this segment using only a compass and straightedge. This is called *duplicating* a segment. When you construct a figure, you may use a ruler as a straightedge, but *not* as a measuring tool.



Draw a ray on your paper that extends longer than \overline{AB} . Label the endpoint of the ray point C. Now think about how you can use *only* your compass to create a segment, \overline{CD} , that is the same length as \overline{AB} . Try constructing \overline{CD} on your own before looking at Step 1 of the investigation in your book. You can use a ruler to check that $\overline{AB} \cong \overline{CD}$.

Step 1 shows the three stages involved in duplicating segment AB. The stages are described below.

Stage 1: Draw a ray that extends longer than \overline{AB} and label the endpoint C.

Stage 2: Put the sharp end of your compass on point A. Open the compass until the other end touches point B, and make an arc.

Stage 3: *Without changing the opening of your compass*, put the sharp end of your compass on point C and make an arc on the ray. Label the point where the arc intersects the ray point D. Segment CD is congruent to segment AB.

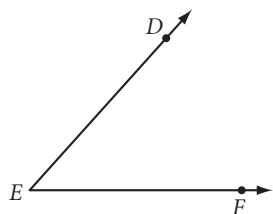
(continued)

Lesson 3.1 • Duplicating Segments and Angles (continued)

To duplicate \overline{AB} using patty paper, simply place the patty paper over the segment and trace it, using a straightedge to ensure the tracing is straight.

Investigation 2: Duplicating an Angle

In this investigation you will copy this angle using a compass and straightedge.



Draw a ray that extends longer than a side of $\angle DEF$. Label the endpoint of the ray G . This ray will be one side of the duplicate angle. Try to figure out how to duplicate $\angle DEF$ on your own before looking at Step 1 in your book. You can use a protractor to check that the angles are congruent.

Step 1 shows the first two stages involved in duplicating $\angle DEF$. The stages are described below.

Stage 1: Use your compass to construct an arc with its center at point E . The arc should intersect both sides of the angle. *Without changing the opening of your compass*, make an arc centered at point G .

Stage 2: On $\angle DEF$, put the sharp end of your compass on the point where the arc intersects \overline{EF} . Adjust the opening so that the other end touches the point where the arc intersects \overline{ED} , and make an arc. *Without changing the opening of your compass*, put the sharp end of your compass on the point where the arc intersects the ray with endpoint G , and make an arc that intersects the original arc.

To finish the construction, draw a ray from point G through the point where the two arcs intersect. Use a protractor to verify that $\angle G$ is congruent to $\angle DEF$.

Practice duplicating other angles until you are sure you understand the steps. Be sure to try duplicating obtuse angles as well as acute angles.

Now try to duplicate $\angle DEF$ using patty paper instead of a compass.

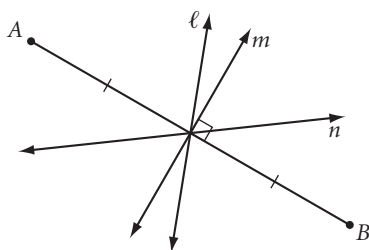
Write a summary of the construction methods you learned in this lesson.

Constructing Perpendicular Bisectors

In this lesson you will

- Construct the **perpendicular bisector** of a segment using patty paper and a straightedge, and using a compass and straightedge
- Complete the **Perpendicular Bisector Conjecture**
- Learn about **medians** and **midsegments** of triangles

A **segment bisector** is a line, ray, or segment that passes through the midpoint of the segment. A line that passes through the midpoint of a segment and that is perpendicular to the segment is the **perpendicular bisector** of the segment. A segment has an infinite number of bisectors, but in a plane it has *only one* perpendicular bisector.



Lines ℓ , m , and n bisect \overline{AB} .
Line m is the perpendicular bisector of \overline{AB} .

Investigation 1: Finding the Right Bisector

Follow Steps 1–3 in your book to construct a perpendicular bisector of \overline{PQ} using patty paper.

Place three points— A , B , and C —on the perpendicular bisector, and use your compass to compare the distances PA and QA , PB and QB , and PC and QC . In each case, you should find that the distances are equal. These findings lead to the following conjecture.

Perpendicular Bisector Conjecture If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints.

C-5

Is the converse of this statement also true? That is, if a point is equidistant from the endpoints of a segment, is it on the segment's perpendicular bisector? If the converse is true, then locating two such points can help you locate the perpendicular bisector.

Investigation 2: Right Down the Middle

In this investigation you will use a compass and straightedge to construct the perpendicular bisector of a segment. First, draw a line segment. Then follow the steps below.

Adjust your compass so that the opening is more than half the length of the segment. Using one endpoint as the center, make an arc on one side of the segment.



(continued)

Lesson 3.2 • Constructing Perpendicular Bisectors (continued)

Without changing the opening of your compass, put the sharp end of your compass on the other endpoint and make an arc intersecting the first arc.

The point where the arcs intersect is equidistant from the two endpoints. Follow the same steps to locate another such point on the other side of the segment. Then draw a line through the two points.

The line you drew is the perpendicular bisector of the segment. You can check this by folding the segment so that the endpoints coincide (as you did in Investigation 1). The line should fall on the crease of the paper.

The construction you did in this investigation demonstrates the conjecture below.

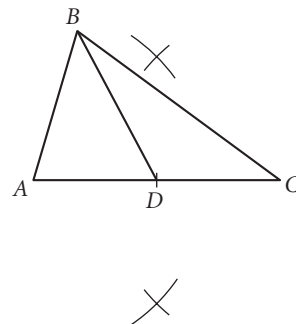
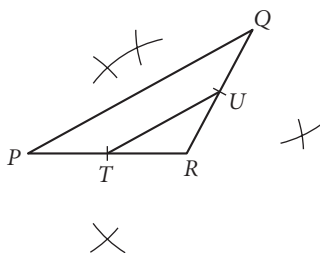
Converse of the Perpendicular Bisector Conjecture If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

C-6

Now that you know how to construct a perpendicular bisector, you can locate the midpoint of any segment. This allows you to construct two special types of segments related to triangles: medians and midsegments.

A **median** is a segment that connects a vertex of a triangle to the midpoint of the opposite side. To construct the median from vertex B , use the perpendicular bisector construction to locate the midpoint of \overline{AC} . Then connect vertex B to this point.

A **midsegment** is a segment that connects the midpoints of two sides of a triangle. To construct a midsegment from \overline{PR} to \overline{QR} , use the perpendicular bisector construction two times to locate midpoints of \overline{PR} and \overline{QR} . Then connect the midpoints.



Write a summary of the construction methods you learned in this lesson.

Constructing Perpendiculars to a Line

In this lesson you will

- Construct the **perpendicular** to a line from a point not on the line
- Complete the **Shortest Distance Conjecture**
- Learn about **altitudes** of triangles

In Lesson 3.2, you learned to construct the perpendicular bisector of a segment.

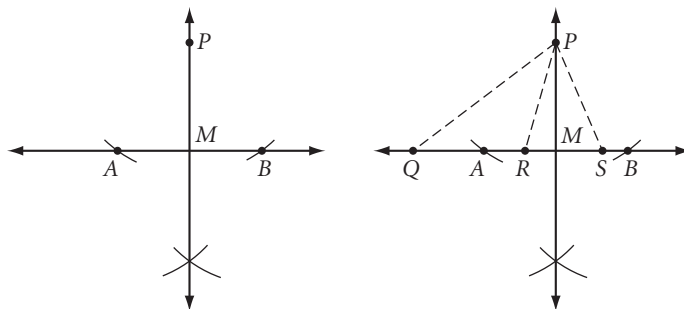
In this lesson you will use what you learned to construct the perpendicular to a line from a point not on the line.

Investigation 1: Finding the Right Line

Draw a line and a point labeled P that is not on the line. With the sharp end of your compass at point P , make two arcs on the line. Label the intersection points A and B .



Note that $PA = PB$, so point P is on the perpendicular bisector of \overline{AB} . Use the construction you learned in Lesson 3.2 to construct the perpendicular bisector of \overline{AB} . Label the midpoint of \overline{AB} point M . You have now constructed a perpendicular to a line from a point not on the line. Now choose any three points on \overline{AB} and label them Q , R , and S . Measure PQ , PR , PS , and PM . Which distance is shortest?



Your observations should lead to this conjecture.

Shortest Distance Conjecture The shortest distance from a point to a line is measured along the perpendicular segment from the point to the line.

C-7

In the next investigation you will use patty paper to create a perpendicular from a point to a line.

(continued)

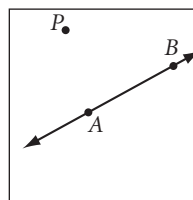
Lesson 3.3 • Constructing Perpendiculars to a Line (continued)

Investigation 2: Patty-Paper Perpendiculars

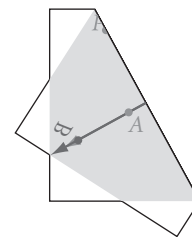
On a piece of patty paper, draw a line \overleftrightarrow{AB} and a point P that is not on \overleftrightarrow{AB} .

Fold the line onto itself. Slide the layers of paper (keeping \overleftrightarrow{AB} aligned with itself) until point P is on the fold.

Crease the paper, open it up, and draw a line on the crease. The line is the perpendicular to \overleftrightarrow{AB} through point P . (Why?)



Step 1



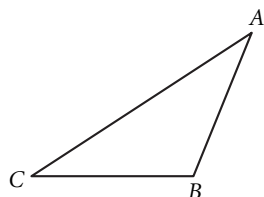
Step 2

Constructing a perpendicular from a point to a line allows you to find the distance from the point to the line, which is defined as follows, “The **distance from a point to a line** is the length of the perpendicular segment from the point to the line.”

The **altitude** of a triangle is a perpendicular segment from a vertex of a triangle to the line containing the opposite side. The length of this segment is the height of the triangle. The illustrations on page 156 of your book show that an altitude can be inside or outside the triangle, or it can be one of the triangle’s sides. A triangle has three different altitudes, so it has three different heights.

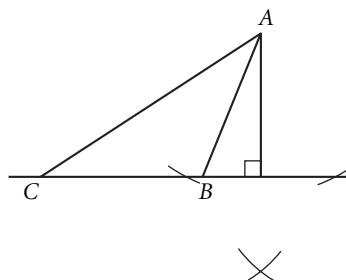
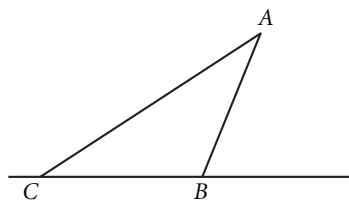
EXAMPLE

Construct the altitude from vertex A of $\triangle ABC$.



► Solution

Extend \overline{CB} to become \overleftrightarrow{CB} and construct a perpendicular segment from point A to \overleftrightarrow{CB} .



Write a summary of the construction methods you learned in this lesson.

3.4

Constructing Angle Bisectors

In this lesson you will

- Construct an **angle bisector** using patty paper and a straightedge, and using a compass and a straightedge
- Complete the **Angle Bisector Conjecture**

An **angle bisector** is a ray that divides an angle into two congruent angles. You can also refer to a segment as an angle bisector if the segment lies on the ray and passes through the angle vertex.

Investigation 1: Angle Bisecting by Folding

Follow Steps 1–3 in your book to construct the bisector of acute $\angle PQR$ using patty paper. You can tell the ray you construct is the angle bisector because the fold forms two angles that coincide.

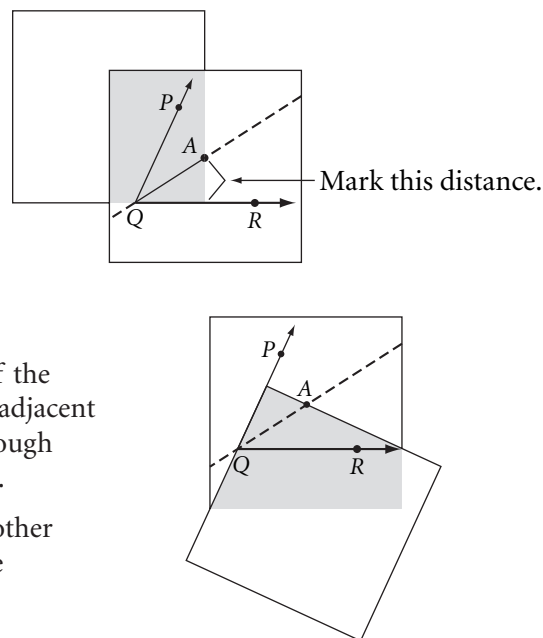
Now construct the bisector of an obtuse angle. Can you use the same method you used to bisect the acute angle?

Does every angle have a bisector? Is it possible for an angle to have more than one bisector? If you are not sure, experiment until you think you know the answers to both of these questions.

Look at the angles you bisected. Do you see a relationship between the points on the angle bisector and the sides of the angle? Choose one of the bisected angles. Choose any point on the bisector and label it A . Compare the distances from A to each of the two sides. (Remember that “distance” means *shortest* distance.) To do this, you can place one edge of a second piece of patty paper on one side of the angle. Slide the edge of the patty paper along the side of the angle until an adjacent perpendicular side of the patty paper passes through the point. Mark this distance on the patty paper.

Compare this distance with the distance to the other side of the angle by repeating the process on the other ray.

Your observations should lead to this conjecture.



Angle Bisector Conjecture If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

C-8

(continued)

Lesson 3.4 • Constructing Angle Bisectors (continued)

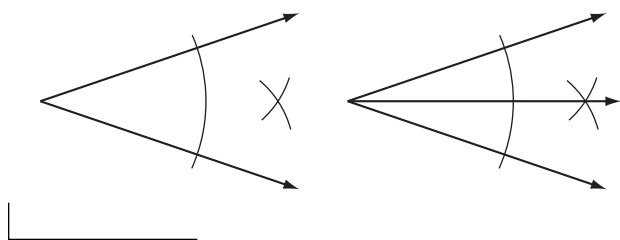
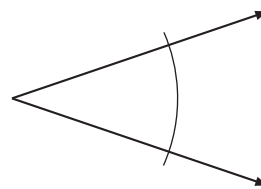
Investigation 2: Angle Bisecting with Compass

You can also construct an angle bisector using a compass and straightedge.

Draw an angle. To start the construction, draw an arc centered at the vertex of the angle that crosses both sides of the angle.

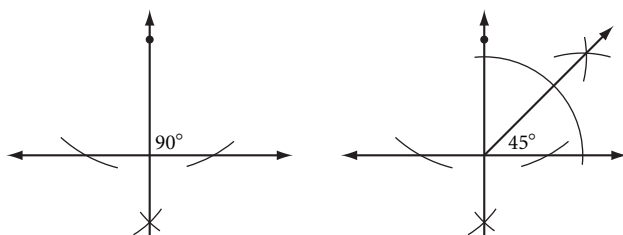
Try to complete the construction on your own before reading the following text. Don't be afraid to experiment. If you make a mistake, you can always start over. When you think you have constructed an angle bisector, fold your paper to check whether the ray you constructed is actually the bisector.

Constructing the angle bisector: Place the sharp end of your compass on one of the points where the arc intersects the angle, and make an arc. *Without changing the opening of your compass*, repeat this process with the other point of intersection. If the two small arcs do not intersect, make your compass opening larger and repeat the last two steps. Draw the ray from the vertex of the angle to the point where the two small arcs intersect.



EXAMPLE | Construct an angle with a measure of exactly 45° using only a compass and straightedge.

► **Solution** | Construct a 90° angle by constructing the perpendicular to a line from a point not on the line. (Look back at Lesson 3.3 if you need to review this construction.) Then use the angle-bisector construction you learned in this lesson to bisect the 90° angle.



Write a summary of the construction methods you learned in this lesson.

3.5

Constructing Parallel Lines

In this lesson you will

- Construct **parallel lines** using patty paper and a straightedge

As you learned in Chapter 1, **parallel lines** are lines that lie in the same plane and do not intersect. So, any two points on one parallel line will be equidistant from the other line. You can use this idea to construct a line parallel to a given line.

Investigation: Constructing Parallel Lines by Folding

Follow Steps 1–3 in your book to construct parallel lines with patty paper. Notice that the pairs of corresponding angles, alternate interior angles, and alternate exterior angles are congruent. (In this case, all are pairs of right angles.)

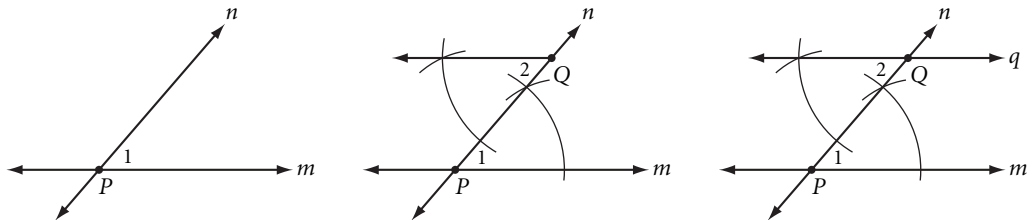
The next example shows another way to construct parallel lines.

EXAMPLE

Use the Converse of the Alternate Interior Angles Conjecture to construct a pair of parallel lines. (Try to do this on your own before reading the solution.)

► Solution

Draw two intersecting lines and label them m and n . Label their point of intersection P and label one of the angles formed $\angle 1$. On line n , label a point Q that is on the same side of line m as $\angle 1$. Using point Q as the vertex and \overrightarrow{QP} as a side, duplicate $\angle 1$ on the opposite side of line n . Label the new angle $\angle 2$. Extend the side of $\angle 2$ into line q .



Notice that line m and line q are cut by a transversal (line n) to form a congruent pair of alternate interior angles ($\angle 1$ and $\angle 2$). According to the converse of the AIA Conjecture, $m \parallel q$.

Now see if you can use the Converse of the Corresponding Angles Conjecture or the Converse of the Alternate Exterior Angles Conjecture to construct a pair of parallel lines.

Write a summary of the construction methods you learned in this lesson.

3.6

Construction Problems

In this lesson you will

- Construct polygons given information about some of the sides and angles

In this chapter you have learned to construct congruent segments and angles, angle and segment bisectors, perpendiculars, perpendicular bisectors, and parallel lines. Once you know these basic constructions, you can create more advanced geometric figures.

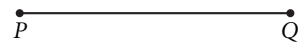
Example A in your book shows you how to construct a triangle if you are given three segments to use as sides. This example also explores an important question: If you are given three segments, how many different-size triangles can you form? Read the example carefully.

Example B shows you how to construct a triangle if you are given three angles. This example shows that three angles do *not* determine a unique triangle. Given three angle measures, you can draw an infinite number of triangles. The triangles will all have the same shape, but they will be different sizes.

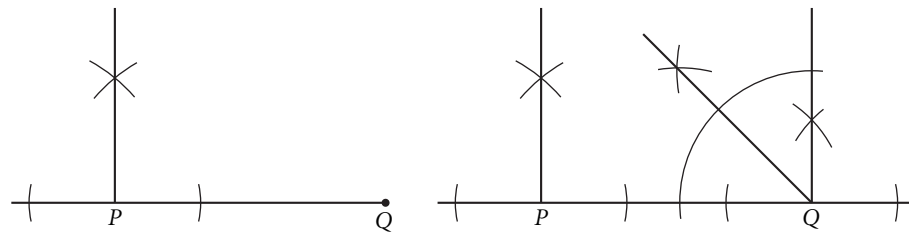
The examples below show some other constructions.

EXAMPLE A

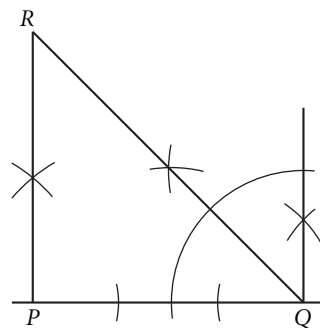
Use a compass and a straightedge to construct $\triangle PQR$ with \overline{PQ} as a side, and with $m\angle P = 90^\circ$ and $m\angle Q = 45^\circ$.

**► Solution**

To construct $\angle P$, extend \overline{PQ} to the left and construct a perpendicular to \overrightarrow{PQ} through point P . To construct $\angle Q$, first construct a perpendicular to \overrightarrow{PQ} through point Q . This creates a right angle with vertex Q . To create a 45° angle, bisect this angle.



To complete the construction, extend the sides of $\angle P$ and $\angle Q$ until they intersect. Label the intersection point R .

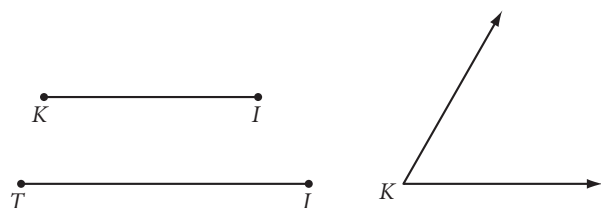


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Lesson 3.6 • Construction Problems (continued)

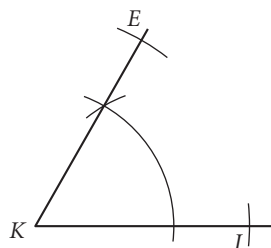
EXAMPLE B

Construct kite $KITE$ with $KI = KE$ and $TI = TE$, using the segments and angle below.

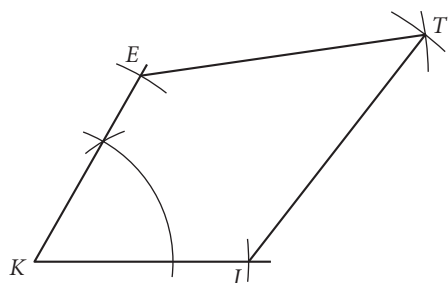


► Solution

Duplicate \overline{KI} and $\angle K$. Because $KI = KE$, duplicate \overline{KI} on the other side of $\angle K$ to create side \overline{KE} .



To locate vertex T , make a large arc with radius TI centered at point I . Vertex T must be on this arc. Because $TI = TE$, draw another large arc with radius TI centered at point E . The intersection of the two arcs is point T . Connect points E and I to point T to complete the kite.



Constructing Points of Concurrence

In this lesson you will

- Construct the **incenter**, **circumcenter**, and **orthocenter** of a triangle
- Make conjectures about the properties of the incenter and circumcenter of a triangle
- **Circumscribe** a circle about a triangle and **inscribe** a circle in a triangle

You can use the constructions you learned in this chapter to construct special segments related to triangles. In this lesson you will construct the angle bisectors and altitudes of a triangle, and the perpendicular bisectors of a triangle's sides. After you construct each set of three segments, you will determine whether they are *concurrent*. Three or more segments, lines, rays, or planes are **concurrent** if they intersect in a single point. The point of intersection is called the **point of concurrency**.

Investigation 1: Concurrence

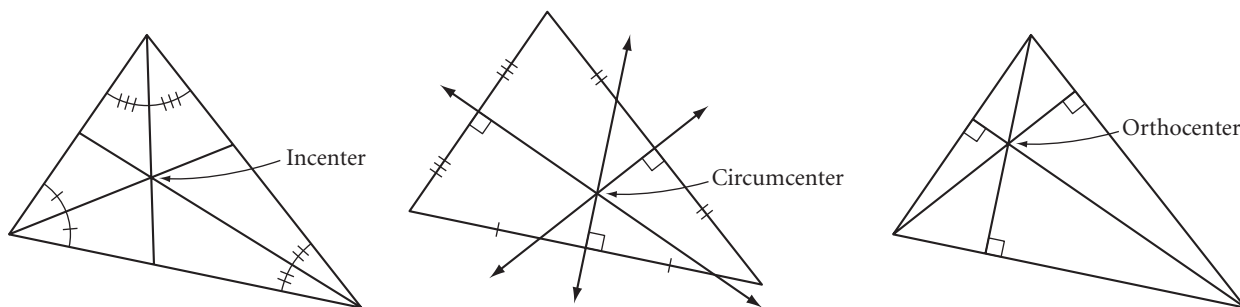
You can perform the constructions in this investigation with patty paper and a straightedge or with a compass and straightedge. Save your constructions to use in Investigation 2.

If you are using patty paper, draw a large acute triangle on one sheet and a large obtuse triangle on another. If you are using a compass, draw the triangles on the top and bottom halves of a piece of paper.

Construct the three angle bisectors of each triangle. You should find that they are concurrent. The point of concurrency is called the **incenter** of the triangle.

Start with two new triangles, one acute and one obtuse, and construct the perpendicular bisector of each side. You should find that, in each triangle, the three perpendicular bisectors are concurrent. The point of concurrency is called the **circumcenter**.

Finally, start with two new triangles and construct the altitude to each side. These segments are also concurrent. The point of concurrency is called the **orthocenter**.



Your observations in this investigation lead to the following conjectures.

Angle Bisector Concurrence Conjecture The three angle bisectors of a triangle are concurrent.

C-9

(continued)

Lesson 3.7 • Constructing Points of Concurrency (continued)

Perpendicular Bisector Concurrency Conjecture The three perpendicular bisectors of a triangle are concurrent. **C-10**

Altitude Concurrency Conjecture The three altitudes (or the lines containing the altitudes) of a triangle are concurrent. **C-11**

For what type of triangle will the incenter, circumcenter, and orthocenter be the same point? If you don't know, experiment with different types of triangles (scalene, isosceles, equilateral, acute, obtuse, right).

Investigation 2: Circumcenter

For this investigation you will need your triangles from Investigation 1 that show the perpendicular bisectors of the sides. For each triangle, measure the distance from the circumcenter to each of the three vertices. Are the distances the same? Now measure the distance from the circumcenter to each of the three sides. Are the distances the same? Use a compass to construct a circle with the circumcenter as center that passes through a vertex of the triangle. What do you notice?

You can state your findings as the Circumcenter Conjecture.

Circumcenter Conjecture The circumcenter of a triangle is equidistant from the three vertices. **C-12**

Investigation 3: Incenter

You will need your triangles from Investigation 1 that show the angle bisectors. For each triangle, measure the distance from the incenter to each of the three vertices. Are the distances the same?

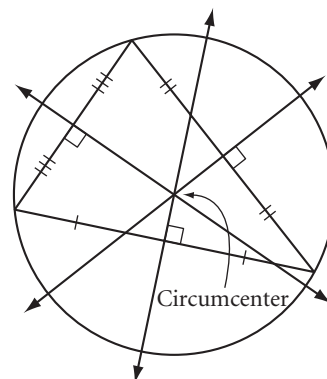
Construct the perpendicular from the incenter to any one of the sides of the triangle. Mark the point of intersection between the perpendicular and the side. Now use a compass to construct a circle with the incenter as center that passes through the point you just marked. What do you notice? What can you conclude about the distance of the incenter from each of the sides?

You can state your findings as the Incenter Conjecture.

Incenter Conjecture The incenter of a triangle is equidistant from the three sides. **C-13**

Read the deductive argument for the Circumcenter Conjecture on pages 180–181 of your book and make sure you understand it.

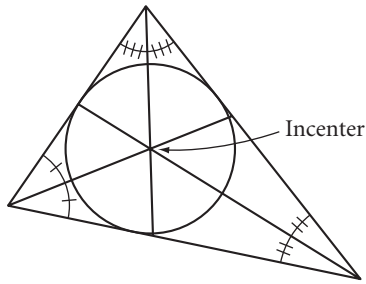
A circle that passes through each vertex of a polygon is **circumscribed** about the polygon. The polygon is **inscribed** in the circle.



(continued)

Lesson 3.7 • Constructing Points of Concurrency (continued)

A circle that is tangent to each side of a polygon is **inscribed** in the polygon. The polygon is **circumscribed** about the circle.

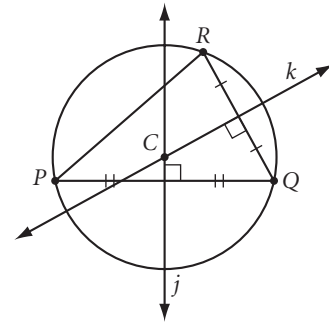


Why does the method for constructing the circumcenter guarantee that it is the center of the circle that is circumscribed about the triangle? Write down your own reasons before you read the logical argument below. You might use the diagram to help you write your argument.

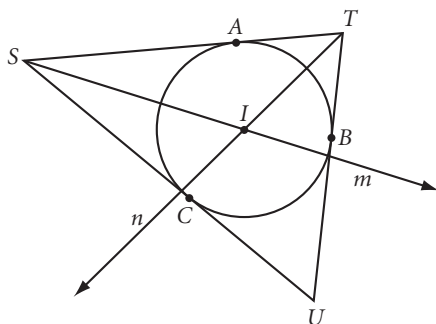
Deductive Argument

A circle is the set of points equidistant from a center point. If a point is located equidistant from the three vertices of a triangle, that point can be used as the center of a circle circumscribed about the triangle.

If the Perpendicular Bisector Conjecture is true, every point on the perpendicular bisector of a segment is equidistant from the two endpoints. So every point on line k is equidistant from points R and Q , and every point on line j is equidistant from points P and Q . Because circumcenter C lies at the intersection of lines j and k , point C must be equidistant from all three vertices, points R , Q , and P . If a circle is constructed with center C that passes through one vertex, for example, point R , it will also pass through the other two because all points on a circle lie the same distance from the center. That circle is the circumscribed circle about $\triangle PQR$.



Use a similar logical argument to explain why the incenter is the center of the inscribed circle. Write down your own logical argument before you read the one on the next page. Use the diagram to help you.



(continued)

Lesson 3.7 • Constructing Points of Concurrency (continued)

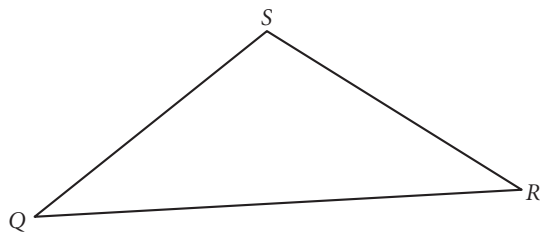
Deductive Argument

A circle is the set of points equidistant from a given center point. If a point is equidistant from the three sides of a triangle, it can be used as the center for an inscribed circle.

If the Angle Bisector Conjecture is true, every point on the angle bisector is equidistant from the two sides of the angle. Therefore, in $\triangle STU$ each point on angle bisector m is equidistant from triangle sides \overline{ST} and \overline{SU} . Similarly, each point on angle bisector n is equidistant from triangle sides \overline{ST} and \overline{TU} . Because incenter I lies on both angle bisectors m and n , it is equidistant from all three sides. Thus, the circle with center at incenter I that is tangent to one of the sides will also be tangent to the other two sides. That circle is the inscribed circle in the triangle.

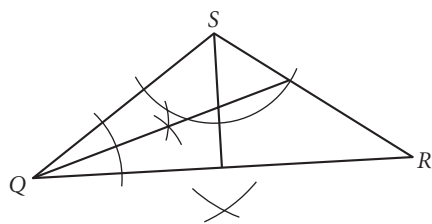
EXAMPLE

Inscribe a circle in $\triangle QRS$.

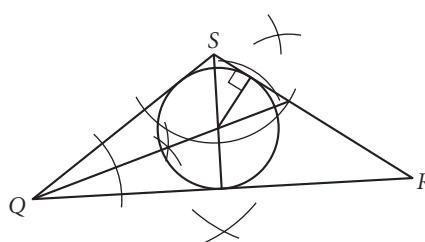
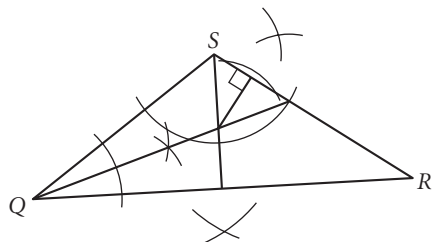


► Solution

To find the center of the circle, construct the incenter. Note that you need to construct only two angle bisectors to locate the incenter. (Why?)



The radius of the circle is the distance from the incenter to each side. To find the radius, construct a perpendicular from the incenter to one of the sides. Here, we construct the perpendicular to \overline{RS} . Now, draw the circle.



3.8

The Centroid

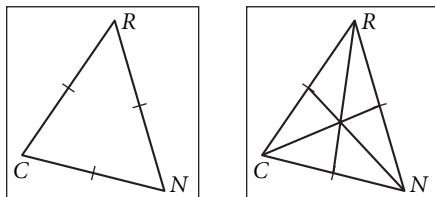
In this lesson you will

- Construct the **centroid** of a triangle
- Make conjectures about the properties of the centroid of a triangle

You have seen that the three angle bisectors, the three perpendicular bisectors of the sides, and the three altitudes of a triangle are concurrent. In this lesson you will look at the three medians of a triangle.

Investigation 1: Are Medians Concurrent?

On a sheet of patty paper, draw a large scalene acute triangle and label it CNR . Locate the midpoints of the three sides and construct the medians. You should find that the medians are concurrent. Save this triangle.



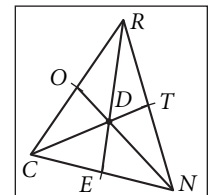
Now start with a scalene obtuse triangle and construct the three medians. Are the medians concurrent? You can state your findings as a conjecture.

Median Concurrency Conjecture The three medians of a triangle are concurrent.

C-14

The point of concurrency of the three medians is the **centroid**. On your acute triangle, label the medians \overline{CT} , \overline{NO} , and \overline{RE} . Label the centroid D .

Use your compass or patty paper to investigate the centroid: Is the centroid equidistant from the three vertices? Is it equidistant from the three sides? Is the centroid the midpoint of each median?



The centroid D divides each median into two segments. For each median, find the ratio of the length of the longer segment to the length of the shorter segment. You should find that, for each median, the ratio is the same. Use your findings to complete this conjecture.

Centroid Conjecture The centroid of a triangle divides each median into two parts so that the distance from the centroid to the vertex is _____ the distance from the centroid to the midpoint of the opposite side.

C-15

In Lesson 3.7, you learned that the circumcenter of a triangle is the center of the circumscribed circle and the incenter is the center of the inscribed circle. In the next investigation you will discover a special property of the centroid.

(continued)

Lesson 3.8 • The Centroid (continued)

Investigation 2: Balancing Act

For this investigation you will need a sheet of cardboard and your scalene acute triangle from Investigation 1.

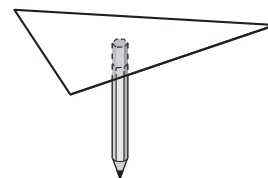
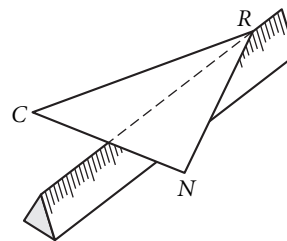
Place your patty-paper triangle on the cardboard. With your compass point, mark the three vertices, the three midpoints, and the centroid on the cardboard. Remove the patty paper and carefully draw the triangle and the medians on the cardboard. Cut out the cardboard triangle.

Try balancing the triangle by placing one of its medians on the edge of a ruler.

You should be able to get the triangle to balance. Repeat the process with each of the other medians. The fact that you can balance the triangle on each median means that each median divides the triangle into two triangular regions of equal area.

Now try to balance the triangle by placing its centroid on the end of a pencil or pen. If you have traced and cut out the triangle carefully, it should balance. Because the triangle balances on its centroid, the centroid is the triangle's **center of gravity**.

You can state your findings as a conjecture.



Center of Gravity Conjecture The centroid of a triangle is the center of gravity of the triangular region.

C-16

Notice that it makes sense that the triangle balances on the centroid because it balances on each median, and the centroid is on each median. As long as the weight of the cardboard is distributed evenly throughout the triangle, you can balance any triangle on its centroid.

4.1

Triangle Sum Conjecture

In this lesson you will

- State a conjecture about the sum of the measures of the angles in a triangle
- Complete a paragraph proof of the **Triangle Sum Conjecture**

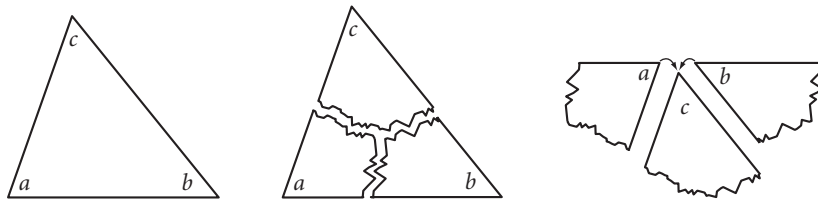
In this chapter you will focus on properties of triangles. To start, you will look at the angle measures of triangles.

Investigation: The Triangle Sum

Draw two large acute triangles with very different shapes and two large obtuse triangles with very different shapes.

For each triangle, measure the three angles as accurately as possible, and then find the sum of the three measures. You should find that the angle sum for each triangle is the same. What is the angle sum?

To check this sum, write the letters a , b , and c in the interiors of the three angles of one of the acute triangles, and carefully cut out the triangle. Then tear off the three angles of the triangle and arrange them so that their vertices meet at a point.



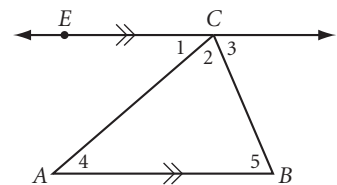
How does the arrangement verify the angle sum you found above?

Your work in this investigation leads to the following conjecture.

Triangle Sum Conjecture The sum of the measures of the angles in every triangle is 180° .

C-17

Next you will write a **paragraph proof** to show why the Triangle Sum Conjecture is true. In your proof, you can use conjectures, definitions, and properties to support your argument. Look at the figure at right. $\triangle ABC$ is any triangle. \overleftrightarrow{EC} is drawn parallel to \overleftrightarrow{AB} . Note: \overleftrightarrow{EC} is an **auxiliary line** (or helping line) because it is an extra line added to the figure to help with the proof. Consider the questions on page 201 of your book.



Copy the diagram and mark angle relationships that might help with your proof. Then use the diagram and your answers to these questions to write a paragraph proof explaining why the Triangle Sum Conjecture is true.

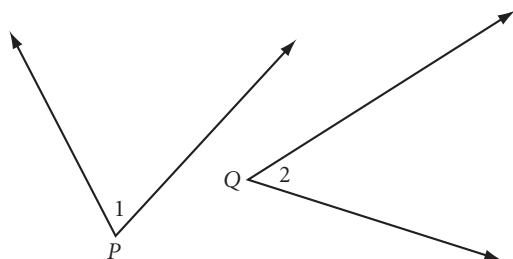
After you have finished, compare your paragraph proof with the one on page 202 of your book.

(continued)

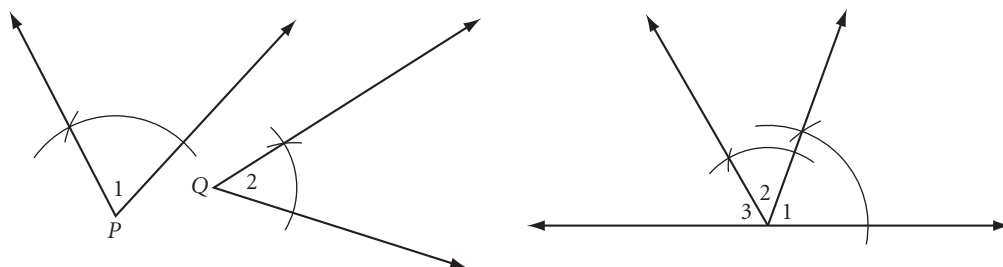
Lesson 4.1 • Triangle Sum Conjecture (continued)

The Triangle Sum Conjecture allows you to construct the third angle of a triangle if you are given two of the angles. Work through the example in your book. The following example shows a slightly different method. Try it to see whether one method is easier. Can you find another method that works?

EXAMPLE A Given $\angle P$ and $\angle Q$ of $\triangle PQR$, construct $\angle R$.

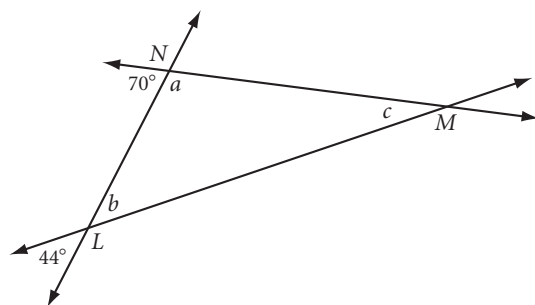


► **Solution** Draw a line and construct $\angle P$ opening to the right on this line. Construct $\angle Q$ so that it shares both the vertex of $\angle P$ and the side of $\angle P$ that is not on the line. The angle labeled 3 on the diagram is $\angle R$, because the sum of the measures of the three angles is 180° .



The following example applies what you have learned in this lesson.

EXAMPLE B Find the lettered angle measures.



► **Solution** The angle labeled a and the 70° angle form a linear pair, so $a + 70^\circ = 180^\circ$. Therefore, $a = 110^\circ$. The angle labeled b and the 44° angle are vertical angles, so $b = 44^\circ$. By the Triangle Sum Conjecture, $110^\circ + 44^\circ + c = 180^\circ$, so $c = 26^\circ$.

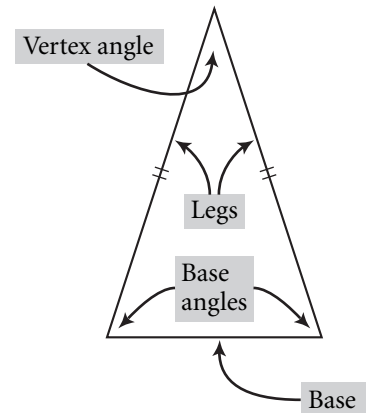
4.2

Properties of Isosceles Triangles

In this lesson you will

- Discover how the angles of an isosceles triangle are related
- Make a conjecture about triangles that have two congruent angles

An *isosceles triangle* is a triangle with at least two congruent sides. The angle between the congruent sides is called the *vertex angle*. The other two angles are called the *base angles*. The side between the base angles is called the *base*. The other two sides are called the *legs*.



Investigation 1: Base Angles in an Isosceles Triangle

Draw an acute angle C on patty paper. Then follow Steps 2 and 3 in your book to construct an isosceles triangle, $\triangle ABC$.

Because \overline{CA} and \overline{CB} are the congruent sides, $\angle C$ is the vertex angle and $\angle A$ and $\angle B$ are base angles. Use your protractor to measure the base angles. How do the measures compare? Confirm your answer by folding your patty paper, keeping \overline{AB} aligned with itself. Is $\angle A \cong \angle B$?

Now draw two more isosceles triangles, one with an obtuse vertex angle and one with a right vertex angle. Compare the base angles in each triangle. Are your findings the same as for the isosceles acute triangle?

Your observations should lead to the following conjecture.

Isosceles Triangle Conjecture If a triangle is isosceles, then its base angles are congruent.

C-18

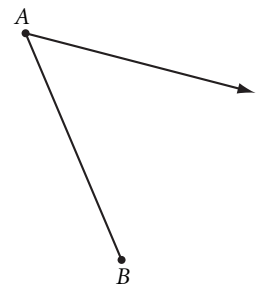
Equilateral triangles are also isosceles triangles, because at least two of their sides are congruent. How do you think the Isosceles Triangle Conjecture applies to equilateral triangles?

As you know, reversing the “if” and “then” parts of a conjecture gives the *converse* of the conjecture. Is the converse of the Isosceles Triangle Conjecture true? In other words, if a triangle has two congruent angles, is it isosceles? To test this statement, you need to draw a triangle with two congruent angles.

Investigation 2: Is the Converse True?

Draw a segment, \overline{AB} , on your paper. Draw an acute angle at point A . In the finished triangle, $\angle A$ and $\angle B$ will be the congruent angles.

Notice that $\angle A$ must be acute. If it were right or obtuse, the sum of the measures of $\angle A$ and $\angle B$ would be greater than or equal to 180° , and, as you know, the sum of all *three* angle measures must be 180° .

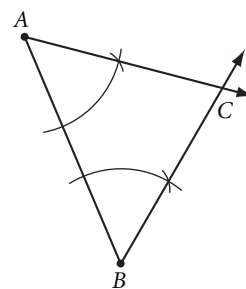


(continued)

Lesson 4.2 • Properties of Isosceles Triangles (continued)

Now copy $\angle A$ at point B on the same side of \overline{AB} . If necessary, extend the sides of the angles until they intersect. Label the point of intersection C .

Use your compass to compare the lengths of sides \overline{AC} and \overline{BC} . Do they appear to be the same length? Check your answer using patty paper. Draw at least one more triangle with two congruent angles, and compare the side lengths. Your findings should provide evidence that the converse of the Isosceles Triangle Conjecture is true.



Converse of the Isosceles Triangle Conjecture If a triangle has two congruent angles, then it is an isosceles triangle.

C-19

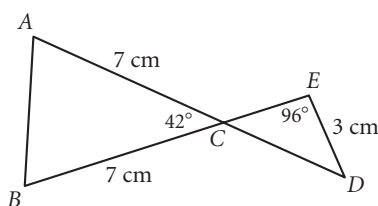
The following example gives you practice applying what you have learned.

EXAMPLE

$$m\angle A = \underline{\hspace{2cm}}$$

$$m\angle D = \underline{\hspace{2cm}}$$

$$EC = \underline{\hspace{2cm}}$$



► Solution

By the Triangle Sum Conjecture, $m\angle A + m\angle B + 42^\circ = 180^\circ$, so $m\angle A + m\angle B = 138^\circ$. Because $\angle A$ and $\angle B$ are the base angles of an isosceles triangle, they are congruent. So, $m\angle A = \frac{1}{2}(138^\circ) = 69^\circ$.

Because $\angle ACB$ and $\angle ECD$ are vertical angles, they are congruent. So, $m\angle ECD = 42^\circ$. By the Triangle Sum Conjecture, $42^\circ + 96^\circ + m\angle D = 180^\circ$. Solving for $m\angle D$ gives $m\angle D = 42^\circ$.

Because $\angle ECD \cong \angle D$, $\triangle CDE$ is isosceles by the Converse of the Isosceles Triangle Conjecture. Therefore, the legs are congruent, so $EC = ED = 3$ cm.

4.3

Triangle Inequalities

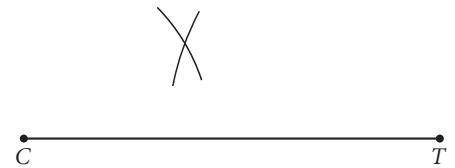
In this lesson you will

- Determine whether you can form a triangle from any three segments
- Discover a relationship between the side lengths and angle measures of a triangle
- Look for a relationship between the measure of the **exterior angle** of a triangle and the measures of the corresponding **remote interior angles**

If you are given three segments, will you always be able to form a triangle with those segments as sides? In the following investigation, you will explore this question.

Investigation 1: What Is the Shortest Path from A to B?

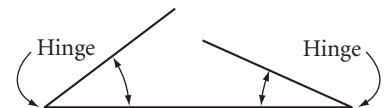
In Step 1 of the investigation, you are given two sets of three segments to use as side lengths of triangles. Consider the first set of segments. To construct $\triangle CAT$, first copy \overline{CT} . To construct the other two sides of the triangle, swing an arc of length AC centered at point C and an arc of length AT centered at point T . Point A is where the two arcs intersect.



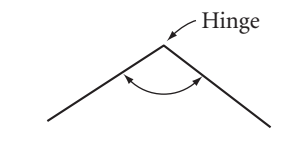
Now try to use the second set of segments to construct $\triangle FSH$. Are you able to do it? Why or why not?

You should have found that the arcs that you made using the lengths of two of the sides did not intersect, so it was not possible to construct $\triangle FSH$. In general, for three segments to form a triangle, the sum of the lengths of any two segments must be greater than the length of the third segment. Here are two ways to visualize this.

Imagine two of the segments connected to the endpoints of the third segment by hinges. To form a triangle, you need to be able to swing the segments so that their unhinged endpoints meet but do not lie completely flat. This is possible only if the combined length of the two segments is greater than the length of the third segment.



Imagine two segments connected by a hinge. To form a triangle, you need to be able to adjust the opening between these sides so that the unhinged endpoints meet the endpoints of the third segment, without lying completely flat. This is possible only if the combined length of the two hinged segments is greater than the length of the third segment.



You can state this idea as a conjecture.

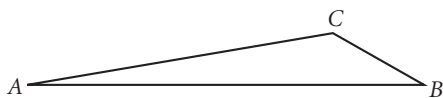
Triangle Inequality Conjecture The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

C-20

(continued)

Lesson 4.3 • Triangle Inequalities (continued)

You can think of the triangle conjecture in a different way: The shortest distance between two points is along the segment connecting them. In other words, the distance from A to C to B cannot be shorter than the distance from A to B .

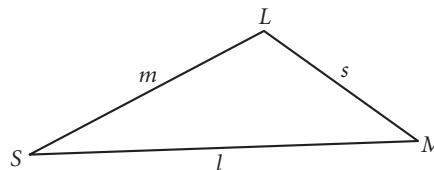


No matter where C is,
 $AC + CB \geq AB$.

Investigation 2: Where Are the Largest and Smallest Angles?

Draw a scalene obtuse triangle. Follow Steps 1 and 2 in your book to label the angles and sides according to their size. Then answer the questions in Step 3.

As in the example at right, you should find that the longest side is opposite the angle with the largest measure, the second longest side is opposite the side with the second largest measure, and the shortest side is opposite the side with the smallest measure.



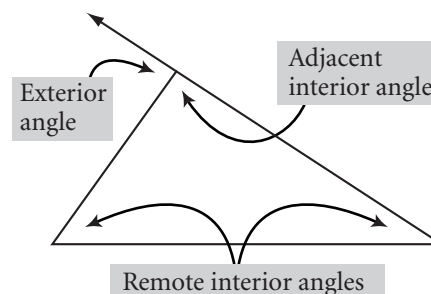
Draw a scalene acute triangle and follow Steps 1–3 again. Are your findings the same?

State your findings as a conjecture. Here is one possible way to word the conjecture.

Side-Angle Inequality Conjecture In a triangle, if one side is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

C-21

So far, you have been focusing on the *interior angles* of triangles. Triangles also have *exterior angles*. To construct an **exterior angle**, extend one side beyond the vertex. Each exterior angle of a triangle has an **adjacent interior angle** and a pair of **remote interior angles**.



Investigation 3: Exterior Angles of a Triangle

In this investigation you will look for a relationship between the measure of an exterior angle and the measure of the two associated remote interior angles. Follow Steps 1–3 in your book for at least two different triangles. You can state your findings as a conjecture.

Triangle Exterior Angle Conjecture The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

C-22

4.4

Are There Congruence Shortcuts?

In this lesson you will

- Look for shortcuts for determining whether two triangles are congruent

If the three sides and three angles of one triangle are congruent to the three sides and three angles of another triangle, then you know the triangles are congruent. But, to determine whether two triangles are congruent, do you really need to make all six comparisons?

In this lesson and the next, you will search for shortcuts that allow you to determine whether two triangles are congruent by making only three comparisons. Page 221 of your book illustrates the six different ways that three parts of one triangle can be congruent to three parts of another. Note that the order in which the parts are listed is important. For example, Side-Angle-Side (SAS) refers to two sides and the angle included *between* the sides, while Side-Side-Angle (SSA) refers to two sides and an angle that is *not between* them.

Investigation 1: Is SSS a Congruence Shortcut?

In this investigation you will explore the following question: If the three sides of one triangle are congruent to the three sides of another triangle, must the triangles be congruent? In other words, is Side-Side-Side (SSS) a congruence shortcut?

Follow Step 1 in your book to construct a triangle using the three given segments. Now try to construct a *different* triangle using the three segments as sides. Are you able to do it? (To determine whether two triangles are the same or different, you can place one on top of the other to see whether they coincide.)

You should find that you can make only one triangle from the three segments. In fact, if you are given *any* three segments (that satisfy the triangle inequality), you will be able to make only one triangle. That is, any two triangles with the same side lengths must be congruent. You can state this observation as a conjecture.

SSS Congruence Conjecture If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

C-23**Investigation 2: Is SAS a Congruence Shortcut?**

Next you will consider the Side-Angle-Side (SAS) case. If two sides and the included angle of one triangle are congruent to two sides and the included angle of another, must the triangles be congruent?

Follow Step 1 in your book to construct a triangle from the three given parts. Now try to construct a *different* triangle from the same three parts. Are you able to do it? (Remember, the angle must be included *between* the sides.)

(continued)

Lesson 4.4 • Are There Congruence Shortcuts? (continued)

You should find that you can make only one triangle from the given parts. In fact, if you are given *any* two sides and an included angle, you will be able to make only one triangle. You can state this observation as a conjecture.

SAS Congruence Conjecture If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

C-24

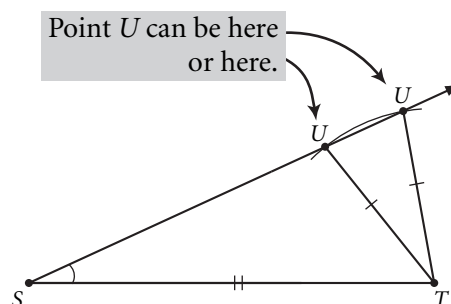
Investigation 3: Is SSA a Congruence Shortcut?

In this investigation you will explore the Side-Side-Angle (SSA) case. If two sides and a non-included angle of one triangle are congruent to the corresponding sides and angle of another triangle, must the triangles be congruent?

Follow Step 1 in your book to construct a triangle from the three given parts. Now try to construct a *different* triangle using the same two sides and non-included angle. Are you able to construct two different triangles using the same parts?

Once you construct \overline{ST} on a side of $\angle S$, there are two possible locations for point U on the other side of the angle.

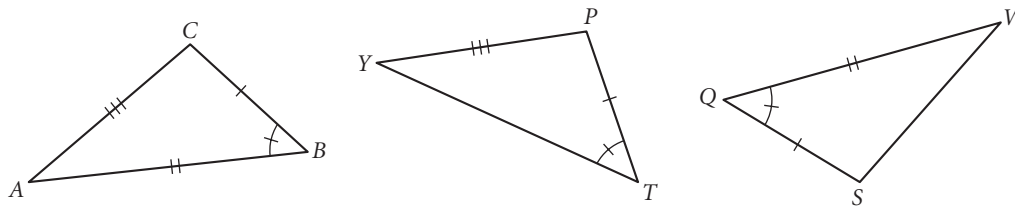
You can state this observation in a conjecture: If two sides and a non-included angle of one triangle are congruent to the corresponding two sides and non-included angle of another triangle, then the triangles are not necessarily congruent.



Here is an example that uses the new conjectures from this lesson.

EXAMPLE

Using *only* the information given, determine which triangles below are congruent and state which congruence shortcut you used.



► Solution

Because $\overline{AB} \cong \overline{WQ}$, $\angle B \cong \angle Q$, and $\overline{BC} \cong \overline{QS}$, $\triangle ABC \cong \triangle WQS$ by SAS.

Although $\overline{BC} \cong \overline{TP}$, $\overline{PY} \cong \overline{CA}$, and $\angle B \cong \angle T$, you cannot conclude that $\triangle ABC \cong \triangle TPY$ because SSA is not a congruence shortcut.

For $\triangle TPY$ and $\triangle QSW$, you know only that $\angle Q \cong \angle T$ and $\overline{QS} \cong \overline{TP}$. This is not enough information to conclude that the triangles are congruent.

Are There Other Congruence Shortcuts?

In this lesson you will

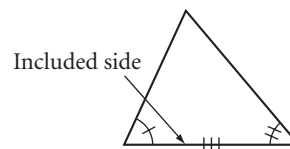
- Look for more shortcuts for determining whether two triangles are congruent

In Lesson 4.4, you saw that there are six ways in which three parts of one triangle can be congruent to three parts of another, and you investigated three of these cases. You learned the following congruence shortcuts.

- SSS: If three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.
- SAS: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

You also learned that if two sides and a non-included angle of one triangle are congruent to two sides and a non-included angle of another, then the triangles are *not* necessarily congruent. Now you will explore the three remaining cases.

A side that is between two given angles of a triangle is called an **included side**, as shown in the diagram at right.



Investigation 1: Is ASA a Congruence Shortcut?

In this investigation you explore the Angle-Side-Angle (ASA) case. If two angles and the included side of one triangle are congruent to two angles and the included side of another, must the triangles be congruent?

Follow Step 1 in your book to construct a triangle using the three given parts. Now try to construct a *different* triangle using the three parts. Are you able to do it? (Remember, to determine whether two triangles are the same or different, you can place one on top of the other to see if they coincide.)

You should find that you can make only one triangle from the three given parts. In fact, if you are given any two angles and an included side, you will be able to make only one triangle. You can state this fact as a conjecture.

ASA Congruence Conjecture If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

C-25

Investigation 2: Is SAA a Triangle Congruence Shortcut?

Now consider Side-Angle-Angle (SAA), where the side is not included between the two angles.

Follow Step 1 in your book to construct a triangle from the three given parts. Now try to construct a *different* triangle using the same three parts. Are you able to do it?

(continued)

Lesson 4.5 • Are There Other Congruence Shortcuts? (continued)

You should find that you can make only one triangle when given two angles and a non-included side. You can now state the following conjecture:

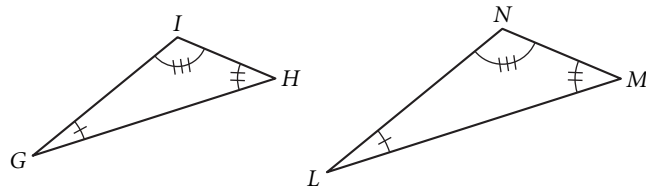
SAA Congruence Conjecture If two angles and a non-included side of one triangle are congruent to the corresponding angles and side of another triangle, then the triangles are congruent.

C-26

Investigation 3: Is AAA a Triangle Congruence Shortcut?

Finally, you will explore the Angle-Angle-Angle (AAA) case. Construct a triangle from the three angles given in your book. Now try to construct a *different* triangle using the same three angles. Are you able to do it?

Because no side lengths are given, you can make the first side any length you like. By using a different length in your second triangle, you get a triangle of a different size.



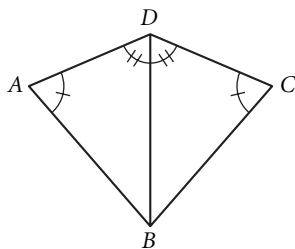
You can now state the following conjecture: If three angles of one triangle are congruent to the three angles of another triangle, then the two triangles are not necessarily congruent.

In the last two lessons, you have found that SSS, SAS, ASA, and SAA are all congruence shortcuts. Add them to your conjecture list. Here is an example.

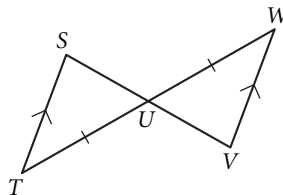
EXAMPLE

Complete each statement and tell which congruence shortcut you used to determine that the triangles are congruent. If the triangles cannot be shown to be congruent, write “cannot be determined.”

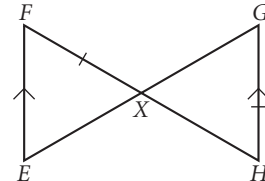
a. $\triangle ADB \cong \triangle$ _____



b. $\triangle STU \cong \triangle$ _____



c. $\triangle EFX \cong \triangle$ _____



► Solution

- Because $\angle A \cong \angle C$, $\angle ADB \cong \angle CDB$, and $\overline{BD} \cong \overline{BD}$, $\triangle ADB \cong \triangle CDB$ by SAA.
- Because \overline{ST} and \overline{WV} are parallel, $\angle S \cong \angle V$ and $\angle T \cong \angle W$. It is given that $\overline{TU} \cong \overline{WU}$. Therefore, $\triangle STU \cong \triangle VWU$ by SAA. You could also reason that $\angle SUT \cong \angle VUW$ because they are vertical angles, so $\triangle STU \cong \triangle VWU$ by ASA.
- Cannot be determined. Because \overline{EF} and \overline{GH} are parallel, $\angle E \cong \angle G$ and $\angle F \cong \angle H$. However, the congruent sides \overline{FX} and \overline{GX} are not corresponding. So, there is not enough information to show that the two triangles are congruent.

Corresponding Parts of Congruent Triangles

In this lesson you will

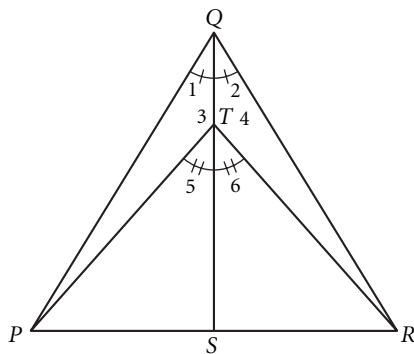
- Use the fact that corresponding parts of congruent triangles are congruent to prove statements
- Learn techniques for keeping track of information when you are writing a proof

In Lessons 4.4 and 4.5, you discovered four shortcuts for showing that two triangles are congruent—SSS, SAS, ASA, and SAA. Once you have established that two triangles are congruent, you know that their corresponding parts are congruent. We will abbreviate the statement *corresponding parts of congruent triangles are congruent* as CPCTC.

Example A in your book uses CPCTC to prove that two segments are congruent. Read this example carefully. Notice that the argument first explains why the triangles AMD and BMC are congruent, and then uses CPCTC to explain why the sides \overline{AD} and \overline{BC} are congruent. Below is another example.

EXAMPLE A

Give a deductive argument to explain why $\overline{PT} \cong \overline{RT}$.



► Solution

First, you'll show that $\triangle PQT \cong \triangle RQT$ and then use CPCTC to show that $\overline{PT} \cong \overline{RT}$. You are given that $\angle 1 \cong \angle 2$. You also know that $\overline{QT} \cong \overline{QT}$ because they are the same segment. Now, because $\angle 3$ and $\angle 5$ are a linear pair, $m\angle 3 + m\angle 5 = 180^\circ$, or equivalently, $m\angle 3 = 180^\circ - m\angle 5$. Because $\angle 5 \cong \angle 6$, you can substitute $m\angle 6$ for $m\angle 5$ to get $m\angle 3 = 180^\circ - m\angle 6$.

However, $\angle 4$ and $\angle 6$ form a linear pair, so $m\angle 4$ is also equal to $180^\circ - m\angle 6$. Therefore, $m\angle 3 = m\angle 4$. That is, $\angle 3 \cong \angle 4$. So, by ASA, $\triangle PQT \cong \triangle RQT$. Because the triangles are congruent, $\overline{PT} \cong \overline{RT}$ by CPCTC.

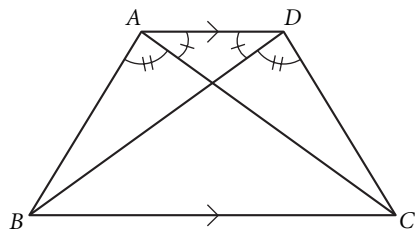
When you are trying to prove that triangles are congruent, it can be hard to keep track of the information. Be sure to mark all the information on the figure. If the triangles are hard to see, you can draw them with different colors or redraw them separately. These techniques are demonstrated in Example B in your book. Read that example and make sure you understand it. Then read the example on the next page.

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Lesson 4.6 • Corresponding Parts of Congruent Triangles (continued)

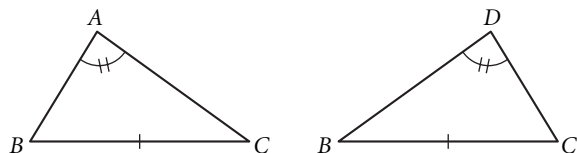
EXAMPLE B

Is $\overline{AC} \cong \overline{DB}$? Write a paragraph proof explaining why.



► Solution

You can draw triangles ABC and DCB separately to see them more clearly. As you discover more information, mark it on the original diagram and the separated triangles.



Paragraph Proof: Show that $\overline{AC} \cong \overline{DB}$.

$\angle BAC \cong \angle CDB$. Also, $\overline{BC} \cong \overline{CB}$ because they are the same segment. Because $\overline{AD} \parallel \overline{BC}$, the alternate interior angles are congruent. Therefore, $\angle ACB \cong \angle DAC$ and $\angle ADB \cong \angle DBC$. Because it is given that $\angle DAC \cong \angle ADB$, it must also be true that $\angle ACB \cong \angle DBC$. (Mark this information on the diagrams.) $\triangle ABC \cong \triangle DCB$ by SAA. By CPCTC, $\overline{AC} \cong \overline{DB}$.

4.7

Flowchart Thinking

In this lesson you will

- Write **flowchart proofs**

So far, you have been writing explanations as deductive arguments or paragraph proofs. Example A in your book shows a paragraph proof. Read this example and make sure you understand the proof.

When a logical argument is complex or includes many steps, a paragraph proof may not be the clearest way to present the steps. In such cases, it is often helpful to organize the steps in the form of a *flowchart*. A **flowchart** is a visual way to organize all the steps in a complicated procedure in their proper order. The steps in the procedure are written in boxes. Arrows connect the boxes to show how facts lead to conclusions.

Creating a **flowchart proof** makes your logic visible so that others can follow your reasoning. Example B in your book presents the argument from Example A in flowchart form. Read the proof carefully. Notice that each statement is written in a box and that the logical reason for each step is written beneath its box.

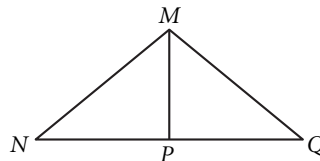
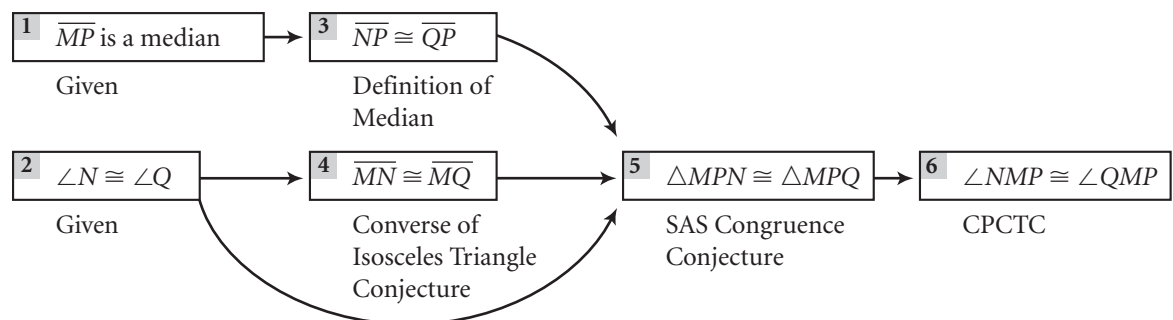
More flowchart proofs are given in the examples below. In each example, try to write a proof yourself before looking at the solution. Remember, there are often several ways to prove a statement. Your proof may not be the same as the one given.

EXAMPLE A**Given:**

\overline{MP} is a median
 $\angle N \cong \angle Q$

Show:

$\angle NMP \cong \angle QMP$

► **Solution** | **Flowchart Proof**

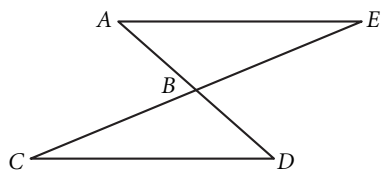
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Lesson 4.7 • Flowchart Thinking (continued)

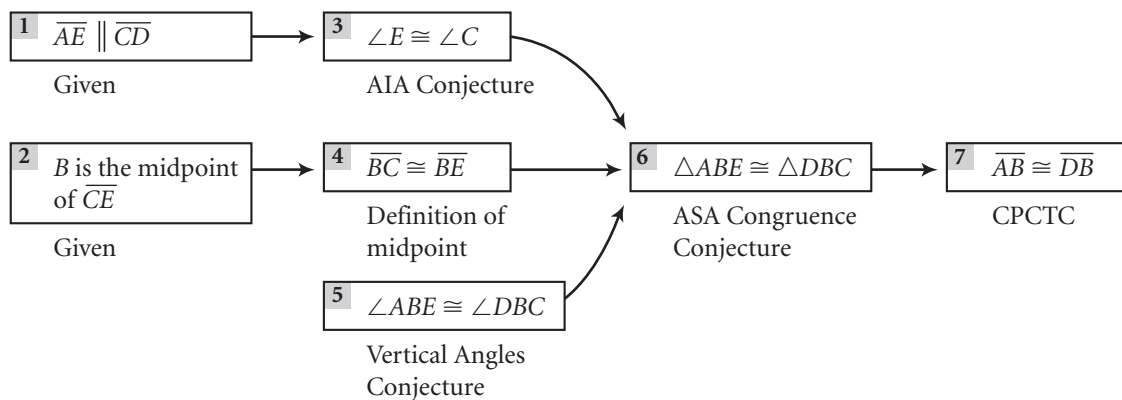
EXAMPLE B

Given: $\overline{AE} \parallel \overline{CD}$
 B is the midpoint of \overline{CE}

Show: $\overline{AB} \cong \overline{DB}$



► Solution | Flowchart Proof

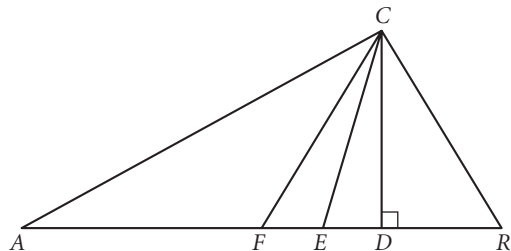


Proving Special Triangle Conjectures

In this lesson you will

- Make a conjecture about the bisector of the vertex angle in an isosceles triangle
- Make and prove a conjecture about equilateral triangles
- Learn about **biconditional** conjectures

In $\triangle ARC$, \overline{CD} is the altitude to the base \overline{AR} , \overline{CE} is the angle bisector of $\angle ACR$, and \overline{CF} is the median to side \overline{AR} . This example illustrates that the angle bisector, the altitude, and the median can all be different segments. Is this always true? Can they all be the same segment? You will explore these questions in the investigation.



Investigation: The Symmetry Line in an Isosceles Triangle

Construct a large isosceles triangle on a sheet of unlined paper. Label it ARK , with K as the vertex angle.

Construct angle bisector \overline{KD} with point D on \overline{AR} . Compare $\triangle ADK$ with $\triangle RDK$. Do they look congruent?

Use your compass to compare \overline{AD} and \overline{RD} . Are they congruent? If so, then D is the midpoint of \overline{AR} , and therefore \overline{KD} is the median to \overline{AR} . Notice that $\angle ADK$ and $\angle RDK$ are a linear pair and are therefore supplementary. Now compare $\angle ADK$ and $\angle RDK$. Are the angles congruent? If so, what must the measure of each angle be? What does this tell you about \overline{KD} and \overline{AR} ?

Your findings should lead to the following conjecture.

Vertex Angle Bisector Conjecture In an isosceles triangle, the bisector of the vertex angle is also the altitude and the median to the base.

C-27

In Chapter 3, you discovered that if a triangle is equilateral, then each angle measures 60° . Therefore, if a triangle is equilateral, then it is equiangular. This is called the Equilateral Triangle Conjecture. A proof of this statement is given on page 245 of your book. Read this proof carefully and make sure you understand each step. The *converse* of this statement is also true. That is, if a triangle is equiangular, then it is equilateral. The following conjecture combines these ideas.

Equilateral/Equiangular Triangle Conjecture Every equilateral triangle is equiangular, and, conversely, every equiangular triangle is equilateral.

C-28

(continued)

Lesson 4.8 • Proving Special Triangle Conjectures (continued)

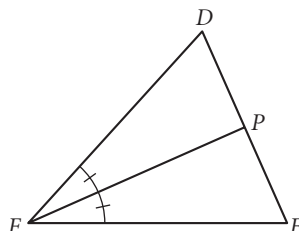
A **biconditional** conjecture is a conjecture in which one condition cannot be true unless the other is also true. In other words, both the statement and its converse are true. The Equilateral/Equiangular Triangle Conjecture is biconditional, so it can be written: A triangle is equilateral *if and only if* it is equiangular.

Here is an example using the new conjectures.

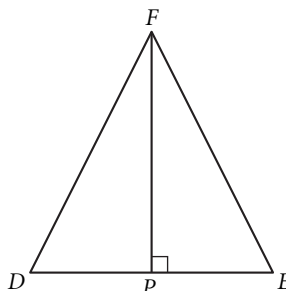
EXAMPLE

$\triangle DEF$ is isosceles with $\overline{DF} \cong \overline{EF}$.

- a. $m\angle D = 67^\circ$
 $DE = 15$ cm
 $m\angle DFP =$ _____
 $DP =$ _____



- b. $m\angle DFE = 54^\circ$
 $DP = 7$ cm
 $m\angle DFP =$ _____
 $DE =$ _____



► Solution

- a. If $m\angle D = 67^\circ$, then $m\angle E = 67^\circ$ because $\angle D$ and $\angle E$ are base angles of an isosceles triangle. Therefore, $m\angle DFE = 180^\circ - (67^\circ + 67^\circ) = 46^\circ$. Because \overline{FP} bisects $\angle DFE$, $m\angle DFP = \frac{1}{2}(46^\circ) = 23^\circ$.

Because \overline{FP} bisects vertex angle DFE , it must also be the median to \overline{DE} .
 Therefore, $DP = \frac{1}{2}DE = \frac{1}{2}(15 \text{ cm}) = 7.5 \text{ cm}$.

- b. Because \overline{FP} is the altitude to \overline{DE} , it must also be the bisector of vertex angle DFE . Therefore, $m\angle DFP = \frac{1}{2}(m\angle DFE) = \frac{1}{2}(54^\circ) = 27^\circ$.

Because \overline{FP} bisects vertex angle DFE , it must also be the median to \overline{DE} .
 Therefore, $DE = 2DP = 2(7 \text{ cm}) = 14 \text{ cm}$.

5.1

Polygon Sum Conjecture

In this lesson you will

- Discover a formula for finding the **sum of the angle measures** for any **polygon**
- Use **deductive reasoning** to explain why the polygon sum formula works

Triangles come in many different shapes and sizes. However, as you discovered in Chapter 4, the sum of the angle measures of any triangle is 180° . In this lesson you will investigate the sum of the angle measures of other polygons. After you find a pattern, you'll write a formula that relates the number of sides of a polygon to the sum of the measures of its angles.

Investigation: Is There a Polygon Sum Formula?

Draw three different quadrilaterals. For each quadrilateral, carefully measure the four angles and then find the sum of the angle measures. If you measure carefully, you should find that all of your quadrilaterals have the same angle sum. What is the sum? Record it in a table like the one below.

Number of sides	3	4	5	6	7	8	...	n
Sum of angle measures	180°				900°	1080°		

Next draw three different pentagons. Carefully measure the angles in each pentagon and find the angle sum. Again, you should find that the angle sum is the same for each pentagon. Record the sum in the table.

Use your findings to complete the conjectures below.

Quadrilateral Sum Conjecture The sum of the measures of the four angles of any quadrilateral is _____.

C-29

Pentagon Sum Conjecture The sum of the measures of the five angles of any pentagon is _____.

C-30

Now draw at least two different hexagons and find their angle sum. Record the sum in the table.

The angle sums for heptagons and octagons have been entered in the table for you, but you can check the sums by drawing and measuring your own polygons.

Look for a pattern in the completed table. Find a general formula for the sum of the angle measures of a polygon in terms of the number of sides, n . (*Hint:* Use what you learned in Chapter 2 about finding the formula for a pattern with a constant difference.) Then complete this conjecture.

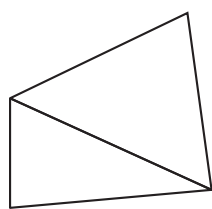
Polygon Sum Conjecture The sum of the measures of the n interior angles of an n -gon is _____.

C-31

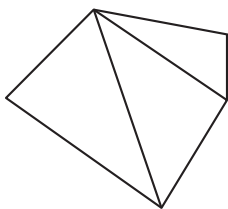
(continued)

Lesson 5.1 • Polygon Sum Conjecture (continued)

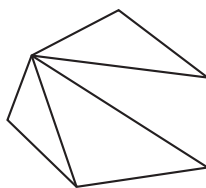
You can use deductive reasoning to see why your formula works. In each polygon below, all the diagonals from one vertex have been drawn, creating triangles. Notice that in each polygon, the number of triangles is 2 less than the number of sides.



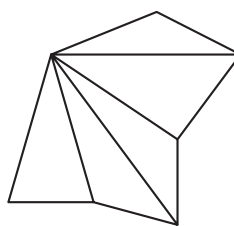
4 sides



5 sides
3 triangles



6 sides
4 triangles



7 sides
5 triangles

The quadrilateral has been divided into two triangles, each with an angle sum of 180° . So, the angle sum for the quadrilateral is $180^\circ \cdot 2$, or 360° . The pentagon has been divided into three triangles, so its angle sum is $180^\circ \cdot 3$, or 540° . The angle sums for the hexagon and the heptagon are $180^\circ \cdot 4$ and $180^\circ \cdot 5$, respectively. In general, if a polygon has n sides, its angle sum is $180^\circ(n - 2)$ or, equivalently, $180^\circ n - 360^\circ$. This should agree with the formula you found earlier.

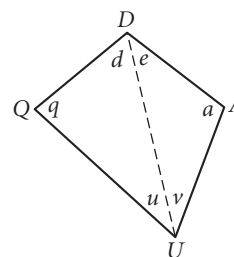
You can use the diagram at right to write a paragraph proof of the Quadrilateral Sum Conjecture. See if you can fill in the steps in the proof below.

Paragraph Proof: Show that $m\angle Q + m\angle U + m\angle A + m\angle D = 360^\circ$.

$q + d + u = 180^\circ$ and $e + a + v = 180^\circ$ by the _____ Conjecture.

By the Addition Property of Equality, $q + d + u + e + a + v = \underline{\hspace{2cm}}$.

Therefore, the sum of the angle measures of a quadrilateral is 360° .

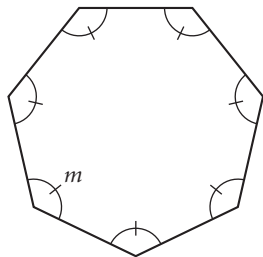


Here is an example using your new conjectures.

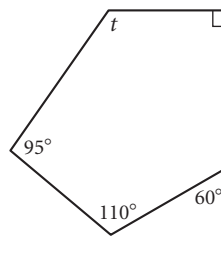
EXAMPLE

Find the lettered angle measures.

a.



b.



► Solution

- The polygon has seven sides, so the angle sum is $180^\circ \cdot 5$, or 900° . Because all the angles have the same measure, the measure of angle m is $900^\circ \div 7$, or about 128.6° .
- The polygon has five sides, so the angle sum is $180^\circ \cdot 3$, or 540° . Therefore, $90^\circ + 120^\circ + 110^\circ + 95^\circ + t = 540^\circ$. Solving for t gives $t = 125^\circ$.

5.2

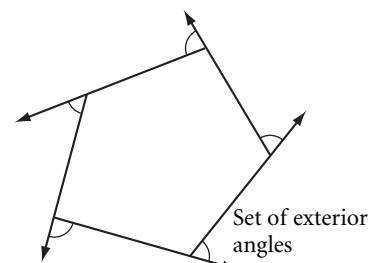
Exterior Angles of a Polygon

In this lesson you will

- Find the sum of the measures of one set of **exterior angles** of a polygon
- Derive two formulas for the measure of each angle of an equiangular polygon

In Lesson 5.1, you discovered a formula for the sum of the measures of the *interior* angles of any polygon. In this lesson, you will find a formula for the sum of the measures of a set of *exterior* angles.

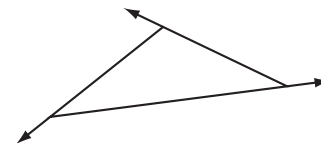
To create a set of exterior angles of a polygon, extend each side of the polygon to form one exterior angle at each vertex.



Investigation: Is There an Exterior Angle Sum?

Draw a triangle and extend its sides to form a set of exterior angles.

Measure two of the *interior* angles of your triangle. Then use the Triangle Sum Conjecture to find the measure of the remaining angle. Label each angle in your triangle with its measure.



Use the Linear Pair Conjecture to calculate the measure of each exterior angle. Then find the sum of the exterior angle measures. Record your result in a table like the one below. Do you think you would get the same result for a different triangle? Draw another triangle and see.

Number of sides	3	4	5	6	7	8	...	n
Sum of exterior angle measures	360°						...	

Next, draw a quadrilateral and create a set of exterior angles. Use a procedure similar to the one you used for triangles to find the sum of the set of exterior angles. Add your result to the table.

Next, find the sum of the exterior angles for a pentagon. Are you starting to see a pattern? Predict the sum of the exterior angle measures for a hexagon. Then draw a hexagon and check your prediction. Add your results to the table.

You should have discovered that, no matter how many sides a polygon has, the sum of its exterior angle measures is 360° . This can be stated as a conjecture.

Exterior Angle Sum Conjecture For any polygon, the sum of the measures of a set of exterior angles is 360° .

C-32

Look at the software construction on page 263 of your book. Notice that the exterior angles stay the same as the polygon shrinks to a point. How does this demonstrate the Exterior Angle Sum Conjecture?

(continued)

Lesson 5.2 • Exterior Angles of a Polygon (continued)

Now you will find two formulas for the measure of each *interior* angle in an equiangular polygon with n sides. Remember, an equiangular polygon is a polygon in which all angles have the same measure.

You can use the Polygon Sum Conjecture to derive the first formula. That conjecture states that the sum of the interior angle measures in a polygon with n sides is $180^\circ(n - 2)$. If the polygon is equiangular, then each of the n angles has the same measure. Use these facts to write a formula for the measure of each angle.

To derive the second formula, you can use the Exterior Angle Sum Conjecture. According to that conjecture, the sum of the measures of the n exterior angles of a polygon is 360° . In an equiangular polygon, each of the exterior angles has the same measure. So, the measure of each exterior angle is $\frac{360^\circ}{n}$. If each exterior angle measures $\frac{360^\circ}{n}$, what must be the measure of each interior angle?

You can state your findings in a conjecture.

Equiangular Polygon Conjecture You can find the measure of each interior angle in an equiangular n -gon by using either of these formulas:

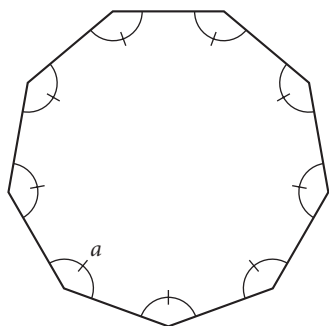
C-33

$$180^\circ - \frac{360^\circ}{n} \text{ or } \frac{180^\circ(n - 2)}{n}.$$

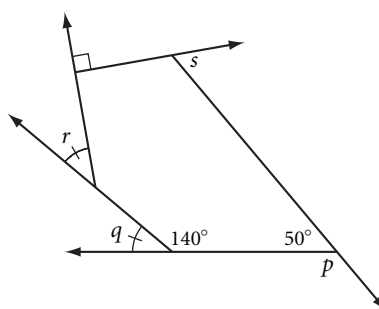
EXAMPLE

Find the lettered angle measures.

a.



b.



► Solution

- a. This is an equiangular 9-gon. You can use the Equiangular Polygon Conjecture to find the measure of a .

$$a = 180^\circ - \frac{360^\circ}{9} = 180^\circ - 40^\circ = 140^\circ$$

- b. By the Linear Pair Conjecture, $p = 130^\circ$ and $q = 40^\circ$ so $r = 40^\circ$ as well. To find s , use the Exterior Angle Sum Conjecture.

$$130^\circ + 40^\circ + 40^\circ + 90^\circ + s = 360^\circ$$

Solving this equation gives $s = 60^\circ$.

5.3

Kite and Trapezoid Properties

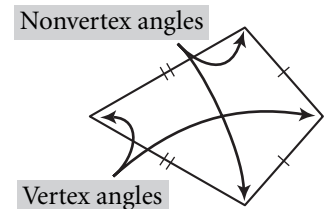
In this lesson you will

- Investigate the properties of **kites**
- Investigate properties of **trapezoids** and **isosceles trapezoids**

In this lesson you will look at two special types of quadrilaterals, kites and trapezoids. Recall that a **kite** is a quadrilateral with two distinct pairs of congruent consecutive sides.

You can make a kite by constructing two different isosceles triangles on opposite sides of a common base and then removing the base. In an isosceles triangle, the angle between the two congruent sides is called the vertex angle. For this reason, we'll call angles between the pairs of congruent sides of a kite **vertex angles**. We'll refer to the other two angles as **nonvertex angles**.

A kite has one line of reflectional symmetry, just like an isosceles triangle. You can use this property to discover other properties of kites.



Investigation 1: What Are Some Properties of Kites?

Follow Step 1 in your book to construct a kite on patty paper.

Compare each angle to the opposite angle by folding. Which angles are congruent, vertex angles or nonvertex angles? Use your findings to complete this conjecture.

Kite Angles Conjecture The _____ angles of a kite are congruent.

C-34

Draw the diagonals of the kite. Fold the kite along one of the diagonals. The two parts of the other diagonal should coincide. What can you conclude about the angle between the diagonals? You are now ready to complete this conjecture.

Kite Diagonals Conjecture The diagonals of a kite are _____.

C-35

Now fold along each diagonal and compare the lengths of the segments on the diagonals. Does either diagonal bisect the other? Complete this conjecture.

Kite Diagonal Bisector Conjecture The diagonal connecting the vertex angles of a kite is the _____ of the other diagonal.

C-36

Fold along the diagonal connecting the vertex angles. Does the diagonal bisect the vertex angles? Now fold along the other diagonal. Does it bisect the nonvertex angles? Complete this conjecture.

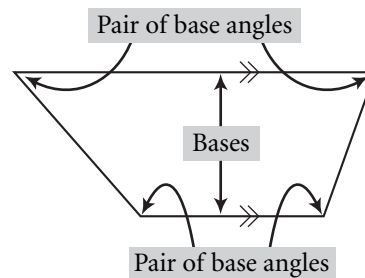
Kite Angle Bisector Conjecture The _____ angles of a kite are bisected by a diagonal.

C-37

(continued)

Lesson 5.3 • Kite and Trapezoid Properties (continued)

Now you will explore some properties of trapezoids. Recall that a **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**. A pair of angles that share a base as a common side are called **base angles**.



Investigation 2: What Are Some Properties of Trapezoids?

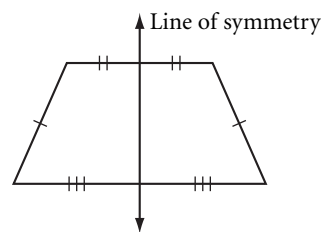
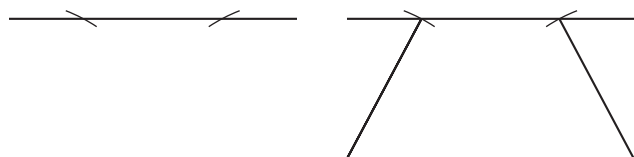
Follow Steps 1 and 2 in your book. Use your findings to complete this conjecture.

Trapezoid Consecutive Angles Conjecture The consecutive angles between the bases of a trapezoid are _____.

C-38

An **isosceles trapezoid** is a trapezoid whose nonparallel sides are the same length. An isosceles trapezoid has a line of symmetry that passes through the midpoints of the two bases.

Use both edges of your straightedge to draw parallel segments. To construct the congruent sides, make identical arcs centered at the endpoints of one of the segments so that each arc intersects the other segment. Then connect points as shown below to form the trapezoid.



Measure each pair of base angles. How do the angles in each pair compare? Complete this conjecture.

Isosceles Trapezoid Conjecture The base angles of an isosceles trapezoid are _____.

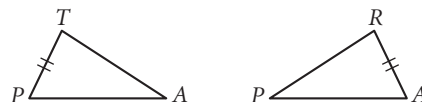
C-39

Now draw the two diagonals. Compare their lengths and complete this conjecture.

Isosceles Trapezoid Diagonals Conjecture The diagonals of an isosceles trapezoid are _____.

C-40

Follow the Developing Proof instructions on page 271 of your book and complete a flowchart proof of the Isosceles Trapezoid Diagonals Conjecture, using the Isosceles Trapezoid Conjecture and congruent triangles. Separating the triangles as shown at right might help you. Remember to mark congruent parts on your diagram.



5.4

Properties of Midsegments

In this lesson you will

- Discover properties of the **midsegment** of a **triangle**
- Discover properties of the **midsegment** of a **trapezoid**

In Chapter 3, you learned that a *midsegment* of a triangle is a segment connecting the midpoints of two sides. In this lesson you will investigate properties of midsegments.

Investigation 1: Triangle Midsegment Properties

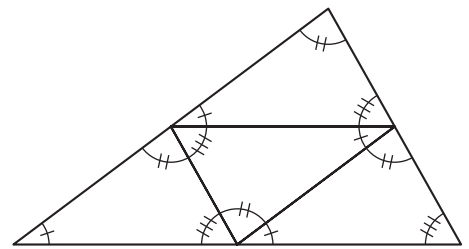
Follow Steps 1–3 in your book. Your conclusions should lead to the following conjecture.

Three Midsegments Conjecture The three midsegments of a triangle divide it into four congruent triangles.

C-41

Mark all the congruent angles in your triangle as shown in this example.

Focus on one of the midsegments and the third side of the triangle (the side the midsegment doesn't intersect). Look at the pairs of alternate interior angles and corresponding angles associated with these segments. What conclusion can you make? Look at the angles associated with each of the other midsegments and the corresponding third side.



Now compare the length of each midsegment to the length of the corresponding third side. How are the lengths related?

State your findings in the form of a conjecture.

Triangle Midsegment Conjecture A midsegment of a triangle is _____ to the third side and _____ the length of the third side.

C-42

The midsegment of a trapezoid is the segment connecting the midpoints of the two nonparallel sides.

Investigation 2: Trapezoid Midsegment Properties

Follow Steps 1–3 in your book. You should find that the trapezoid's base angles are congruent to the corresponding angles at the midsegment. What can you conclude about the relationship of the midsegment to the bases?



Now follow Steps 5–7. You should find that the midsegment fits twice onto the segment representing the sum of the two bases. That is, the length of the

(continued)

Lesson 5.4 • Properties of Midsegments (continued)

midsegment is half the sum of the lengths of the two bases. Another way to say this is: The length of the midsegment is the average of the lengths of the bases.

Use what you have learned about the midsegment of a trapezoid to complete this conjecture.

Trapezoid Midsegment Conjecture The midsegment of a trapezoid is _____ to the bases and equal in length to _____.

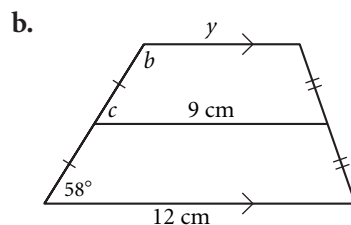
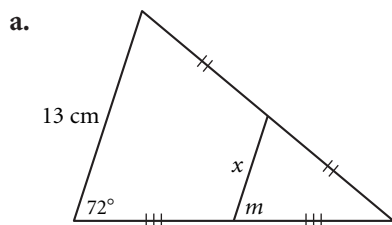
C-43

Read the text below the investigation on page 277 of your book and study the software construction. Make sure you understand the relationship between the Trapezoid and Triangle Midsegment Conjectures.

Work through the following example yourself before checking the solution.

EXAMPLE

Find the lettered measures.



► Solution

- a. By the Triangle Midsegment Conjecture, $x = \frac{1}{2}(13 \text{ cm}) = 6.5 \text{ cm}$.

The Triangle Midsegment Conjecture also tells you that the midsegment is parallel to the third side. Therefore, the corresponding angles are congruent, so $m = 72^\circ$.

- b. By the Trapezoid Midsegment Conjecture, $\frac{1}{2}(12 + y) = 9$. Solving for y gives $y = 6$.

The Trapezoid Midsegment Conjecture also tells you that the midsegment is parallel to the bases. Therefore, the corresponding angles are congruent, so $c = 58^\circ$.

By the Trapezoid Consecutive Angles Conjecture, $b + 58^\circ = 180^\circ$, so $b = 122^\circ$.

5.5

Properties of Parallelograms

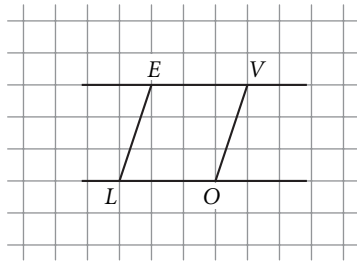
In this lesson you will

- Discover how the **angles** of a parallelogram are related
- Discover how the **sides** of a parallelogram are related
- Discover how the **diagonals** of a parallelogram are related

You have explored properties of kites and trapezoids and of the midsegments of triangles and trapezoids. In this lesson you will explore properties of parallelograms.

Investigation: Four Parallelogram Properties

Follow the directions in Step 1 in your book to construct and label a parallelogram.



Use patty paper or a protractor to compare the measures of the opposite angles. Then use your findings to complete this conjecture.

Parallelogram Opposite Angles Conjecture The opposite angles of a parallelogram are _____.

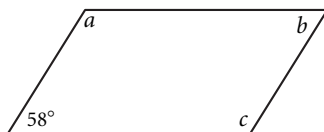
C-44

Consecutive angles are angles that share a common side. In parallelogram *LOVE*, $\angle LOV$ and $\angle EVO$ are one pair of consecutive angles. There are three other pairs. Find the sum of the measures of each pair of consecutive angles. You should find that the sum is the same for all four pairs. What is the sum? Complete this conjecture.

Parallelogram Consecutive Angles Conjecture The consecutive angles of a parallelogram are _____.

C-45

Suppose you are given the measure of one angle of a parallelogram. Describe how you can use the conjectures above to find the measures of the other three angles. If you don't know, look at this particular figure. What are the values of a , b , and c ? (Remember all your parallel lines conjectures.)



(continued)

Lesson 5.5 • Properties of Parallelograms (continued)

Use a compass or patty paper to compare the lengths of the opposite sides of your parallelogram. How are the lengths related? Complete this conjecture.

Parallelogram Opposite Sides Conjecture The opposite sides of a parallelogram are _____.

C-46

Now draw the diagonals of your parallelogram. Label the point where the diagonals intersect M . How do LM and VM compare? How do EM and OM compare? What does this tell you about the relationship between the diagonals? Complete this conjecture.

Parallelogram Diagonals Conjecture The diagonals of a parallelogram _____ each other.

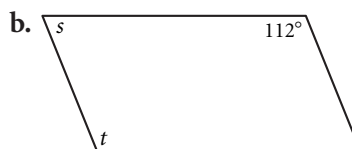
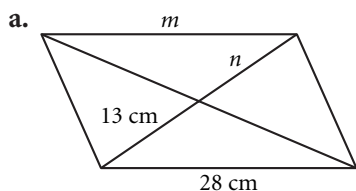
C-47

In your book, read the text about **vectors** that follows the investigation.

Here is an example using your new conjectures.

EXAMPLE

In parts a and b, the figures are parallelograms. Find the lettered measures and state which conjectures you used.



► Solution

- a. By the Parallelogram Opposite Sides Conjecture, $m = 28\text{ cm}$.
By the Parallelogram Diagonals Conjecture, $n = 13\text{ cm}$.
- b. By the Parallelogram Opposite Angles Conjecture, $t = 112^\circ$.
By the Parallelogram Consecutive Angles Conjecture, $s = 180^\circ - 112^\circ = 68^\circ$.

Properties of Special Parallelograms

In this lesson you will

- Discover properties of **rhombuses** and their diagonals
- Discover properties of **rectangles** and their diagonals
- Discover properties of **squares** and their diagonals

In Lesson 5.5, you investigated parallelograms. In this lesson you focus on three special parallelograms—rhombuses, rectangles, and squares.

Investigation 1: What Can You Draw With the Double-Edged Straightedge?

Follow Steps 1–3 in your book. You should find that all the sides of the parallelogram you create are the same length. Use your findings to complete this conjecture.

Double-Edged Straightedge Conjecture If two parallel lines are intersected by a second pair of parallel lines that are the same distance apart as the first pair, then the parallelogram formed is a _____.

C-48

Now that you know a quick way to construct a rhombus, you will explore some special properties of rhombuses.

Investigation 2: Do Rhombus Diagonals Have Special Properties?

In this investigation you will look at the diagonals of a rhombus. Follow Steps 1 and 2 in your book. Then complete this conjecture.

Rhombus Diagonals Conjecture The diagonals of a rhombus are _____ and they _____ each other.

C-49

Follow Step 3 to compare the two angles formed at each vertex by a diagonal and the sides. Then complete this conjecture.

Rhombus Angles Conjecture The diagonals of a rhombus _____ the angles of the rhombus.

C-50

You have just explored rhombuses, parallelograms with four congruent sides. Now you will look at rectangles, parallelograms with four congruent angles.

By the Quadrilateral Sum Conjecture, you know that the sum of the angle measures of a rectangle is 360° . Because all the angles have the same measures, each angle must have measure 90° . In other words, a rectangle has four right angles.

(continued)

Lesson 5.6 • Properties of Special Parallelograms (continued)

Investigation 3: Do Rectangle Diagonals Have Special Properties?

Follow Steps 1 and 2 in your book. What do you notice about the lengths of the two diagonals? Because a rectangle is a parallelogram, you also know that the diagonals bisect each other. You can use your compass to confirm this for your rectangle. Combine these two observations to complete the conjecture.

Rectangle Diagonals Conjecture The diagonals of a rectangle are _____ and _____ each other.

C-51

A square is a parallelogram that is both equiangular and equilateral. Here are two definitions of a square.

A **square** is an equiangular rhombus.

A **square** is an equilateral rectangle.

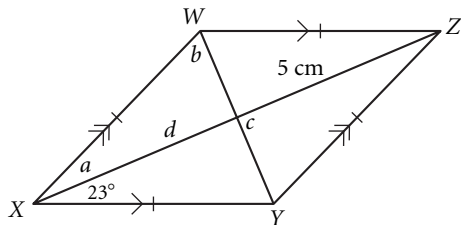
Because a square is a parallelogram, a rhombus, and a rectangle, all the properties of these quadrilaterals are also true for squares. Look back at what you know about the diagonals of each of these quadrilaterals, and use your findings to complete this conjecture.

Square Diagonals Conjecture The diagonals of a square are _____, _____, and _____.

C-52

EXAMPLE

Find the lettered measures.



► Solution

The figure is a rhombus, so by the Rhombus Angles Conjecture, the diagonals bisect the angles. Therefore, $a = 23^\circ$.

By the Parallelogram Consecutive Angles Conjecture, $\angle WXY$ and $\angle XWZ$ are supplementary, so $m\angle XWZ + 46^\circ = 180^\circ$. Therefore, $m\angle XWZ = 134^\circ$. So, using the Rhombus Angles Conjecture, $b = \frac{1}{2}(134^\circ) = 67^\circ$.

By the Rhombus Diagonals Conjecture, the diagonals are perpendicular and bisect each other, so $c = 90^\circ$ and $d = 5$ cm.

5.7

Proving Quadrilateral Properties

In this lesson you will

- Learn about the “thinking backward” strategy for writing proofs
- Prove many of the quadrilateral conjectures from this chapter

In this chapter you have made many conjectures about the properties of quadrilaterals. In this lesson you will write proofs for several of these conjectures.

Look at the illustration of the firefighters in your book. The firefighter holding the hose has asked the other firefighter to turn on one of the hydrants. Which one should he turn on? One way to solve this problem is to start with the end of the hose in the first firefighter’s hand and trace it backward to a hydrant. You can use a similar strategy to help you write proofs.

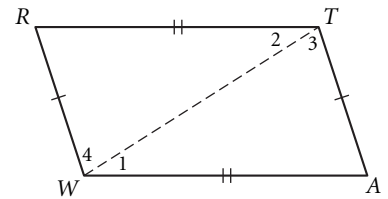
To plan a proof, it often helps to start with the conclusion (that is, the statement you want to prove) and work your way back to the beginning, one step at a time. Making a flowchart can help you visualize the flow of reasoning. The example in your book illustrates how this strategy works. Read this example carefully. Then read the example below, which shows how to prove the statement in Exercise 3.

EXAMPLE

Prove the conjecture: If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Given: Quadrilateral $WATR$ with $\overline{WA} \cong \overline{RT}$ and $\overline{WR} \cong \overline{AT}$, and diagonal \overline{WT}

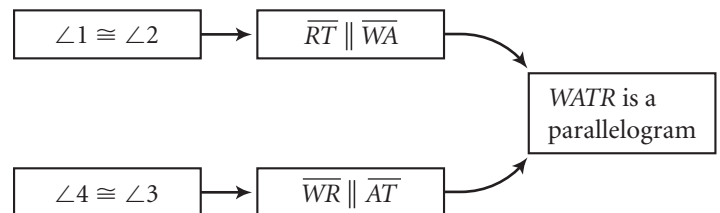
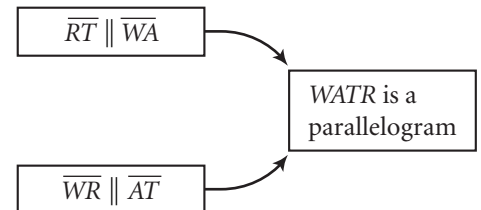
Show: $WATR$ is a parallelogram

**► Solution**

Construct your proof by working backward. Your thinking might go something like this:

“I can show that $WATR$ is a parallelogram if I can show that the opposite sides are parallel. That is, I need to show that $\overline{RT} \parallel \overline{WA}$ and $\overline{WR} \parallel \overline{AT}$.”

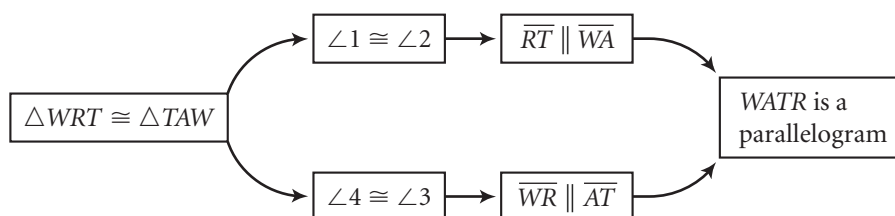
“I can show that $\overline{RT} \parallel \overline{WA}$ if I can show that the alternate interior angles $\angle 1$ and $\angle 2$ are congruent. I can show that $\overline{WR} \parallel \overline{AT}$ if I can show that the alternate interior angles $\angle 4$ and $\angle 3$ are congruent.”



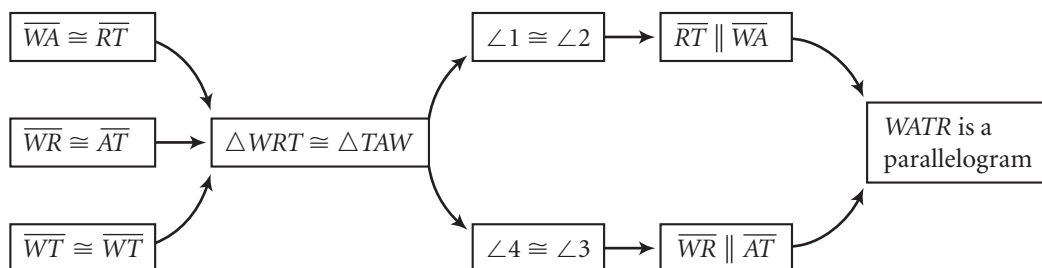
(continued)

Lesson 5.7 • Proving Quadrilateral Properties (continued)

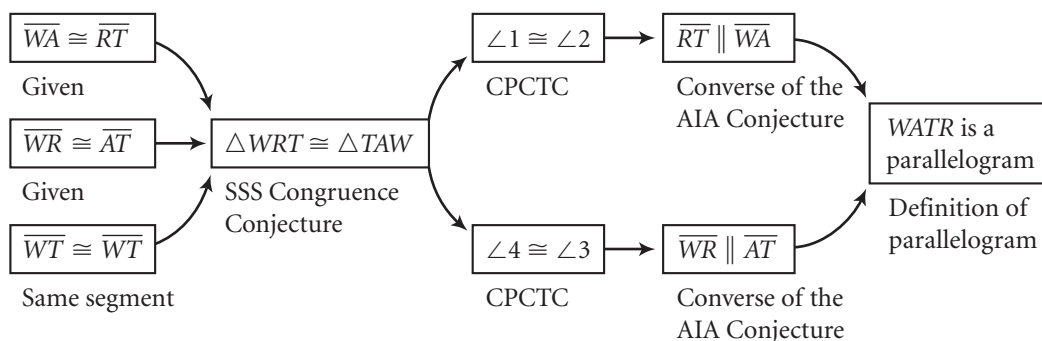
“I can show that $\angle 1 \cong \angle 2$ and $\angle 4 \cong \angle 3$ if they are corresponding parts of congruent triangles.”



“Can I show that $\triangle WRT \cong \triangle TAW$? Yes, I can, by SSS, because it is given that $\overline{WA} \cong \overline{RT}$ and $\overline{WR} \cong \overline{AT}$, and $\overline{WT} \cong \overline{WT}$ because it is the same segment in both triangles.”



By adding the reason for each statement below each box, you can make the flowchart into a complete flowchart proof.



If you prefer, you can write the proof in paragraph form:

“It is given that $\overline{WA} \cong \overline{RT}$ and $\overline{WR} \cong \overline{AT}$. $\overline{WT} \cong \overline{WT}$ because it is the same segment. So, $\triangle WRT \cong \triangle TAW$ by the SSS Congruence Conjecture. By CPCTC, $\angle 1 \cong \angle 2$ and $\angle 4 \cong \angle 3$. By the converse of the Alternate Interior Angles Conjecture, $\overline{RT} \parallel \overline{WA}$ and $\overline{WR} \parallel \overline{AT}$. Therefore, WATR is a parallelogram because, by definition, a parallelogram is a quadrilateral in which the opposite sides are parallel. Q.E.D.”

For more practice with working backward, work through the Finding the Square Route Investigation on page 300 of your book.

6.1

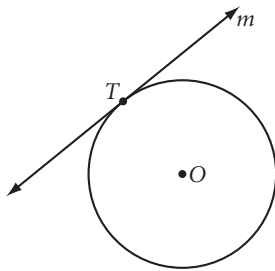
Tangent Properties

In this lesson you will

- Review terms associated with **circles**
- Discover how a tangent to a circle and the radius to the **point of tangency** are related
- Make a conjecture about **tangent segments** to a circle from a point outside the circle

You have already learned some terms related to circles. Review them by matching the terms and figures on page 310 of your book.

The **tangent** to a circle is a line in the plane of the circle that intersects the circle in exactly one point. The point where the tangent touches the circle is the **point of tangency**.



Line m is a **tangent** to circle O .
Point T is the **point of tangency**.

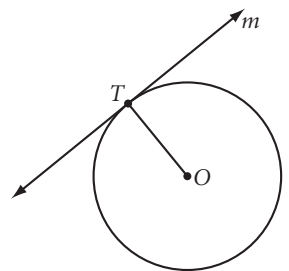
Investigation 1: Going Off on a Tangent

Construct a large circle. Label the center O .

Use a straightedge to draw a line that appears to touch the circle at one point. Label the point T and draw radius \overline{OT} .

Measure the angles formed by the radius and the tangent line.
How do the angles compare?

Will you get the same result with a different tangent? Draw another tangent and see. Use your findings to complete this conjecture.



Tangent Conjecture A tangent to a circle is _____ to the radius drawn to the point of tangency.

C-53

Investigation 2: Tangent Segments

Follow Steps 1–3 in your book. How do the lengths of the tangent segments compare?

Will you get the same results if you start with a different point outside the circle? Choose another point and see. Use your findings to complete this conjecture.

Tangent Segments Conjecture Tangent segments to a circle from a point outside the circle are _____.

C-54

(continued)

Lesson 6.1 • Tangent Properties (continued)

Read the information about minor and major arcs on page 313 of your book, and study the example. Then read about **internally tangent circles** and **externally tangent circles**.

Now read the example below.

EXAMPLE

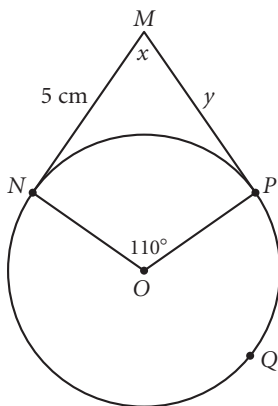
- a. \overline{MN} and \overline{MP} are tangents to circle O .

$$x = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$

$$m\widehat{NP} = \underline{\hspace{2cm}}$$

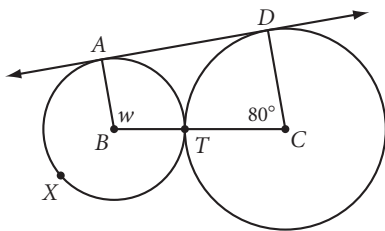
$$m\widehat{PQN} = \underline{\hspace{2cm}}$$



- b. \overleftrightarrow{AD} is tangent to both circle B and circle C .

$$w = \underline{\hspace{2cm}}$$

$$m\widehat{AXT} = \underline{\hspace{2cm}}$$



- c. Refer to the figure for part a. What type of quadrilateral is $MNOP$?

► Solution

- a. By the Tangent Conjecture, $m\angle MNO = 90^\circ$ and $m\angle MPO = 90^\circ$. By the Quadrilateral Sum Conjecture, $90^\circ + 90^\circ + 110^\circ + x = 360^\circ$, so $x = 70^\circ$. By the Tangent Segments Conjecture, $\overline{MN} \cong \overline{MP}$, so $y = 5$ cm.

Because the measure of a minor arc equals the measure of its central angle, $m\widehat{NP} = 110^\circ$. Subtract this measure from 360° to find the measure of the major arc, giving $m\widehat{PQN} = 250^\circ$.

- b. By the Tangent Conjecture, $m\angle BAD = 90^\circ$ and $m\angle CDA = 90^\circ$. By the Quadrilateral Sum Conjecture, $90^\circ + 90^\circ + 80^\circ + w = 360^\circ$, so $w = 100^\circ$.

The measure of minor arc AT equals the measure of the central angle $\angle ABT$, so $m\widehat{AT} = 100^\circ$. Subtract this measure from 360° to find the measure of its major arc: $m\widehat{AXT} = 360^\circ - 100^\circ = 260^\circ$.

- c. $MNOP$ is a kite because $\overline{MN} \cong \overline{MP}$ and $\overline{ON} \cong \overline{OP}$.

6.2

Chord Properties

In this lesson you will

- Define **central angle** and **inscribed angle**
- Investigate properties of **chords** of a circle

Investigation 1: Defining Angles in a Circle

In your book, look at the examples and non-examples of central angles. What do the central angles have in common? What characteristics do the central angles have that the other angles do not have? Use your observations to complete this definition of a central angle.

A **central angle** is an angle whose vertex is _____.

Now look at the examples and non-examples of inscribed angles. Notice that each inscribed angle has its vertex on the circle. However, so does $\angle VWX$, which is *not* an inscribed angle. What makes the inscribed angles different from $\angle VWX$? Use your findings to complete this definition.

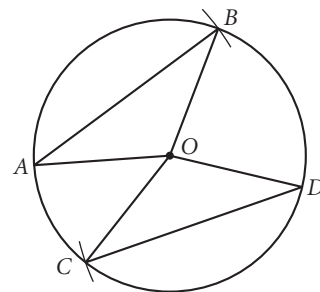
An **inscribed angle** is an angle whose vertex is _____ and whose sides _____.

Investigation 2: Chords and Their Central Angles

Construct a large circle and label its center O . Use your compass to construct two congruent chords. Label the chords \overline{AB} and \overline{CD} . Construct radii \overline{OA} , \overline{OB} , \overline{OC} , and \overline{OD} .

Use a protractor to measure $\angle BOA$ and $\angle COD$. How do the measures compare?

See if you get the same results if you start with a different circle. Then complete this conjecture.



Chord Central Angles Conjecture If two chords in a circle are congruent, then they determine two central angles that are _____.

C-55

You learned in Lesson 6.1 that a minor arc has the same measure as its central angle. If chords \overline{AB} and \overline{CD} are congruent and central angles $\angle BOA$ and $\angle COD$ are congruent, what can you conclude about their intercepted arcs, \widehat{AB} and \widehat{CD} ?

You can fold your paper to confirm your answer. Now complete the conjecture.

Chord Arcs Conjecture If two chords in a circle are congruent, then their _____ are congruent.

C-56

(continued)

Lesson 6.2 • Chord Properties (continued)

Investigation 3: Chords and the Center of the Circle

Follow Step 1 in your book. The perpendicular to each chord divides the chord into two segments. How do the lengths of the segments compare? Use your findings to complete this conjecture.

Perpendicular to a Chord Conjecture The perpendicular from the center of a circle to a chord is the _____ of the chord. **C-57**

Now use your compass to compare the distances (measured along the perpendicular) from the center to the chords. How do the distances compare?

Draw a circle of a different size and follow Step 1 in your book for the new circle. Compare the distances from the center of the circle to each chord. Are your findings the same as they were for the first circle? Complete the conjecture.

Chord Distance to Center Conjecture Two congruent chords in a circle are _____ from the center of the circle. **C-58**

Investigation 4: Perpendicular Bisector of a Chord

Construct a large circle and mark the center. Draw two nonparallel chords that are *not* diameters. Construct the perpendicular bisector of each chord and extend the bisectors until they intersect. Where is the point of intersection?

Construct another circle that is larger or smaller than the first circle you drew. Draw two chords and construct their perpendicular bisectors. Where do the perpendicular bisectors intersect? Complete this conjecture.

Perpendicular Bisector of a Chord Conjecture The perpendicular bisector of a chord passes through the _____ of the circle. **C-59**

Now you can find the center of any circle and the vertex of the central angle of any arc. All you have to do is construct the perpendicular bisectors of nonparallel chords.

6.3

Arcs and Angles

In this lesson you will

- Make conjectures about **inscribed angles** in a circle
- Investigate relationships among the angles in a **cyclic quadrilateral**
- Compare the arcs formed when two parallel lines intersect a circle

In this lesson you'll discover properties of arcs and the angles associated with them.

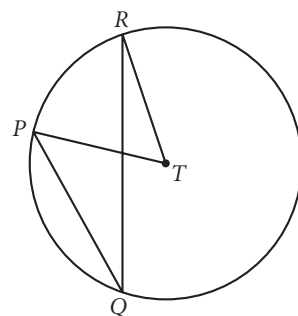
Investigation 1: Inscribed Angle Properties

Refer to the diagram of circle O in your book. Notice that $\angle COR$ is a *central angle*, while $\angle CAR$ is an *inscribed angle*. Both angles intercept \widehat{CR} .

Recall that the measure of a minor arc is equal to the measure of its central angle. Find $m\widehat{CR}$ by measuring $\angle COR$. Then find the measure of $\angle CAR$. How does the measure of the inscribed angle compare with the measure of its intercepted arc?

Construct your own circle T . Draw an inscribed angle PQR and its corresponding central angle PTR . An example is shown at right.

What is the measure of \widehat{PR} ? What is the measure of $\angle PQR$? How do the measures compare? Use your findings from this investigation to complete the conjecture.



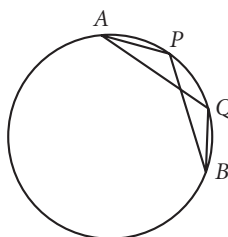
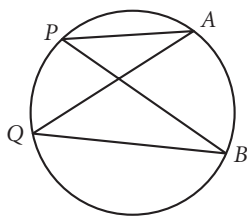
Inscribed Angle Conjecture The measure of an angle inscribed in a circle is _____ the measure of its intercepted arc.

C-60

Investigation 2: Inscribed Angles Intercepting the Same Arc

Follow Steps 1 and 2 in your book. Note that selecting P and Q on the major arc creates two angles, $\angle APB$ and $\angle AQB$, that intercept the same *minor arc*.

Draw another circle. Follow Step 4 in your book. Note that selecting P and Q on the minor arc creates two angles, $\angle APB$ and $\angle AQB$, that intercept the same *major arc*.



In each case, how do the measures of the two inscribed angles compare? Use your findings to complete the conjecture.

Inscribed Angles Intercepting Arcs Conjecture Inscribed angles that intercept the same arc are _____.

C-61

(continued)

Lesson 6.3 • Arcs and Angles (continued)

Investigation 3: Angles Inscribed in a Semicircle

Follow Steps 1 and 2 in your book to construct and measure three angles inscribed in the same semicircle. All the angles should have the same measure. What is the measure? Complete this conjecture.

Angles Inscribed in a Semicircle Conjecture Angles inscribed in a semicircle are _____.

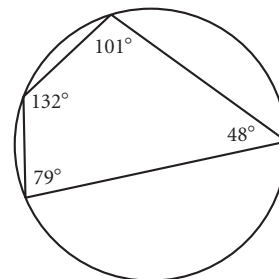
C-62

Investigation 4: Cyclic Quadrilaterals

A quadrilateral inscribed in a circle is called a **cyclic quadrilateral**.

Construct a large circle. Choose four points on the circle and connect them to form a cyclic quadrilateral. Label each angle with its measure. In another circle, construct a second cyclic quadrilateral and label its angles with their measures. At right is one more example.

In each cyclic quadrilateral, find sums of pairs of consecutive angles and sums of pairs of opposite angles. What do you discover? Complete this conjecture.



Cyclic Quadrilateral Conjecture The _____ angles of a cyclic quadrilateral are _____.

C-63

Investigation 5: Arcs by Parallel Lines

Read the definition of **secant** and follow Steps 1–3 in your book. Then complete the conjecture.

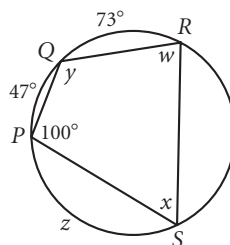
Parallel Lines Intercepted Arcs Conjecture Parallel lines intercept _____ arcs on a circle.

C-64

Read the example below.

EXAMPLE

Find each lettered measure.



► Solution

By the Cyclic Quadrilateral Conjecture, $w + 100^\circ = 180^\circ$, so $w = 80^\circ$. $\angle PSR$ intercepts \widehat{PR} . $m\widehat{PR} = 47^\circ + 73^\circ = 120^\circ$, so by the Inscribed Angle Conjecture, $x = \frac{1}{2}(120^\circ) = 60^\circ$.

By the Cyclic Quadrilateral Conjecture, $x + y = 180^\circ$. Substituting 60° for x and solving the equation gives $y = 120^\circ$. By the Inscribed Angle Conjecture, $w = \frac{1}{2}(47^\circ + z)$. Substituting 80° for w and solving the equation gives $z = 113^\circ$.

6.4

Proving Circle Conjectures

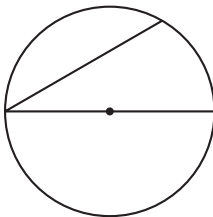
In this lesson you will

- Prove the **Inscribed Angle Conjecture**
- Prove other conjectures related to circles

In Lesson 6.3, you discovered the Inscribed Angle Conjecture.

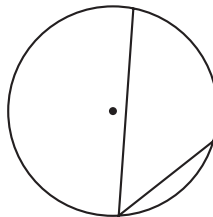
The measure of an inscribed angle in a circle equals half the measure of its intercepted arc.

In this lesson you will prove this conjecture. First, note that there are three ways an inscribed angle can relate to the circle's center.



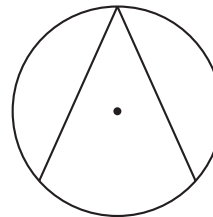
Case 1

The circle's center is on the angle.



Case 2

The center is outside the angle.



Case 3

The center is inside the angle.

These are the only possibilities, so if you can prove the conjecture is true for each case, you will have proved the conjecture is always true.

Case 1: The circle's center is on the inscribed angle.

Look at the drawing for Case 1 on page 330 of your book. The dashed radius \overline{OA} has been added to form $\triangle AOB$. To prove the conjecture is true for this case, you need to show that $\angle ABC = \frac{1}{2}m\widehat{AC}$ (that is, $x = \frac{1}{2}m\widehat{AC}$).

Notice the following things:

- $\angle AOC$ is the central angle for \widehat{AC} . (How are z and $m\widehat{AC}$ related?)
- $\angle AOC$ is an exterior angle for $\triangle AOB$. (How are x , y , and z related?)
- $\triangle AOB$ is isosceles. (How are x and y related?)

Combine these observations to write a flowchart proof for Case 1. Then, compare your proof with the one in your book.

Case 2: The circle's center is outside the inscribed angle.

You can use Case 1 to help you prove Case 2. Look at the drawing for Case 2 on page 331 of your book. A dashed diameter, \overline{DB} , has been added. To prove the conjecture is true for this case, you must show that $m\angle ABC = \frac{1}{2}m\widehat{AC}$ (that is, $x = \frac{q}{2}$).

Think about the following things:

- How are x , y , and z related?
- How are p , q , and $m\widehat{DC}$ related?
- Using Case 1, how is y related to p ? How is z related to $p + q$?

(continued)

Lesson 6.4 • Proving Circle Conjectures (continued)

Combine these observations to write a flowchart proof for Case 2. Then compare your proof with the one in your book.

Case 3: The circle's center is inside the inscribed angle.

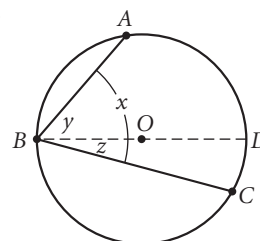
Use the given information and the marked figure in Example A below to write a flowchart proof for this case. You may want to use your flowchart from Case 2 as a guide for this proof. Write your proof before you look at the solution to Example A. Then compare your proof with the one given.

EXAMPLE A

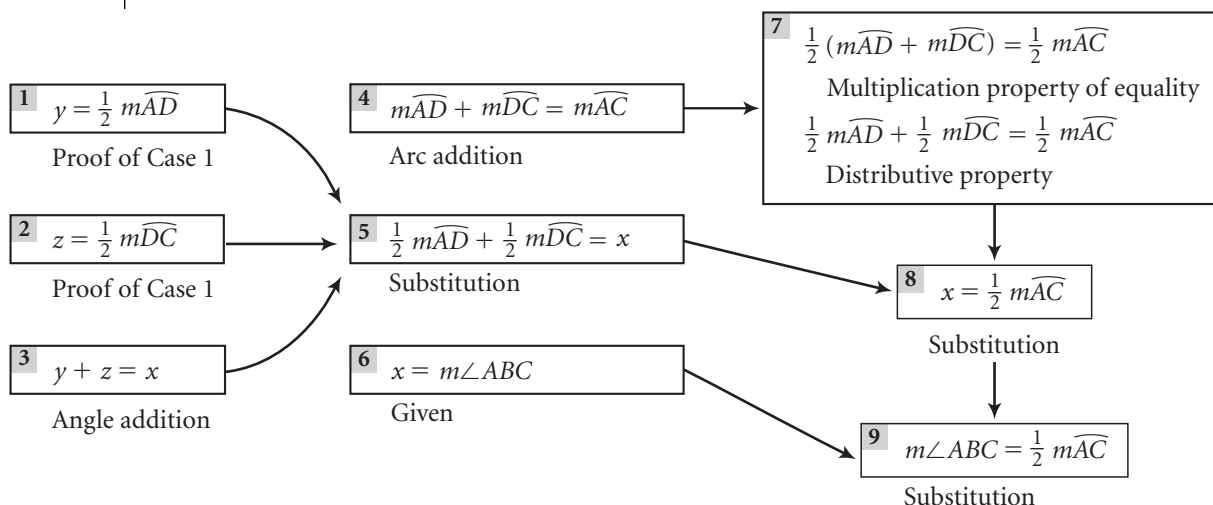
Prove Case 3 of the Inscribed Angle Conjecture: The measure of an inscribed angle in a circle equals half the measure of its intercepted arc when the center of the circle is inside the angle.

Given: Circle O with inscribed angle ABC whose sides lie on either side of diameter BD

Show: $m\angle ABC = \frac{1}{2}m\widehat{AC}$, or $x = \frac{1}{2}m\widehat{AC}$



► Solution

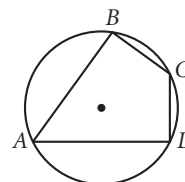


EXAMPLE B

Write a paragraph proof or a flowchart proof of the conjecture: If a kite is inscribed in a circle, then the nonvertex angles are right angles.

Given: Circle with inscribed kite $ABCD$ where $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{CD}$

Show: $\angle B$ and $\angle D$ are right angles



► Solution

By the Kite Angles Conjecture, $\angle B \cong \angle D$. By the Cyclic Quadrilateral Conjecture, $\angle B$ and $\angle D$ are supplementary. Because $\angle B$ and $\angle D$ are both equal and supplementary, they must have measures of 90° . Therefore, $\angle B$ and $\angle D$ are right angles.

6.5

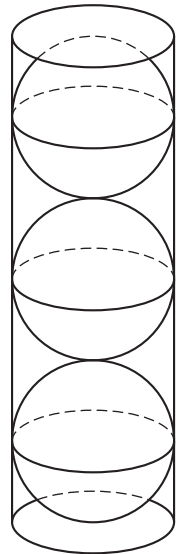
The Circumference/Diameter Ratio

In this lesson you will

- Discover the relationship between the **diameter** and the **circumference** of a circle

The distance around a polygon is called the perimeter. The distance around a circle is called the **circumference**.

Think of a can of three tennis balls. Which do you think is greater, the height of the can or the circumference of the can? The height is about three tennis-ball diameters, which is about the same as three can diameters. If you have a tennis-ball can, wrap a string around the can to measure the circumference, and then compare the measurement with the height. You should find that the circumference of the can is slightly greater than the height. That is, the circumference is a little more than three diameters. In the next investigation you will discover the relationship between the diameter and circumference of any circle.



Investigation: A Taste of Pi

For this investigation you will need several round objects, along with either a metric measuring tape or a string and a meterstick.

Wrap the measuring tape or string around each object to measure its circumference. Then measure the diameter of each object. Make all measurements to the nearest millimeter (tenth of a centimeter). The table below includes some sample measurements. Copy the table and add your findings to it.

Object	Circumference (C)	Diameter (d)	Ratio $\frac{C}{d}$
Quarter	7.8 cm	2.5 cm	
Compact disc	37.7 cm	12.0 cm	
Mug	25.9 cm	8.2 cm	
Small plate	47.4 cm	15.1 cm	

Calculate the ratio $\frac{C}{d}$ for each object. You should find that, in each case, the ratio is a little more than 3. In fact, the ratio $\frac{C}{d}$ is exactly the same number for every circle. This constant ratio is denoted by the Greek letter π (pi). So, $\frac{C}{d} = \pi$.

If you solve this equation for C, you get the formula for the circumference of a circle in terms of its diameter. Because the diameter of a circle is twice the radius, you can also write a formula for the circumference in terms of the radius.

Circumference Conjecture If C is the circumference and d is the diameter of a circle, then there is a number π such that $C =$ _____.
If $d = 2r$ where r is the radius, then $C =$ _____.

C-65

(continued)

Lesson 6.5 • The Circumference/Diameter Ratio (continued)

The number π is an *irrational number*, so its decimal form never ends and its digits follow no pattern. Your calculator probably gives an approximation of π to eight or nine decimal places. If you don't have access to a calculator, you can use the value 3.14 as an approximation for π . If you are asked to give an exact answer, state your answer in terms of π . The examples in your book show you how you can apply the Circumference Conjecture. Read the examples carefully. Here are two more examples.

EXAMPLE A | A circle has a radius of 6.5 meters. What is its circumference? Use a calculator and give your answer to the nearest 0.1 meter.

► **Solution** | $C = 2\pi r$ The formula for circumference.
 $C = 2\pi(6.5)$ Substitute 6.5 for r .
 $C = 13\pi \approx 40.8$ m Evaluate and round to the nearest 0.1 m.

EXAMPLE B | To find your hat size, you measure the circumference of your head in inches, and then use the circumference formula to find the diameter to the nearest eighth of an inch. The circumference of Tameka's head is about $23\frac{1}{2}$ in. What is her hat size?

► **Solution** | $C = \pi d$ The formula for circumference.
 $23\frac{1}{2} = \pi d$ Substitute $23\frac{1}{2}$ for C .
 $7\frac{1}{2} \approx d$ Divide both sides by π and round to the nearest eighth of an inch.
Tameka's hat size is $7\frac{1}{2}$.

6.6

Around the World

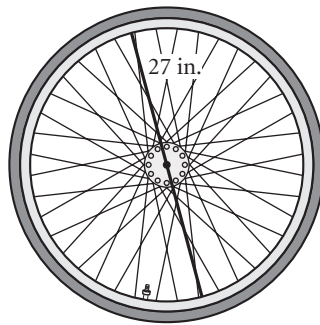
In this lesson you will

- Solve application problems involving radius, diameter, and circumference

Now that you have learned about π and the formula for circumference, you can solve many real-world problems involving circles. The example in your book shows the solution to a problem related to the novel *Around the World in Eighty Days*, by Jules Verne. Read this example carefully. Then read the examples below.

EXAMPLE A

The wheel on Devin's unicycle has a diameter of 27 inches.



- How far will the unicycle travel in 100 revolutions of the wheel? Give your answer to the nearest whole foot.
- Devin rides his unicycle 2 miles to school each day. About how many revolutions does the wheel make during his trip? (5280 feet = 1 mile)

► Solution

- In one revolution, the wheel covers a distance equal to its circumference.

$$C = \pi d \quad \text{The formula for circumference.}$$

$$C = \pi(27) \quad \text{Substitute 27 for } d.$$

The unicycle travels 27π inches in one revolution. So, in 100 revolutions, it covers $(100) \cdot 27\pi \approx 8482$ inches, or about 707 feet.

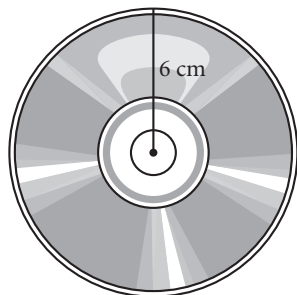
- Two miles is 10,560 feet, or 126,720 inches. The wheel makes one revolution every 27π inches, so it makes $126,720 \div 27\pi$, or about 1494 revolutions during Devin's trip to school.

(continued)

Lesson 6.6 • Around the World (continued)

EXAMPLE B

An audio CD spins at a rate of 200 rotations per minute. If the radius of a CD is 6 cm, how far does a point on the outer edge travel during the playing of a 57-minute CD?



► Solution

The point makes 200 rotations in 1 minute. So, in 57 minutes, it will make $200 \cdot 57$, or 11,400 rotations.

The distance the point travels in one rotation is equal to the circumference of the CD. Use the circumference formula to find this distance.

$$C = 2\pi r \quad \text{The formula for circumference.}$$

$$C = 2\pi(6) \quad \text{Substitute 6 for } r.$$

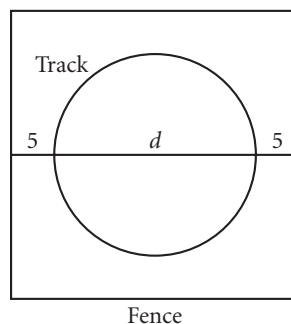
The point travels 12π cm in one rotation. So, in 11,400 rotations, it will travel $11,400 \cdot 12\pi$, or 429,770 cm. This is equal to about 4.3 kilometers.

EXAMPLE C

The city of Rutledge wants to build a circular walking track that is a quarter-mile long. To keep vehicles off the track, the city plans to build a fence around the track. The fenced area will be square and the fence, at its closest point, will be 5 feet from the edge of the circular track. How many feet of fencing will be needed? Give your answer to the nearest foot and ignore any gates or openings. (1 mile = 5280 feet)

► Solution

Refer to the sketch at right.



First find the diameter of the track. The circumference of the track is $5280/4$, or 1320 feet. Using $C = \pi d$, solve for d to get $d = C/\pi$. So, $d = 1320/\pi$, or $d \approx 420.2$ feet. Each side of the square fence is 10 feet longer than the diameter of the track. Therefore, each side of the square must be about 430.2 feet. The amount of fencing needed is about $4 \cdot 430.2 = 1720.8$ feet, or to the nearest foot, 1721 feet.

6.7

Arc Length

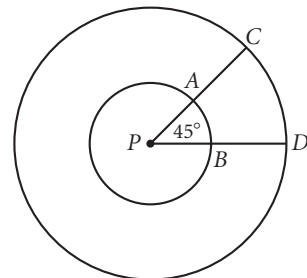
In this lesson you will

- Learn the difference between **arc length** and **arc measure**
- Find a method for calculating arc length
- Solve problems involving arc length

In the figure at right, \widehat{AB} and \widehat{CD} have the same *measure* (45°) because their central angles, $\angle APB$ and $\angle CPD$, are the same angle. However, the *lengths* of \widehat{AB} and \widehat{CD} are clearly different— \widehat{CD} is longer than \widehat{AB} . In this lesson you'll learn how to find the length of an arc.

Read Example A in your book. It illustrates that if an arc has measure x , then the arc is $\frac{x}{360^\circ}$ of the circle. For example, each of the 45° arcs in the figure at right is $\frac{45^\circ}{360^\circ}$, or $\frac{1}{8}$, of a complete circle.

Think about how the fractions in Example A are related to the length of the arc. If you travel a quarter of the way around a circle, you cover $\frac{1}{4}$ of its circumference. If you travel halfway around, you cover $\frac{1}{2}$ the circumference. The length of an arc, or **arc length**, is a fraction of the circumference of its circle. While arc measure is expressed in degrees, arc length is expressed in a unit of distance.



Investigation: Finding the Arcs

Look at the figures in Step 1 in your book. Find the fraction of the circle each arc makes up. You should get the following results.

\widehat{AB} is $\frac{1}{4}$ of circle T

\widehat{CED} is $\frac{1}{2}$ of circle O

\widehat{GH} is $\frac{140^\circ}{360^\circ}$, or $\frac{7}{18}$, of circle P

Now find the circumference of each circle using the formula $C = \pi d$ or $C = 2\pi r$. For example, because circle P has radius 36 feet, its circumference is $2\pi(36)$, or 72π feet.

Use the fractions you found in Step 1 and the circumference of each circle to find the length of each arc. For example, \widehat{GH} makes up $\frac{7}{18}$ of circle P , and circle P has a circumference of 72π . So,

$$\text{Length of } \widehat{GH} = \frac{7}{18}(72\pi) = 28\pi$$

The length of \widehat{GH} is 28π feet.

Generalize this method for finding the length of an arc as a conjecture.

Arc Length Conjecture The length of an arc equals the circumference times C-66
the arc measure divided by _____.

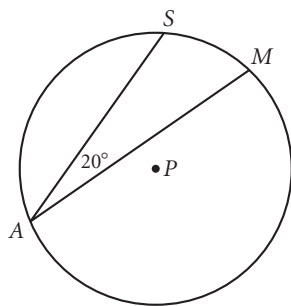
(continued)

Lesson 6.7 • Arc Length (continued)

Examples B and C in your book show how you can apply the Arc Length Conjecture. Read these examples carefully. Then read the example below.

EXAMPLE

The length of \widehat{SM} is 6π centimeters. What is the radius of circle P ?



► Solution

$m\angle SAM = 20^\circ$, so $m\widehat{SM} = 40^\circ$ by the Inscribed Angle Conjecture. This means that \widehat{SM} is $\frac{40^\circ}{360^\circ}$, or $\frac{1}{9}$, of the circumference of the circle.

Arc length $\widehat{SM} = \frac{1}{9}C$ Use the Arc Length Conjecture.

$$6\pi = \frac{1}{9}C \quad \text{Substitute } 6\pi \text{ for the arc length.}$$

$$54\pi = C \quad \text{Multiply both sides by 9.}$$

The circumference is 54π centimeters. Use the circumference formula to find the radius.

$$C = 2\pi r \quad \text{The circumference formula.}$$

$$54\pi = 2\pi r \quad \text{Substitute } 54\pi \text{ for the arc length.}$$

$$27 = r \quad \text{Divide both sides by } 2\pi.$$

The radius is 27 centimeters.

7.1

Transformations and Symmetry

In this lesson you will

- Learn about three types of **rigid transformations**—**translation**, **rotation**, and **reflection**
- Use patty paper to model reflection
- Learn how to identify figures with **rotational symmetry** or **reflectional symmetry**

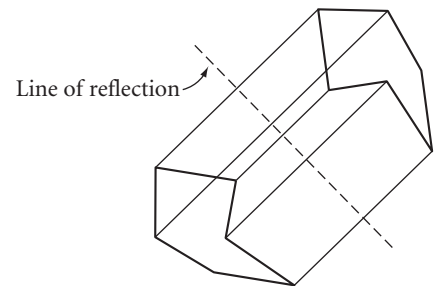
In your book, read the text before the investigation. Below is a summary of some of the key points.

1. A transformation that creates an image that is congruent to the original figure is called a **rigid transformation**, or **isometry**. Three types of rigid transformations are **translation**, **rotation**, and **reflection**.
2. A transformation that changes the size or shape of a figure is a **nonrigid transformation**.
3. A **translation** slides a figure along a straight-line path, moving each point the same distance in the same direction. You can describe a translation using a **translation vector**, which specifies both the distance and the direction.
4. A **rotation** turns a figure about a fixed point, rotating each point the same number of degrees. You define a rotation by giving the center point, the number of degrees, and the direction (clockwise or counterclockwise). When a direction is not specified, the rotation is assumed to be counterclockwise.
5. A **reflection** flips a figure across a line, creating the mirror image of the figure. You define a reflection by specifying the **line of reflection**.

Investigation: The Basic Property of a Reflection

Follow Steps 1 and 2 in your book to create a figure and its reflected image. Then draw segments connecting each vertex with its image point.

Measure the angles where the segments intersect the line of reflection. What do you find? The line of reflection divides each connecting segment into two segments. How do the lengths of the two segments compare? Use your findings to complete this conjecture.



Reflection Line Conjecture The line of reflection is the _____ of every segment joining a point in the original figure with its image.

C-67

(continued)

Lesson 7.1 • Transformations and Symmetry (continued)

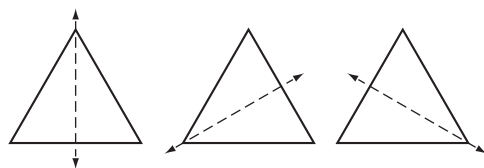
Read the remaining text in this lesson. Below is a summary of the key points.

1. A figure that can be reflected across a line so that the resulting image coincides with the original has **reflectional symmetry**. The reflection line is called a **line of symmetry**. A figure can have more than one line of symmetry. A figure with n lines of symmetry has n -fold reflectional symmetry.
2. A figure that can be rotated *less than a full turn* about a point so that the rotated image coincides with the original has **rotational symmetry**. If the image coincides with the original figure n times during a full turn, then the figure is said to have n -fold rotational symmetry. A figure that has 2-fold rotational symmetry is also said to have **point symmetry**.

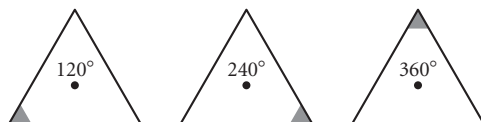
EXAMPLE | Describe all the symmetries of an equilateral triangle.

► **Solution**

An equilateral triangle has three reflectional symmetries. There is a line of reflection through each vertex and the midpoint of the opposite side.



An equilateral triangle has three rotational symmetries (that is, it has 3-fold rotational symmetry). It can be rotated 120° , 240° , and 360° about its center, and it will coincide with itself.



7.2

Properties of Isometries

In this lesson you will

- Use **ordered pair rules** to transform polygons in the coordinate plane
- Learn the ordered pair rules that correspond to various **isometries**
- Discover how to determine the **minimal path** from a point to a line to another point on the same side of the line

You can use an ordered pair rule to transform figures in the coordinate plane. An **ordered pair rule** describes how each point in an original figure is relocated to create an image.

Example A in your book illustrates that the rule $(x, y) \rightarrow (x + 2, y - 3)$ is a translation that moves each point of a figure right 2 units and down 3 units. The translation vector is written $\langle 2, -3 \rangle$. Read Example A. In general, the rule $(x, y) \rightarrow (x + h, y + k)$ is a translation of h units horizontally and k units vertically. The general translation vector is written $\langle h, k \rangle$.

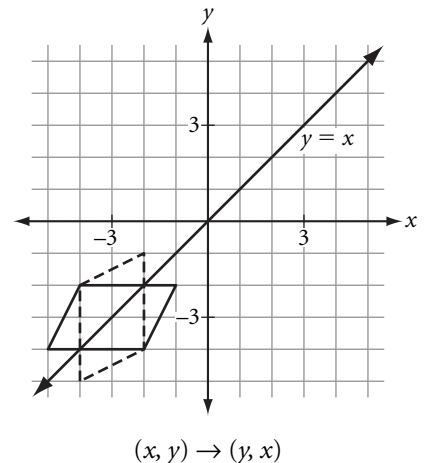
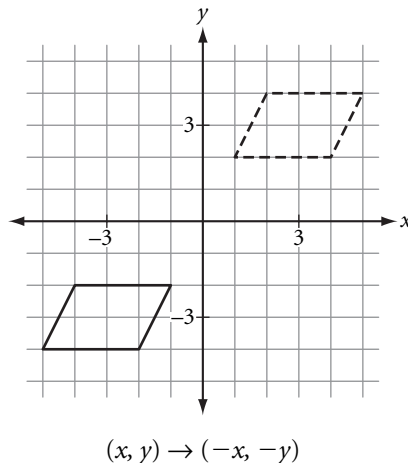
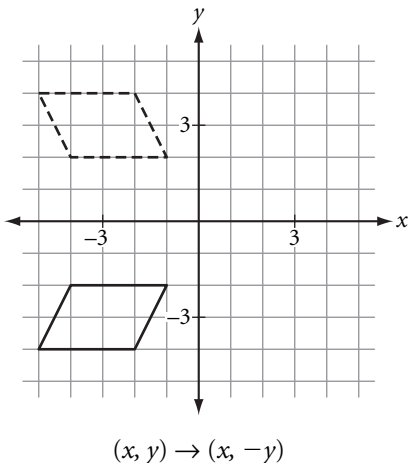
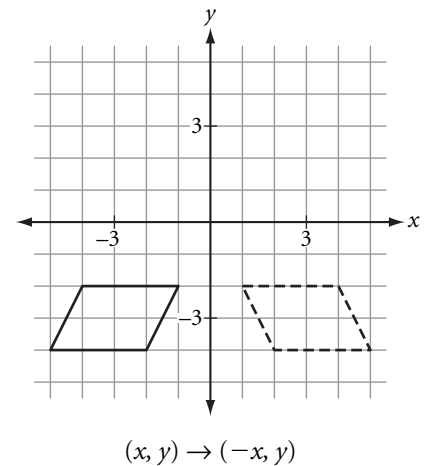
Investigation 1: Transformations on a Coordinate Plane

In this investigation you will explore four ordered pair rules.

Follow Steps 1 and 2 in your book. Draw your original polygon in Quadrant I, II, or IV. (Here are some examples in which the original polygon is in Quadrant III.)

Using your drawings and some patty paper, determine whether each transformation is a reflection, translation, or rotation. Identify the lines of reflection and centers and angles of rotation.

Now consider the examples. In each graph, the solid figure in Quadrant III is the original, and the dashed figure is the image. (Note: The line $y = x$ has been added to the last graph.) Determine how the original figure has been transformed to create the image. Do you get the same results you got for your polygon?



(continued)

Lesson 7.2 • Properties of Isometries (continued)

Use your findings to complete the Coordinate Transformations Conjecture in your book.

Now you will revisit “poolroom geometry.” Recall that when a ball rolls without spin into a cushion, the outgoing angle is congruent to the incoming angle.

Investigation 2: Finding a Minimal Path

Follow Steps 1–4 of the investigation in your book. Then unfold the paper and draw $\overline{AB'}$. Notice that $\overline{AB'}$ passes through point C .

Measure the length of the path from A to B' . Measure the length of the two-part path from A to C to B . You should find that the lengths of the paths are the same.

The path from A to C to B is the shortest path, or the **minimal path**, from A to the cushion to B . To see why, choose any other point D on the cushion. The path from A to D to B is the same length as the path from A to D to B' . $\overline{AB'}$ is shorter than the path from A to D to B' (why?), so it is also shorter than the path from A to D to B . Because $\overline{AB'}$ is the same length as the path from A to C to B , the path from A to C to B is shorter than the path from A to D to B . This argument is given in symbols below.

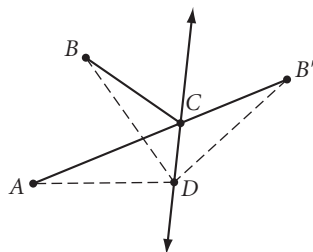
$$AD + DB = AD + DB'$$

$$AB' < AD + DB'$$

$$AB' < AD + DB$$

$$AB' = AC + CB$$

$$AC + CB < AD + DB$$



Complete this conjecture.

Minimal Path Conjecture If points A and B are on one side of line ℓ , then the minimal path from point A to line ℓ to point B is found by reflecting point _____ across line ℓ , drawing segment _____, then drawing segments AC and _____, where point C is the point of intersection of segment _____ and line ℓ .

C-69

Your findings in the investigation show that if you want to hit a ball from point A off the cushion so it passes through point B , you should visualize point B reflected across the cushion and then aim at the reflected image.

Example B in your book applies what you learned in Investigation 2 to solve a problem about miniature golf. Read this example.

7.3

Compositions of Transformations

In this lesson you will

- Find the single transformation equivalent to the **composition of two translations**
- Find the single transformation equivalent to the **composition of reflections across two parallel lines**
- Find the single transformation equivalent to the **composition of reflections across two intersecting lines**

When you apply a transformation to a figure and then apply another transformation to its image, the resulting transformation is called a **composition** of transformations.

In the example in your book, a figure is translated by one rule and then its image is translated by a different rule. The composition of the two translations is equivalent to a single translation. Read this example carefully and make sure you understand it.

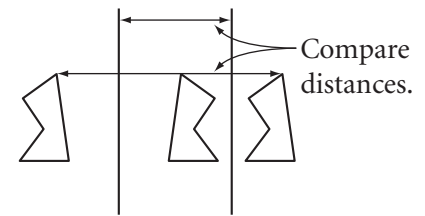
Investigation 1: Reflections across Two Parallel Lines

Follow Steps 1–4 to reflect a figure across one line and then reflect the image across a second line, parallel to the first.

What single transformation would take the original figure to the final image?
(*Hint:* How does the orientation of the final image compare with the orientation of the original?)

Use a compass or patty paper to compare the distance between the parallel lines to the distance between a point on the original figure and the corresponding point on the final image.

Use your findings to complete this conjecture.



Reflections across Parallel Lines Conjecture A composition of two reflections across two parallel lines is equivalent to a single _____.

C-70

In addition, the distance from any point to its second image under the two reflections is _____ the distance between the parallel lines.

Read the example on the next page.

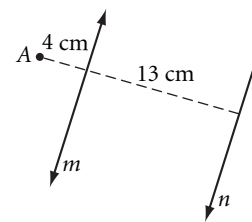
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Lesson 7.3 • Compositions of Transformations (continued)

EXAMPLE

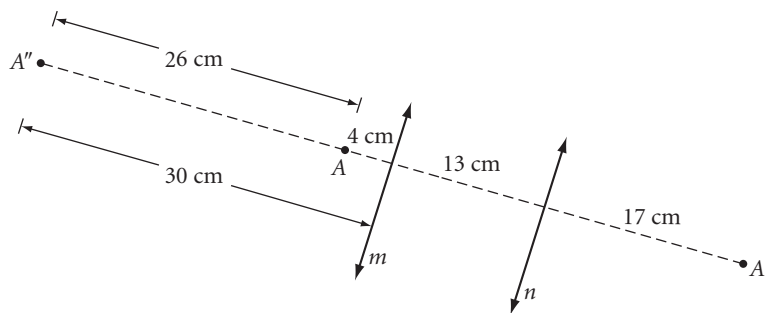
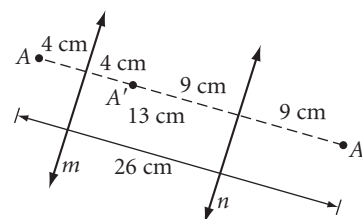
Lines m and n are parallel and 13 cm apart.

- Point A is 4 cm from line m and 17 cm from line n . Point A is reflected across line m , and then its image, A' , is reflected across line n to create a second image, point A'' . How far is point A from point A'' ?
- What if A is reflected across n , and then its image is reflected across m ? Find the new image and distance.



► Solution

- By the Reflections across Parallel Lines Conjecture, the distance between A and A'' is 26 cm, twice the distance between the lines. A drawing verifies this.
- By the Reflections across Parallel Lines Conjecture, the distance between A and A'' is 26 cm. A drawing verifies this.

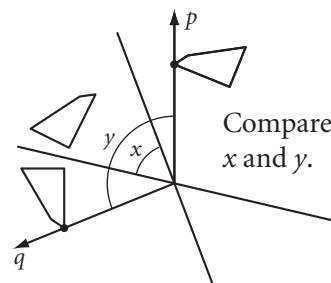


Investigation 2: Reflections across Two Intersecting Lines

Follow Steps 1–4 to reflect a figure across one line and then reflect the image across a second line that intersects the first.

Draw two rays, p and q , that start at the point of intersection of the two lines and that pass through corresponding points on the original figure and its second image. What single transformation would take the original figure to the final image?

You should have found that the two reflections are equivalent to a single rotation. Use a protractor to compare the angle of rotation (that is, the angle created by rays p and q) with the acute angle formed by the intersecting lines. Use your findings to complete this conjecture.



Reflections across Intersecting Lines Conjecture A composition of two reflections across a pair of intersecting lines is equivalent to a single _____. The angle of _____ is _____ the acute angle between the pair of intersecting reflection lines.

C-71

There are many other ways to combine transformations. For example, the composition of a translation and a reflection is called a **glide reflection**. Page 386 of your book shows examples of glide reflection.

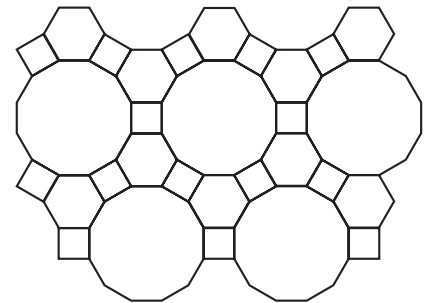
Tessellations with Regular Polygons

In this lesson you will

- Learn about the three **regular tessellations**
- Discover all the possible **semiregular tessellations**

An arrangement of shapes that covers a plane completely without gaps or overlaps is called a **tessellation** or **tiling**. Read the text in your book before the investigation. Here is a summary of the main points.

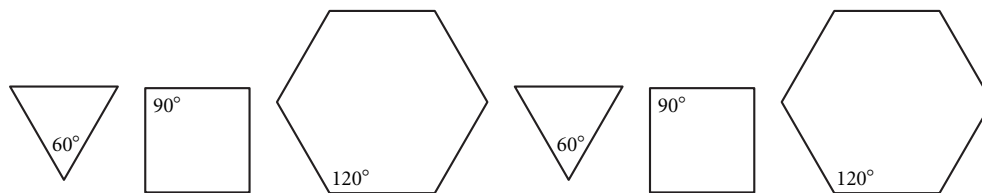
1. For shapes to create a tessellation, their angles, when arranged around a point, must have measures that add to exactly 360° .
2. A tiling that uses only one shape is called a **monohedral tessellation**.
3. A monohedral tessellation of congruent regular polygons is called a **regular tessellation**. The only polygons that create a regular tessellation are equilateral triangles, squares, and regular hexagons. (These are the regular polygons with angle measures that are factors of 360° .)
4. When the same combination of two or more regular polygons meet in the same order at each vertex of a tessellation, it is called a **semiregular tessellation**.
5. You can describe a tessellation by giving its **vertex arrangement**, or **numerical name**. To name a tessellation, list the number of sides of each shape, in order as you move around a vertex. For example, each vertex of the tessellation at right is surrounded by a square (4 sides), a hexagon (6 sides), and a dodecagon (12 sides). So, the numerical name for this tessellation is 4.6.12.



Investigation: The Semiregular Tessellations

There are eight different semiregular tessellations. Your book shows three ($4.8.8$, or 4.8^2 , $4.6.12$, and $3.12.12$, or 3.12^2). In this investigation you will find the other five. All five are made from combinations of triangles, squares, and hexagons.

You will need triangles, squares, and hexagons either from a set of pattern blocks or traced or copied from the set below and cut out. If you have geometry software available, you may use that instead.

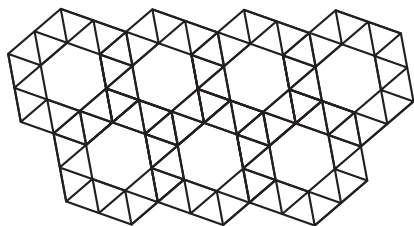


First, look for combinations of *two* polygons that can be used to create a semiregular tessellation. Start by finding combinations of angle measures that add to 360° . For example, because $4 \cdot 60^\circ + 120^\circ = 360^\circ$, four triangles and one hexagon could be arranged around a vertex. Try to find a way to arrange the shapes so the pattern can be continued indefinitely. (Remember, the polygons must meet in the *same order* at each vertex.)

(continued)

Lesson 7.4 • Tessellations with Regular Polygons (continued)

Here is the tessellation, labeled with its numerical name.



3.3.3.3.6 or $3^4.6$

Find the remaining four semiregular tessellations that can be made with two different polygons. Sketch each one and label it with its numerical name.

Now look for combinations of *three* polygons that can be used to create semiregular tessellations. Again, first find combinations of angle measures that add to 360° , and then make the tessellation. Find at least one semiregular tessellation that can be made with three different polygons. Sketch it and label it with its numerical name.



Read the remaining text in this lesson. Here is a summary of the key points.

1. The three regular and eight semiregular tessellations are called the **Archimedean tilings**.
2. The regular and semiregular tessellations are also called *1-uniform tilings* because all the vertices are identical. A tessellation with two types of vertices is called *2-uniform*, a tessellation with three types of vertices is called *3-uniform*, and so on. (See the illustrations in your book for examples.)

Tessellations with Nonregular Polygons

In this lesson you will

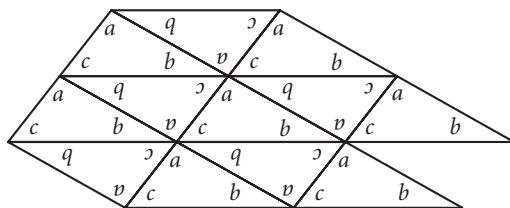
- Determine whether all triangles tessellate
- Determine whether all quadrilaterals tessellate
- Look at some examples of pentagonal tessellations

In Lesson 7.4, you investigated tessellations that were formed from regular polygons. Now you will try to create tessellations from nonregular polygons.

Investigation 1: Do All Triangles Tessellate?

Follow the steps for “Making Congruent Triangles” on page 394 in your book to create and label 12 congruent scalene triangles. Try to form a tessellation using the triangles. Here is an example.

Look at your tessellation and the tessellation below. You should find that, in both tessellations, each angle fits twice around each vertex point.



The sum of the measures of the angles of a triangle is 180° . Because each angle fits twice around each point, the sum of the measures of the angles around each point is $2(180^\circ)$, or 360° . As you saw in Lesson 7.4, this is the sum of the angles surrounding each vertex in *any* tessellation.

Do you think you would be able to create a tessellation using an isosceles triangle? Try it and see.

Your findings from this investigation lead to the following conjecture.

Tessellating Triangles Conjecture Any triangle will create a monohedral tessellation.

C-72

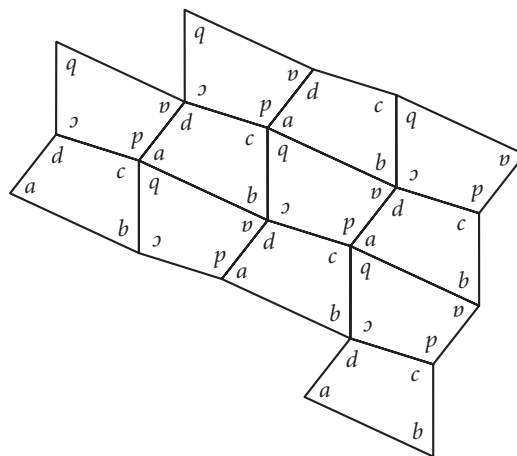
You know that squares and rectangles can tile a plane, and you can probably visualize tiling with parallelograms. Will any quadrilateral tessellate? You will explore this question in the next investigation.

(continued)

Investigation 2: Do All Quadrilaterals Tessellate?

Create 12 congruent quadrilaterals (not parallelograms or trapezoids), and label the corresponding angles in each quadrilateral a , b , c , and d . Try to form a tessellation using the quadrilaterals. Here is an example.

Look at your tessellation and the tessellation at right. You should find that, in both tessellations, each angle fits once around each vertex point. This makes sense because the sum of the angle measures of a quadrilateral is 360° . Your findings lead to this conjecture.



Tessellating Quadrilaterals Conjecture Any quadrilateral will create a monohedral tessellation.

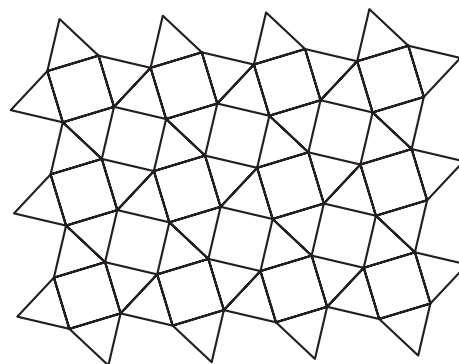
C-73

You know that a regular pentagon will not tessellate. However, it is possible to create tessellations from other types of pentagons. Pages 395 and 396 of your book show some examples. So far, 14 types of pentagons have been shown to tessellate. No one currently knows if there are more.

In the next example, you will create a pentagonal tessellation. This example is Exercise 2 in your book. Refer to Exercises 9 and 10 in Lesson 7.4 for a description of *dual* tessellations.

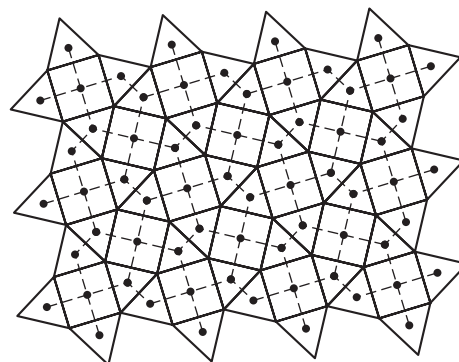
EXAMPLE

Produce a pentagonal tessellation by creating the *dual* of the semiregular tessellation at right.



► Solution

Recall that the dual is created by connecting the centers of the polygons that surround each vertex point. Here is the result.



8.1

Areas of Rectangles and Parallelograms

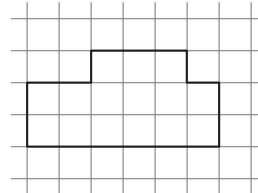
In this lesson you will

- Review the formula for the area of a rectangle
- Use the area formula for rectangles to find areas of other shapes
- Discover the formula for the area of a parallelogram

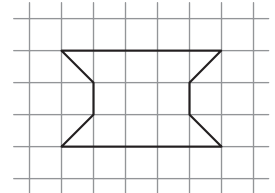
The **area** of a plane figure is the number of square units that can be arranged to fill the figure completely.

You probably already know several area formulas. The investigations in this chapter will help you understand and remember the formulas.

Pages 422 and 423 of your book discuss the formula for the area of a rectangle. Read this text carefully. Make sure that you understand the meaning of **base** and **height** and that the area formula makes sense to you, then complete the Rectangle Area Conjecture in your book. Example A in your book shows how the area formula for rectangles can help you find areas of other shapes. Here is another example.



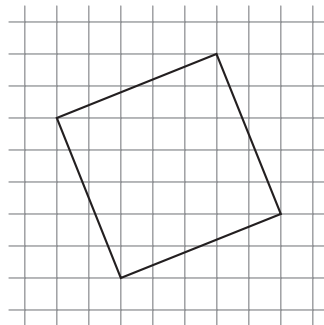
Area = 15 square units



Area = 11 square units

EXAMPLE A

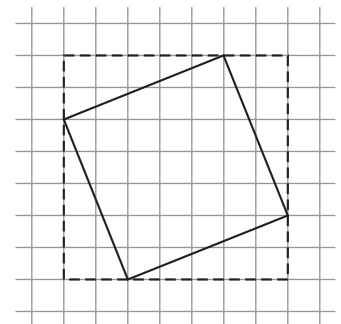
Find the area of this square.



► Solution

Surround the given “slanted” square with a 7-by-7 square with horizontal and vertical sides. Then subtract the area of the four right triangles formed from the area of the surrounding square.

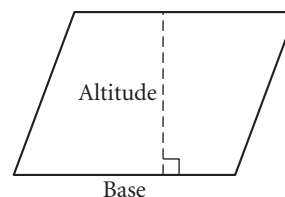
Each of the four triangles is half of a 2-by-5 rectangle, so each has area $\frac{1}{2} \cdot 2 \cdot 5$, or 5 square units. Therefore, the area of the original square is $(7 \cdot 7) - (4 \cdot 5) = 29$ square units.



(continued)

Lesson 8.1 • Areas of Rectangles and Parallelograms (continued)

Just as with a rectangle, any side of a parallelogram can be called the *base*. An **altitude** of a parallelogram is any segment from one side of the parallelogram, perpendicular to that side, to a line through the opposite side. The *height* of a parallelogram is the length of the altitude. Study the diagrams of altitudes on page 424 of your book.

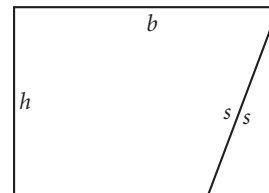


Investigation: Area Formula for Parallelograms

Follow Steps 1 and 2 of the investigation in your book. In Step 2, each new shape you form has the same area as the original parallelogram because you have simply rearranged pieces, without adding or removing any cardboard.

Form a rectangle with the two pieces.

Notice that the base and height of the rectangle are the same as the base and height of the original parallelogram. Because the area of the rectangle and the parallelogram are the same, the area of the parallelogram is bh . This can be summarized as a conjecture.



Parallelogram Area Conjecture The area of a parallelogram is given by the formula $A = bh$, where A is the area, b is the length of the base, and h is the height of the parallelogram.

C-75

If the dimensions of a figure are measured in inches, feet, or yards, the area is measured in in^2 (square inches), ft^2 (square feet), or yd^2 (square yards). If the dimensions are measured in centimeters or meters, the area is measured in cm^2 (square centimeters) or m^2 (square meters). Read Example B in your book, and then read the example below.

EXAMPLE B | A parallelogram has height 5.6 ft and area 70 ft^2 . Find the length of the base.

► **Solution**

$$A = bh \quad \text{Write the formula.}$$

$$70 = b(5.6) \quad \text{Substitute the known values.}$$

$$\frac{70}{5.6} = b \quad \text{Solve for the base length.}$$

$$12.5 = b \quad \text{Divide.}$$

The length of the base is 12.5 ft.

Areas of Triangles, Trapezoids, and Kites

In this lesson you will

- Discover area formulas for triangles, trapezoids, and kites

You can use the area formulas you already know to derive new area formulas.

In the first investigation you will focus on triangles.

Investigation 1: Area Formula for Triangles

Follow Step 1 in your book to create and label a pair of congruent triangles.

You know the area formula for rectangles and parallelograms. Arrange the two congruent triangles to form one of these figures. Write an expression for the area of the entire figure. Then write an expression for the area of one of the triangles.

Summarize your findings by completing the conjecture below.

Triangle Area Conjecture The area of a triangle is given by the formula _____, where A is the area, b is the length of the base, and h is the height of the triangle.

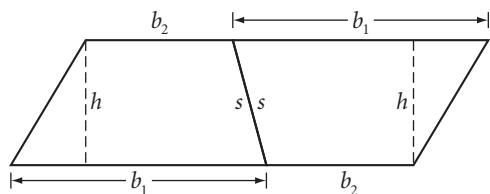
C-76

Next, you'll consider the area of a trapezoid.

Investigation 2: Area Formula for Trapezoids

Follow Steps 1 and 2 in your book to make and label two congruent trapezoids.

You can arrange the trapezoids to form a parallelogram.



What is the base length of the parallelogram? What is the height? Use your answers to the questions to write an expression for the area of the parallelogram. Then use the expression for the area of the parallelogram to write an expression for the area of one trapezoid.

Summarize your findings by completing this conjecture.

Trapezoid Area Conjecture The area of a trapezoid is given by the formula _____, where A is the area, b_1 and b_2 are the lengths of the two bases, and h is the height of the trapezoid.

C-77

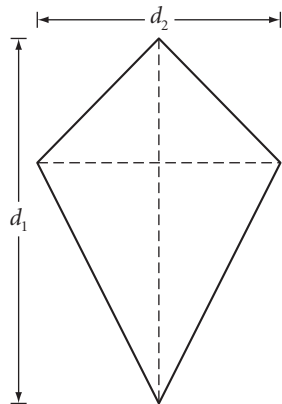
(continued)

Lesson 8.2 • Areas of Triangles, Trapezoids, and Kites (continued)

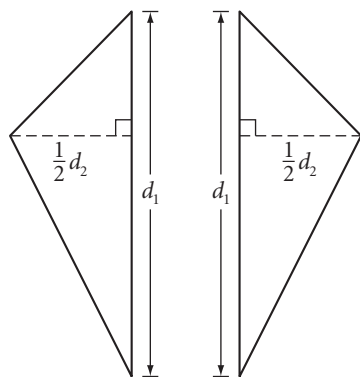
Finally, you will consider the area of a kite.

Investigation 3: Area Formula for Kites

Draw a kite. Draw its diagonals. Let d_1 be the length of the diagonal connecting the vertex angles, and let d_2 be the length of the other diagonal.



Recall that the diagonal connecting the vertex angles of a kite divides it into two congruent triangles. Consider the diagonal labeled d_1 to be the base of one of the triangles. Then, because the diagonal connecting the vertex angles of a kite is the perpendicular bisector of the other diagonal, the height of the triangle is $\frac{1}{2}d_2$.



Write an expression for the area of one of the triangles. Then use the expression for the area of the triangle to write an expression for the area of the kite.

Summarize your findings by completing this conjecture.

Kite Area Conjecture The area of a kite is given by the formula _____, where A is the area and d_1 and d_2 are the lengths of the diagonals.

C-78

8.3

Area Problems

In this lesson you will

- Use a variety of strategies to approximate the areas of irregularly shaped figures
- Use the area formulas from the previous two lessons to find the areas of more complex figures

You have discovered formulas for areas of rectangles, parallelograms, triangles, trapezoids, and kites. In this lesson you will use these formulas, along with other methods, to find the approximate areas of irregularly shaped figures.

Investigation: Solving Problems with Area Formulas

On the next page, you'll find eight geometric figures. For each figure, find a way to calculate the approximate area. Then record the area and write a sentence or two explaining how you found it. It may help to trace a figure onto another sheet of paper.

Below are some hints for how you might find the area of each figure. Read these hints only if you get stuck. There are lots of ways to find each area. The methods you use may be very different from those described here.

Figure A Divide the figure into rectangles.

Figure B This figure is a kite. Use what you learned in Lesson 8.2 to find the area.

Figure C This figure is a parallelogram. Use what you learned in Lesson 8.1 to find the area.

Figure D Divide the figure into triangles.

Figure E This figure is a trapezoid. Use what you learned in Lesson 8.2 to find the area.

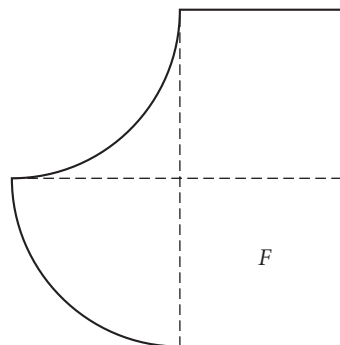
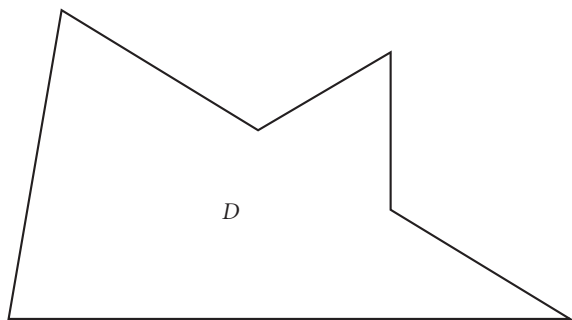
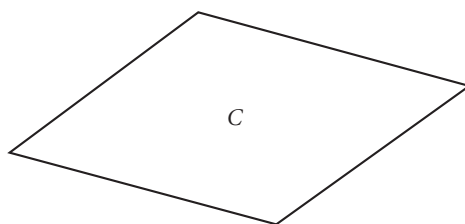
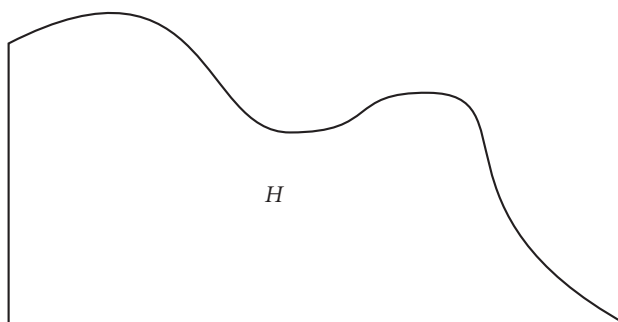
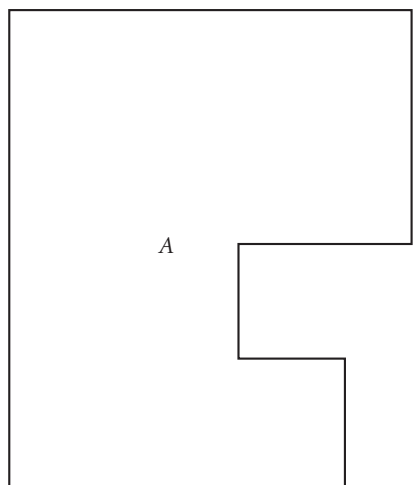
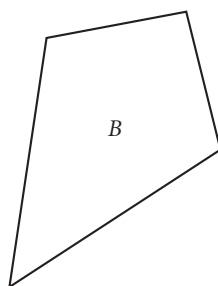
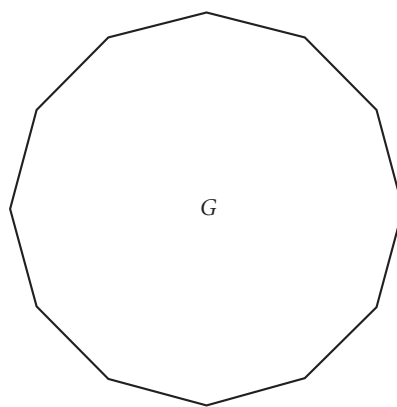
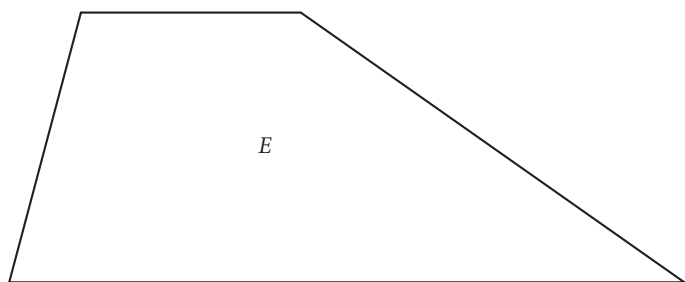
Figure F Find the area of the two squares. Cut out the other two pieces and rearrange them to form a recognizable shape.

Figure G Divide this dodecagon into 12 identical, isosceles triangles with vertex angles at the "center" of the polygon.

Figure H Trace the figure onto a sheet of graph paper. Estimate the number of squares that fit inside the figure. Or draw as large a rectangle as will fit in the shape. Cut off the remaining pieces and arrange them to form recognizable shapes.

(continued)

Lesson 8.3 • Area Problems (continued)



8.4

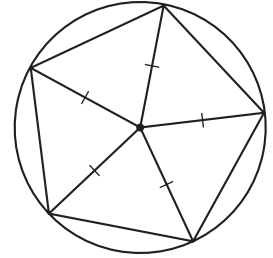
Areas of Regular Polygons

In this lesson you will

- Discover the area formula for regular polygons

You can divide a regular polygon into congruent isosceles triangles by drawing segments from the center of the polygon to each vertex. The center of the polygon is actually the center of the circumscribed circle, so each of these congruent segments is called a radius of the regular polygon.

In the investigation you will divide regular polygons into triangles. Then you will write a formula for the area of any regular polygon.



Investigation: Area Formula for Regular Polygons

The **apothem** of a regular polygon is a perpendicular segment from the center of the polygon's circumscribed circle to a side of the polygon. The apothem is also the length of the segment. Follow the steps in your book to find the formula for the area of a regular n -sided polygon with sides of length s and apothem a . Your findings can be summarized in this conjecture.

Regular Polygon Area Conjecture The area of a regular polygon is given by the formula $A = \frac{1}{2}asn$ or $A = \frac{1}{2}aP$, where A is the area, P is the perimeter, a is the apothem, s is the length of each side, and n is the number of sides.

C-79

The examples below show you how to apply your new formulas.

EXAMPLE A The area of a regular nonagon is about 302.4 cm^2 and the apothem is about 9.6 cm. Find the approximate length of each side.

► **Solution**

Because you are trying to find the side length, s , it is probably easier to use the formula $A = \frac{1}{2}asn$. You could also use $A = \frac{1}{2}aP$, solve for P , and then divide the result by 9 (the number of sides).

$$A = \frac{1}{2}asn \quad \text{Write the formula.}$$

$$302.4 \approx \frac{1}{2}(9.6)(s)(9) \quad \text{Substitute the known values.}$$

$$302.4 \approx 43.2s \quad \text{Multiply.}$$

$$\frac{302.4}{43.2} \approx s \quad \text{Solve for } s.$$

$$7 \approx s \quad \text{Divide.}$$

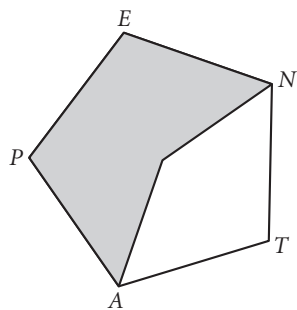
Each side is about 7 cm long.

(continued)

Lesson 8.4 • Areas of Regular Polygons (continued)

EXAMPLE B

Find the shaded area of the regular pentagon *PENTA*. The apothem measures about 2.0 cm. Segment *PE* measures about 2.9 cm.



► Solution

First, find the area of the entire pentagon.

$$A = \frac{1}{2}asn \quad \text{Write the formula.}$$

$$A \approx \frac{1}{2}(2.0)(2.9)(5) \quad \text{Substitute the known values.}$$

$$A \approx 14.5 \quad \text{Multiply.}$$

The area of the pentagon is about 14.5 cm². The shaded portion makes up $\frac{3}{5}$ of the pentagon. (If you divide the pentagon into five isosceles triangles, three will be shaded.) So, the shaded area is about $\frac{3}{5} \cdot 14.5$ cm², or 8.7 cm².

8.5

Areas of Circles

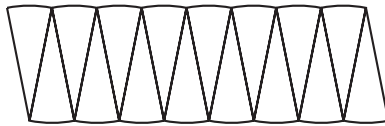
In this lesson you will

- Discover the area formula for circles

A rectangle has straight sides, while a circle is entirely curved. So, it may surprise you that you can use the area formula for rectangles to help you find the area formula for circles. In the next investigation you'll see how.

Investigation: Area Formula for Circles

Follow Steps 1–3 in your book to create a figure like the one below.



The figure resembles a parallelogram with two bumpy sides. If you cut the circle into more wedges, you could arrange these thinner wedges to look more like a rectangle. You would not lose or gain any area in this change, so the area of this new “rectangle” would be the same as the area of the original circle. If you could cut infinitely many wedges, you’d actually have a rectangle with smooth sides.

The two longer sides of the rectangle would be made up of the circumference, C , of the circle. (Each side would be half the circumference.) Consider one of these sides to be the base. Recall the formula for the circumference of a circle that you learned in Chapter 6. Now use this formula to write the length of the base of the rectangle in terms of r , the radius of the original circle.

How is the height of the rectangle related to the original circle?

Remember, the area of the rectangle is the same as the area of the original circle. Use this idea and your findings to complete this conjecture.

Circle Area Conjecture The area of a circle is given by the formula $A = \underline{\hspace{2cm}}$, where A is the area and r is the radius of the circle.

C-80

Examples A and B in your book show how to use your new conjecture. Read these examples, and then read the examples on the next page.

(continued)

Lesson 8.5 • Areas of Circles (continued)

EXAMPLE A | The circumference of a circle is 22π ft. What is the area of the circle?

► **Solution**

Use the circumference formula to find the radius. Then use the area formula to find the area.

$$C = 2\pi r \quad \text{Write the formula for circumference.}$$

$$22\pi = 2\pi r \quad \text{Substitute the known values.}$$

$$11 = r \quad \text{Solve for } r.$$

$$A = \pi r^2 \quad \text{Write the formula for area.}$$

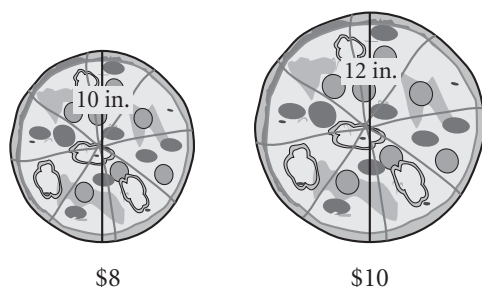
$$A = \pi(11)^2 \quad \text{Substitute the known values.}$$

$$A = 121\pi \quad \text{Simplify.}$$

The area is 121π ft², or about 380.1 ft².

EXAMPLE B

At Maria's Pizzeria, a pepperoni pizza with diameter 10 inches costs \$8, and a pepperoni pizza with diameter 12 inches costs \$10. Which size is a better buy?



► **Solution**

Find the area of each pizza, and then find the price per square inch.

10-inch pizza

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(5)^2 \\ &= 25\pi \end{aligned}$$

The area is 25π in². To find the cost per square inch, divide the price by the area.

$$\frac{8}{25\pi} \approx 0.10$$

The 10-inch pizza costs about 10¢ per square inch.

12-inch pizza

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(6)^2 \\ &= 36\pi \end{aligned}$$

The area is 36π in². To find the cost per square inch, divide the price by the area.

$$\frac{10}{36\pi} \approx 0.09$$

The 12-inch pizza costs about 9¢ per square inch.

The 12-inch pizza costs less per square inch, so the 12-inch pizza is a better buy.

8.6

Any Way You Slice It

In this lesson you will

- Learn how to find the area of a sector, a segment, and an annulus of a circle

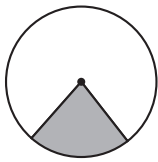
In Lesson 8.5, you discovered the formula for calculating the area of a circle. In this lesson you'll learn how to find the areas of three types of sections of a circle.

A **sector of a circle** is the region between two radii and an arc of the circle.

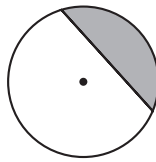
A **segment of a circle** is the region between a chord and an arc of the circle.

An **annulus** is the region between two concentric circles.

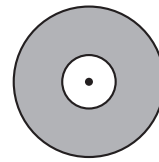
Examples of the three types of sections are pictured below.



Sector of a circle



Segment of a circle



Annulus

The “picture equations” below show you how to calculate the area of each type of section.

$$\frac{a}{360} \cdot \pi r^2 = A_{\text{sector}}$$

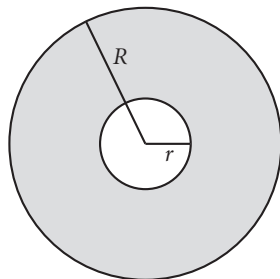
$$\frac{a}{360} \pi r^2 - \frac{1}{2} b h = A_{\text{segment}}$$

$$\pi R^2 - \pi r^2 = A_{\text{annulus}}$$

Read the examples in your book carefully. Then read the examples below.

EXAMPLE A

$R = 9$ cm and $r = 3$ cm. Find the area of the annulus.



(continued)

Lesson 8.6 • Any Way You Slice It (continued)

► Solution

$$A = \pi R^2 - \pi r^2 \quad \text{The area formula for an annulus.}$$

$$= \pi(9)^2 - \pi(3)^2 \quad \text{Substitute the values for } R \text{ and } r.$$

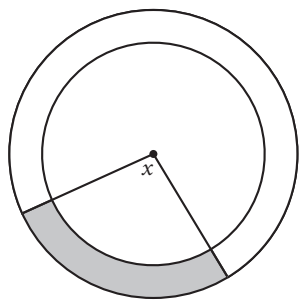
$$= 81\pi - 9\pi \quad \text{Evaluate the exponents.}$$

$$= 72\pi \quad \text{Subtract.}$$

The area of the annulus is $72\pi \text{ cm}^2$, or about 226 cm^2 .

EXAMPLE B

The shaded area is $21\pi \text{ cm}^2$. The radius of the large circle is 12 cm, and the radius of the small circle is 9 cm. Find x , the measure of the central angle.



► Solution

First, find the area of the whole annulus.

$$A = \pi R^2 - \pi r^2 \quad \text{The area formula for an annulus.}$$

$$= \pi(12)^2 - \pi(9)^2 \quad \text{Substitute the values for } R \text{ and } r.$$

$$= 63\pi \quad \text{Simplify.}$$

The shaded area, $21\pi \text{ cm}^2$, is $\frac{x}{360}$ of the area of the annulus. Use this information to write and solve an equation.

$$21\pi = \frac{x}{360} \cdot 63\pi$$

$$360 \cdot \frac{21\pi}{63\pi} = x$$

$$120 = x$$

The measure of the central angle is 120° .

8.7

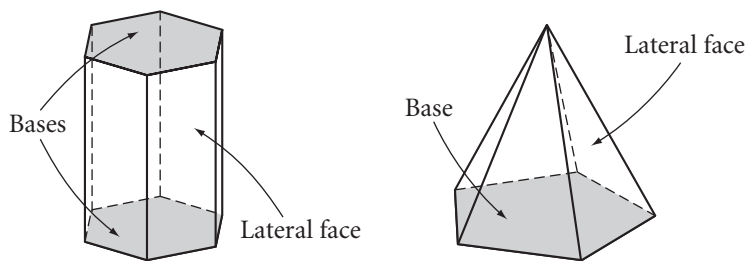
Surface Area

In this lesson you will

- Learn how to find the surface areas of prisms, pyramids, cylinders, and cones

You can use what you know about finding the areas of plane figures to find the surface areas of prisms, pyramids, cylinders, and cones. The **surface area** of each of these solids is the sum of the areas of all the faces or surfaces that enclose the solid. The faces include the solid's **bases** and its **lateral faces**.

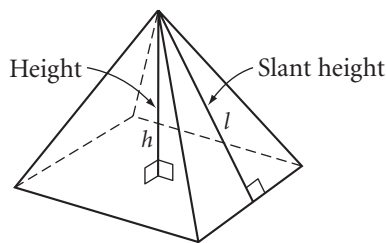
In a prism, the bases are two congruent polygons and the lateral faces are rectangles or other parallelograms. In a pyramid, the base can be any polygon and the lateral faces are triangles.



Read the “Steps for Finding Surface Area” on page 462 of your book. Example A shows how to find the surface area of a rectangular prism. Read the example carefully.

Then read Example B, which shows how to find the surface area of a cylinder. Notice that, to find the area of the cylinder’s lateral surface, you need to imagine cutting the surface and laying it flat to get a rectangle. Because the rectangle wraps exactly around the circular base, the length of the rectangle’s base is the circumference of the circular base.

The surface area of a pyramid is the area of the base, plus the areas of the triangular faces. The height of each triangular face is called the **slant height**. Use l for the slant height and h for the height of the pyramid.

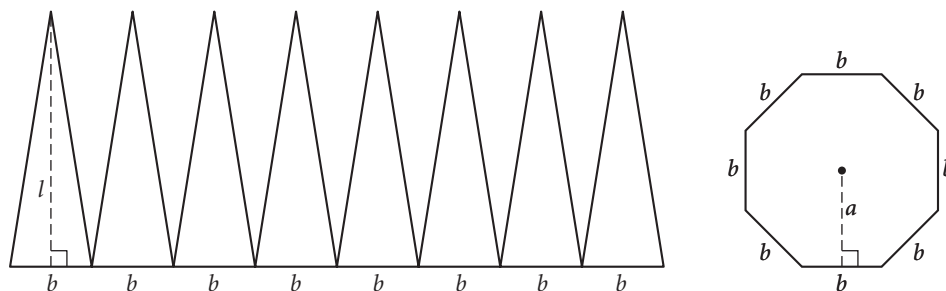


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Lesson 8.7 • Surface Area (continued)

Investigation 1: Surface Area of a Regular Pyramid

The lateral faces of a regular pyramid are identical isosceles triangles, and the base is a regular polygon.



Each lateral face is a triangle with base length b and height l . What is the area of each face?

If the base is an n -gon, then there are n lateral faces. What is the total lateral surface area of the pyramid?

What is the area of the base in terms of a , b , and n ?

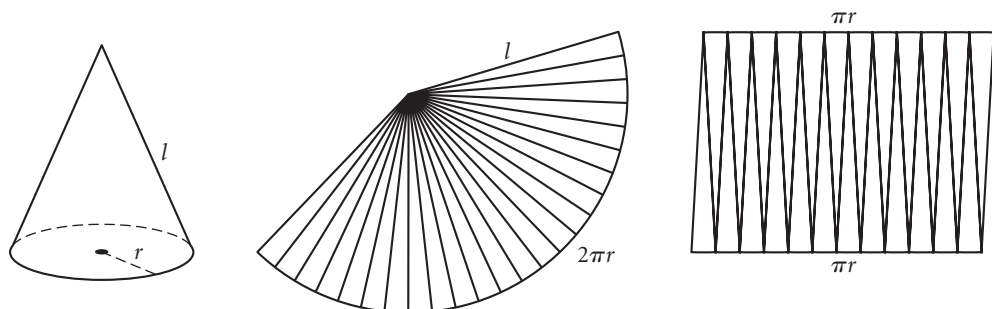
Use your expressions to write a formula for the surface area of a regular n -gon pyramid in terms of the number of sides n , base length b , slant height l , and apothem a .

Using the fact that the perimeter of the base is nb , write another formula for the surface area of a regular n -gon pyramid in terms of slant height l , apothem a , and perimeter of the base, P .

In the next investigation you will find the surface area of a cone with radius r and slant height l .

Investigation 2: Surface Area of a Cone

As the number of faces of a pyramid increases, it begins to look like a cone. You can think of the lateral surface as many thin triangles or as a sector of a circle. You can rearrange the triangles to form a rectangle.



Use the diagrams to help you write a formula for the lateral surface area in terms of r and l .

Using the expression for the lateral surface area and an expression for the area of the base, write a formula for the surface area of the cone.

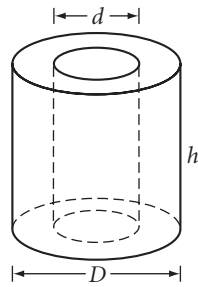
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Lesson 8.7 • Surface Area (continued)

Example C in your book shows you how to apply the formula for the surface area of a cone. Read Example C carefully. Then read the example below.

EXAMPLE

Find the surface area of this solid. $D = 10$, $d = 6$, $h = 14$.



► Solution

The surface area is the lateral surface area of the outside cylinder, plus the lateral surface area of the inside cylinder, plus the area of the two bases, which are annuluses.

$$\text{Lateral surface area of outside cylinder} = 2\pi\left(\frac{D}{2}\right)h = 2\pi\left(\frac{10}{2}\right)(14) = 140\pi \text{ cm}^2$$

$$\text{Lateral surface area of inside cylinder} = 2\pi\left(\frac{d}{2}\right)h = 2\pi\left(\frac{6}{2}\right)(14) = 84\pi \text{ cm}^2$$

$$\begin{aligned}\text{Area of one base} &= \pi\left(\frac{D}{2}\right)^2 - \pi\left(\frac{d}{2}\right)^2 \\ &= \pi\left(\frac{10}{2}\right)^2 - \pi\left(\frac{6}{2}\right)^2 = 16\pi \text{ cm}^2\end{aligned}$$

So,

$$\text{Total surface area} = 140\pi + 84\pi + 2(16\pi) = 256\pi \text{ cm}^2 \approx 804 \text{ cm}^2.$$

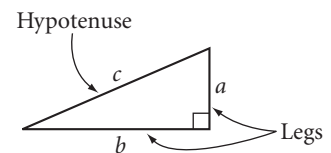
9.1

The Theorem of Pythagoras

In this lesson you will

- Learn about the **Pythagorean Theorem**, which states the relationship between the lengths of the legs and the length of the hypotenuse of a right triangle
- Solve a **dissection puzzle** that helps you understand the Pythagorean Theorem
- Read a **proof** of the Pythagorean Theorem
- Use the Pythagorean Theorem to **solve problems**

In a right triangle, the side opposite the right angle is called the **hypotenuse** and the other sides are called **legs**. In the figure, a and b are the lengths of the legs of a right triangle, and c is the length of the hypotenuse. There is a special relationship between the lengths of the legs and the length of the hypotenuse. This relationship is known as the Pythagorean Theorem.



Investigation: The Three Sides of a Right Triangle

In this investigation you will solve a geometric puzzle that will help you understand the Pythagorean Theorem. You will use a **dissection**, cutting apart a figure and putting the pieces back together to form a new figure.

Construct a scalene right triangle in the middle of a piece of paper. Label the two legs a and b , and label the hypotenuse c . Construct a square on each side of the triangle so that the squares do not overlap the triangle. What is the area of each square in terms of its side length?

Now follow Steps 2–4 in your book.

After you successfully solve the puzzle in Step 4, explain the relationship among the areas of the three squares. Then use this relationship to complete the statement of the Pythagorean Theorem.

The Pythagorean Theorem In a right triangle, the sum of the squares of the lengths of the legs equals _____.

C-81

If a and b are the lengths of the two legs of a right triangle and c is the length of the hypotenuse, then a convenient way to write the Pythagorean Theorem is $a^2 + b^2 = c^2$.

A **theorem** is a conjecture that has been proved. There are over 200 known proofs of the Pythagorean Theorem. Your book gives one proof. Read the proof on page 479 of your book and make sure you can explain each step.

(continued)

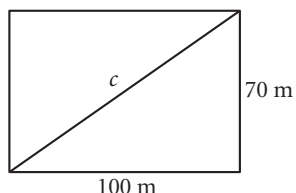
Lesson 9.1 • The Theorem of Pythagoras (continued)

Page 480 of your book gives some examples that illustrate that the Pythagorean relationship, $a^2 + b^2 = c^2$, does not hold for acute or obtuse triangles.

You can use the Pythagorean Theorem to solve problems involving right triangles. Read Examples A and B in your book, and then read the examples below.

EXAMPLE A | An Olympic soccer field is a rectangle 100 meters long and 70 meters wide. How long is the diagonal of the field?

► **Solution** | The diagonal is the hypotenuse of a right triangle with leg lengths 70 m and 100 m. You can use the Pythagorean Theorem to find its length.



$$a^2 + b^2 = c^2 \quad \text{The Pythagorean formula.}$$

$$70^2 + 100^2 = c^2 \quad \text{Substitute the known values.}$$

$$4,900 + 10,000 = c^2 \quad \text{Square the terms.}$$

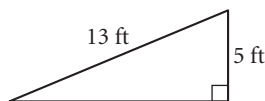
$$14,900 = c^2 \quad \text{Add.}$$

$$122 \approx c \quad \text{Take the positive square root of each side.}$$

The diagonal is about 122 meters long.

EXAMPLE B | What is the area of a right triangle with a leg of length 5 feet and a hypotenuse of length 13 feet?

► **Solution** | You can consider the two legs to be the base and the height of the triangle. The length of one leg is 5 feet. To find the length of the other leg, use the Pythagorean Theorem.



$$a^2 + b^2 = c^2 \quad \text{The Pythagorean formula.}$$

$$5^2 + b^2 = 13^2 \quad \text{Substitute.}$$

$$25 + b^2 = 169 \quad \text{Square the terms.}$$

$$b^2 = 144 \quad \text{Subtract 25 from both sides.}$$

$$b = 12 \quad \text{Take the positive square root of each side.}$$

The other leg has length 12, so the area is $\frac{1}{2} \cdot 5 \cdot 12$, or 30 square feet.

The Converse of the Pythagorean Theorem

In this lesson you will

- Experiment with **Pythagorean triples** to determine whether the converse of the Pythagorean Theorem appears to be true
- Prove the **Converse of the Pythagorean Theorem**
- Use the converse of the Pythagorean Theorem to determine whether a triangle is a right triangle

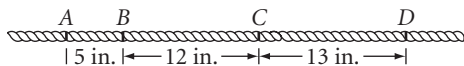
In Lesson 9.1, you learned the Pythagorean Theorem, which states that if a triangle is a right triangle, then the square of the length of its hypotenuse is equal to the sum of the squares of the lengths of the two legs. Do you think the converse is also true? In other words, if the side lengths of a triangle work in the Pythagorean equation, must the triangle be a right triangle? You'll explore this question in the investigation.

Investigation: Is the Converse True?

For this investigation you will need string, three paper clips, and two helpers. If no one is available to help you, you will need string, some pins or tacks, and a large sheet of thick cardboard.

A set of three positive integers that satisfy the Pythagorean formula is called a **Pythagorean triple**. For example, the integers 3, 4, and 5 are a Pythagorean triple because $3^2 + 4^2 = 5^2$.

Page 484 of your book lists nine examples of Pythagorean triples. Select one triple from the list, and mark off four points—*A*, *B*, *C*, and *D*—on a string to create three consecutive lengths from your set of triples. (Leave some string to the left of *A* and to the right of *D* so that you will be able to tie a knot.) For example, if you choose 5, 12, 13, you could mark your string like this:



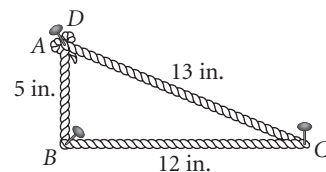
Tie the ends of the string together so that points *A* and *D* meet.

If you are working with two other people:

- Loop three paper clips onto the string.
- Three people should each pull a paper clip at point *A*, *B*, or *C* to stretch the string tight and form a triangle. (See the photograph in your book.)
- Measure the largest angle of the triangle. What type of triangle is formed?

If you are working by yourself:

- Pin the string to the cardboard at one of the marked points.
- Stretch the part of the string between the pinned-down point and the next marked point tight. Pin that point down. Then pull on the third marked point to stretch the string tight and form a triangle, and pin down that point.
- Measure the largest angle of the triangle. What type of triangle is formed?



(continued)

Lesson 9.2 • The Converse of the Pythagorean Theorem (continued)

Select at least one more triple from the list and repeat the experiment.

Your results can be summarized as a conjecture.

Converse of the Pythagorean Theorem If the lengths of the three sides of a triangle satisfy the Pythagorean equation, then the triangle is a right triangle.

C-82

On page 486 of your book, read the beginning of a proof of the Converse of the Pythagorean Theorem. Then complete the proof using the model below.

$\triangle DEF$ is a right triangle. So, by the Pythagorean Theorem, $a^2 + b^2 = x^2$.

But we were given that $a^2 + b^2 = c^2$.

Therefore, by substitution, $x^2 =$ _____.

Take the square root of each side to get _____ = _____.

We now know that $\triangle ABC \cong \triangle DEF$ by _____, so $\angle C \cong \angle F$ by _____.

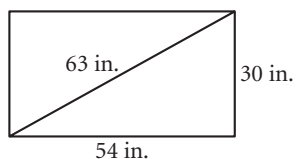
Thus, $m\angle C = m\angle F = 90^\circ$, so $\triangle ABC$ is a right triangle.

EXAMPLE

Les wanted to build a rectangular pen for his guinea pig. When he finished, he measured the bottom of the pen. He found that one side was 54 inches long, the adjacent side was 30 inches long, and one diagonal was 63 inches long. Is the pen really rectangular?

► Solution

If the pen is rectangular, then two adjacent sides and a diagonal will form a right triangle. To see if this is the case, check whether the measurements form a Pythagorean triple.



$$30^2 + 54^2 = 900 + 2916 = 3816 \text{ and } 63^2 = 3969$$

Because $30^2 + 54^2 \neq 63^2$, the measurements are not a Pythagorean triple, so the triangle is not a right triangle. Therefore, the pen is not rectangular.

9.3

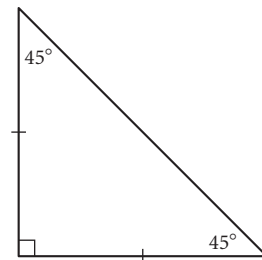
Two Special Right Triangles

In this lesson you will

- Discover a shortcut for finding an unknown side length in an **isosceles right triangle** (also called a **45°-45°-90° triangle**)
- Discover a shortcut for finding an unknown side length in a **30°-60°-90° triangle**

An isosceles right triangle is sometimes called a 45°-45°-90° triangle because of its angle measures. Note that an isosceles right triangle is half of a square.

In the next investigation you will discover the relationship among the side lengths of an isosceles right triangle.



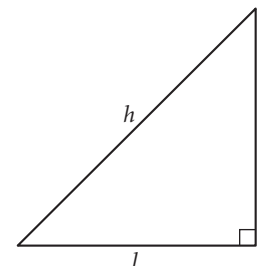
Investigation 1: Isosceles Right Triangles

The isosceles right triangle at right has legs of length l and a hypotenuse of length h . If you know the value of l , you can use the Pythagorean Theorem to find h . Here are two examples.

- If l is 5, then $h^2 = 5^2 + 5^2 = 50$, so $h = \sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$.
- If l is 8, then $h^2 = 8^2 + 8^2 = 128$, so $h = \sqrt{128} = \sqrt{64 \cdot 2} = 8\sqrt{2}$.

Find the value of h for at least three more integer values of l . Simplify the square root, but leave it in radical form.

Look for a pattern in the relationship between l and h . Summarize your findings by completing this conjecture.



Isosceles Right Triangle Conjecture In an isosceles right triangle, if the legs have length l , then the hypotenuse has length _____.

C-83

If you fold an equilateral triangle along one of its lines of symmetry, you get a 30°-60°-90° triangle, as shown at right. Triangle ABC is equilateral and \overline{CD} is an angle bisector.

Prove that $\triangle ACD \cong \triangle BCD$ and then answer the questions posed on page 492 of your book. You can check your answers with the sample answers below.

1. Why must the angles in $\triangle BCD$ be 30°, 60°, and 90°?

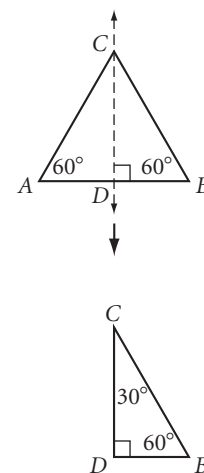
$m\angle B = 60^\circ$ because $\triangle ABC$ is equilateral and therefore equiangular. By the same reasoning, $m\angle ACB = 60^\circ$. Angle bisector \overline{CD} cuts $\angle ACB$ into two congruent angles, so $m\angle ACD = 30^\circ$ and $m\angle BCD = 30^\circ$. Using the Triangle Sum Conjecture, $m\angle CDB = 90^\circ$.

2. How does BD compare to AB ? How does BD compare to BC ?

By the Vertex Angle Bisector Conjecture, \overline{CD} is a median to \overline{AB} , so $AD = BD$ and $BD = \frac{1}{2}AB$. Because $\triangle ABC$ is equilateral, $AB = BC$, so $BD = \frac{1}{2}BC$.

3. In any 30°-60°-90° triangle, how does the length of the hypotenuse compare to the length of the shorter leg?

The length of the hypotenuse is twice the length of the shorter leg.



(continued)

Lesson 9.3 • Two Special Right Triangles (continued)

Investigation 2: 30°-60°-90° Triangles

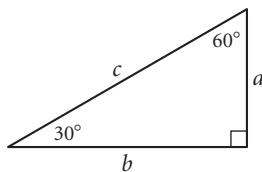
Below is a 30°-60°-90° triangle. If you know the length of the shorter leg, a , you can find the length of the other sides. For example, if a is 3, then the hypotenuse, c , is 6. Use the Pythagorean formula to find the length of the other leg, b .

$$3^2 + b^2 = 6^2$$

$$9 + b^2 = 36$$

$$b^2 = 27$$

$$b = \sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$$



The length of leg b is $3\sqrt{3}$ units.

Copy the table from Step 2 of Investigation 2, and repeat the procedure above to complete the table. Write the length of the longer leg in simplified radical form. Look for a pattern between the lengths of the legs of each triangle. Use your observations to complete the conjecture.

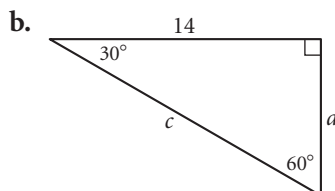
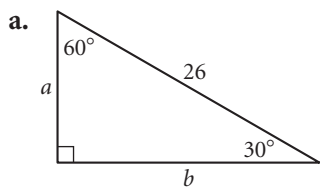
30°-60°-90° Triangle Conjecture In a 30°-60°-90° triangle, if the shorter leg has length a , then the longer leg has length _____ and the hypotenuse has length $2a$.

C-84

Page 493 of your book gives a proof of the 30°-60°-90° Triangle Conjecture. Read the proof and make sure you understand it. Then read the example below.

EXAMPLE

Find the lettered side lengths. All lengths are in centimeters.



► Solution

- The length of the shorter leg is half the length of the hypotenuse, so $a = 13$ cm. The length of the longer leg is the length of the shorter leg times $\sqrt{3}$, so $b = 13\sqrt{3}$ cm, or about 22.5 cm.
- The length of the longer leg is the length of the shorter leg times $\sqrt{3}$, so $14 = a\sqrt{3}$ and $a = \frac{14}{\sqrt{3}}$ cm, or about 8.1 cm. The length of the hypotenuse is twice the length of the shorter leg, so $c = \frac{28}{\sqrt{3}}$ cm, or about 16.2 cm.

9.4

Story Problems

In this lesson you will

- Use the Pythagorean Theorem to **solve problems**

You can use the Pythagorean Theorem to solve many problems involving right triangles.

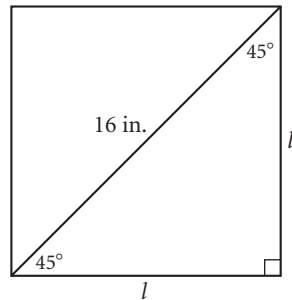
Read the example in your text. Notice that the problem in that example requires applying the Pythagorean Theorem twice, first to find the diagonal of the bottom of the box and then to find the diagonal of the box.

In the examples below, try to solve each problem on your own before reading the solution.

EXAMPLE A | A square has a diagonal of length 16 inches. What is the area of the square?

► **Solution**

The diagonal of a square divides the square into two 45° - 45° - 90° triangles. To find the area of the square, you need to know the leg length, l , of the triangles.



By the Isosceles Right Triangle Conjecture (or by the Pythagorean Theorem), you know that $l \cdot \sqrt{2} = 16$, so $l = \frac{16}{\sqrt{2}}$ in. Therefore,

$$\begin{aligned} \text{Area of square} &= \frac{16}{\sqrt{2}} \cdot \frac{16}{\sqrt{2}} \\ &= \frac{256}{2} \\ &= 128 \end{aligned}$$

So, the area of the square is 128 in^2 .

(continued)

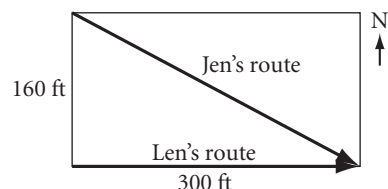
Lesson 9.4 • Story Problems (continued)

EXAMPLE B

The Clementina High School Marching Band is practicing on the school football field. The field is 300 feet long from west to east and 160 feet wide from north to south. Len starts at the southwest corner and marches at a rate of 5 feet per second toward the southeast corner. At the same time, Jen begins marching diagonally from the northwest corner toward the southeast corner. If they want to meet at the corner at the same instant, how fast does Jen need to march?

► Solution

To start, make a sketch to illustrate the problem.



Len marches 300 feet at a rate of 5 feet per second, so it will take him $300 \div 5$, or 60 seconds, to reach the southeast corner.

For them to meet at the same time, Jen must also cover her route in 60 seconds. To find the distance Jen must march, use the Pythagorean Theorem.

$$160^2 + 300^2 = x^2$$

$$25,600 + 90,000 = x^2$$

$$115,600 = x^2$$

$$340 = x$$

Jen must cover 340 feet in 60 seconds, so she must march at a rate of $340 \div 60$, or about 5.7 feet per second.

9.5

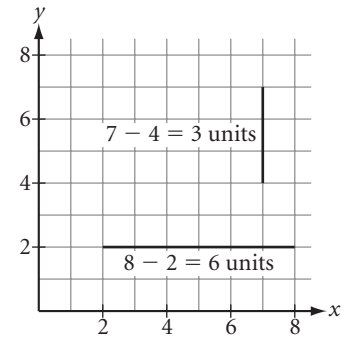
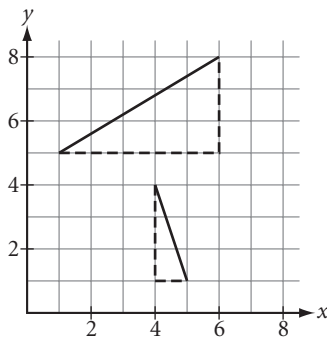
Distance in Coordinate Geometry

In this lesson you will

- Learn a formula for finding the **distance between two points** on a coordinate plane
- Discover the **general equation of a circle**

On a coordinate plane, you can find the length of a segment in the x -direction by counting grid squares or by subtracting x -coordinates. Similarly, you can find the length of a segment in the y -direction by counting or by subtracting y -coordinates.

You can think of any segment that is not in the x - or y -direction as the hypotenuse of a right triangle with legs in the x - and y -directions. This allows you to use the Pythagorean Theorem to find the length of the segment.



In the next investigation you'll use this idea to develop a formula for the distance between any two points on a coordinate plane.

Investigation: The Distance Formula

Step 1 in your book shows four segments on coordinate planes. Find the length of each segment by considering it to be the hypotenuse of a right triangle. For example, the segment in part a is the hypotenuse of a right triangle with legs of lengths 2 units and 4 units, so, using the Pythagorean Theorem,

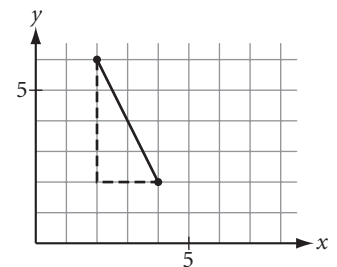
$$\text{length}^2 = 2^2 + 4^2$$

$$= 20$$

$$\text{length} = \sqrt{20} = 2\sqrt{5} \approx 4.5 \text{ units}$$

In Step 2, you must plot and connect the points and then find the distance between them. You can find the distance using the procedure you used in Step 1.

Consider the points $A(15, 34)$ and $B(42, 70)$. It wouldn't be practical to plot these points on a grid, so how can you find the distance between them?



(continued)

Lesson 9.5 • Distance in Coordinate Geometry (continued)

Recall that you can find a horizontal distance by subtracting x -coordinates and a vertical distance by subtracting y -coordinates. Use this idea to complete Steps 3–5 in your book and find the distance between points $A(15, 34)$ and $B(42, 70)$.

You can generalize your findings from this investigation as a conjecture.

Distance Formula The distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula $(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ or $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

C-85

Example A in your book shows how to apply the distance formula. Read Example A.

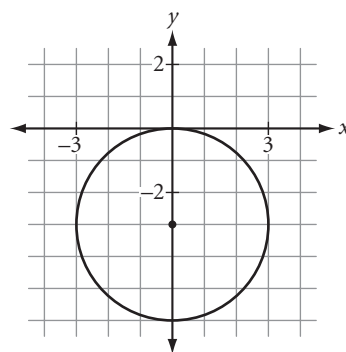
Example B shows how to use the distance formula to write the equation of a circle with center $(5, 4)$ and radius 7 units. The solution uses the fact that the circle is the set of all points (x, y) that are 7 units from the fixed point $(5, 4)$. Read Example B and then read the example below.

EXAMPLE | Write the equation for the circle at right.

► **Solution**

The circle has center $(0, -3)$ and radius 3 units. Let (x, y) represent any point on the circle. The distance from (x, y) to the circle's center, $(0, -3)$, is 3. Substitute this information in the distance formula.

$$(x - 0)^2 + (y - (-3))^2 = 3^2, \text{ or } x^2 + (y + 3)^2 = 3^2$$



In the Mini-Investigation in Exercise 11, you'll develop the general equation of a circle.

Circles and the Pythagorean Theorem

In this lesson you will

- Use **circle conjectures** and the **Pythagorean Theorem** to solve problems

In Chapter 6, you discovered a number of properties of circles that involved right angles. Here are two of the conjectures from that chapter.

Tangent Conjecture: A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

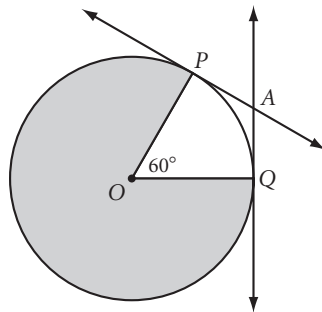
Angles Inscribed in a Semicircle Conjecture: Angles inscribed in a semicircle are right angles.

You can use these and other circle conjectures and the Pythagorean Theorem to solve some challenging problems.

Examples A and B in your book use circle conjectures, dissections, special right triangle relationships, and the Pythagorean Theorem. Read these examples and follow each step in the solutions. Below are two more examples.

EXAMPLE A

\overrightarrow{AP} and \overrightarrow{AQ} are tangent to circle O , and $AP = 3$ cm. Find the area of the shaded region.

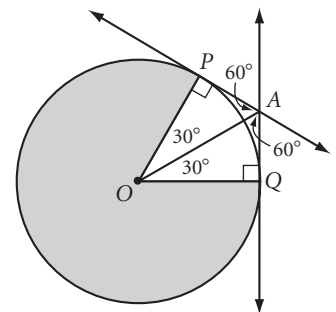


► Solution

You can draw \overline{OA} to create two 30° - 60° - 90° triangles. (How do you know the segment bisects $\angle O$ to create two 30° angles?)

In $\triangle APO$, the shorter leg, \overline{AP} , has length 3 cm, so the longer leg, which is the radius of the circle, has length $3\sqrt{3}$ cm.

Because the radius is $3\sqrt{3}$ cm, the area of the entire circle is 27π cm². The area of the shaded region is $\frac{360 - 60}{360}$, or $\frac{5}{6}$, of the area of the circle. So, the shaded area is $\frac{5}{6}(27\pi) = \frac{45}{2}\pi$ cm², or about 70.7 cm².

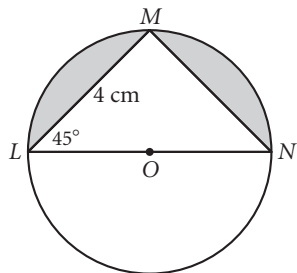


(continued)

Lesson 9.6 • Circles and the Pythagorean Theorem (continued)

EXAMPLE B

Find the area of the shaded region.



► Solution

The area of the shaded region is the area of the semicircle minus the area of $\triangle LMN$.

Because $\angle LMN$ is inscribed in a semicircle, it is a right angle. Using the Triangle Sum Conjecture, $m\angle N = 45^\circ$. Therefore, $\triangle LMN$ is an isosceles right triangle (a 45° - 45° - 90° triangle) with a leg of length 4 cm .

The length of the hypotenuse, which is the diameter of the circle, is $4\sqrt{2}\text{ cm}$. The radius of the circle is then $2\sqrt{2}\text{ cm}$, so the area of the entire circle is $\pi(2\sqrt{2})^2$, or $8\pi\text{ cm}^2$. Therefore, the area of the semicircle is $4\pi\text{ cm}^2$. The area of $\triangle LMN$ is $\frac{1}{2} \cdot 4 \cdot 4$, or 8 cm^2 . So, the area of the shaded region is $(4\pi - 8)\text{ cm}^2$, or about 4.6 cm^2 .

10.1

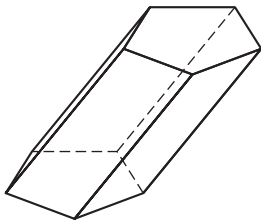
The Geometry of Solids

In this lesson you will

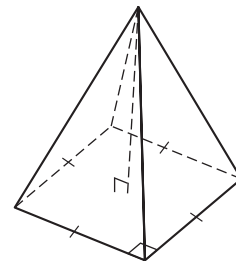
- Learn about **polyhedrons**, including **prisms** and **pyramids**
- Learn about solids with curved surfaces, including **cylinders**, **cones**, and **spheres**

In this chapter you will study three-dimensional solid figures. Lesson 10.1 introduces various types of three-dimensional solids and the terminology associated with them. Read the lesson. Then review what you read by completing the statements and answering the questions below.

1. A polyhedron is a solid formed by _____ that enclose a single region of space.
2. A segment where two faces of a polyhedron intersect is called a(n) _____.
3. A polyhedron with six faces is called a(n) _____.
4. A tetrahedron has _____ faces.
5. If each face of a polyhedron is enclosed by a regular polygon, and each face is congruent to the other faces, and the faces meet each vertex in exactly the same way, then the polyhedron is called a(n) _____.
6. A(n) _____ is a polyhedron with two faces, called bases, that are congruent, parallel polygons.
7. The faces of a prism that are not bases are called _____.
8. What is the difference between a right prism and an oblique prism?
9. What type of solid is shown below?



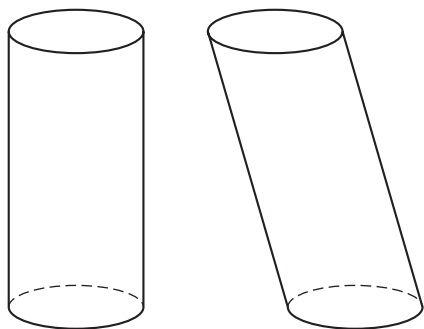
10. How many bases does a pyramid have?
11. The point that all the lateral faces of a pyramid have in common is the _____ of the pyramid.
12. What is the difference between an altitude of a pyramid and the height of a pyramid?
13. What type of solid is shown at right?



(continued)

Lesson 10.1 • The Geometry of Solids (continued)

14. Name three types of solids with curved surfaces.
15. A(n) _____ is the set of all points in space at a given distance from a given point.
16. A circle that encloses the base of a hemisphere is called a(n) _____.
17. Give an example of a real object that is shaped like a cylinder. Explain how you know it is a cylinder.
18. Tell which cylinder below is an oblique cylinder and which is a right cylinder. For each cylinder, draw and label the *axis* and the *altitude*.



19. The base of a cone is a(n) _____.
20. If the line segment connecting the vertex of a cone with the center of the base is perpendicular to the base, then the cone is a(n) _____.

After you have finished, check your answers below.

- | | |
|---|--|
| 1. polygons | 13. square pyramid |
| 2. edge | 14. possible answers: cylinder, cone, sphere, hemisphere |
| 3. hexahedron | 15. sphere |
| 4. four | 16. great circle |
| 5. regular polygon | 17. The example will vary. Reason: It has two parallel bases and both bases are circles. |
| 6. prism | 18. The figure on the left is a right cylinder, and the figure on the right is an oblique cylinder. See page 523 of the book for labeled pictures of both. |
| 7. lateral faces | 19. circle |
| 8. In a right prism all lateral faces are rectangles and are perpendicular to the base. In an oblique prism they are not. | 20. right cone |
| 9. pentagonal prism (it may be oblique or right) | |
| 10. one | |
| 11. vertex | |
| 12. The altitude is a segment from the vertex of the pyramid to the plane of the base that is perpendicular to the plane of the base. The height is the length of the altitude. | |

10.2

Volume of Prisms and Cylinders

In this lesson you will

- Discover the volume formula for **right rectangular prisms**
- Extend the volume formula to **right prisms** and **right cylinders**
- Extend the volume formula to **oblique prisms** and **oblique cylinders**

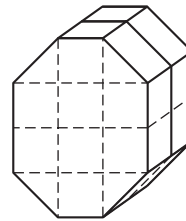
Volume is the measure of the amount of space contained in a solid. You use cubic units to measure volume: cubic inches (in^3), cubic feet (ft^3), cubic yards (yd^3), cubic centimeters (cm^3), cubic meters (m^3), and so on. The volume of an object is the number of unit cubes that completely fill the space within the object.



Length: 1 unit



Volume: 1 cubic unit



Volume: 20 cubic units

Investigation: The Volume Formula for Prisms and Cylinders

Read Step 1 of the investigation in your book.

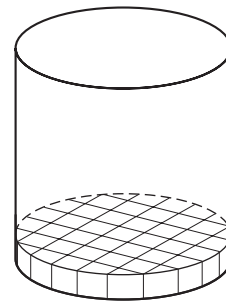
Notice that the number of cubes in the base layer is the number of square units in the area of the base and that the number of layers is the height of the prism. So, you can use the area of the base and the height to find the volume of a right rectangular prism.

Rectangular Prism Volume Conjecture If B is the area of the base of a right rectangular prism and H is the height of the solid, then the formula for the volume is $V =$ _____.

C-86a

You can find the volume of any other right prism or cylinder the same way—by multiplying the area of the base by the height. For example, to find the volume of this cylinder, find the area of the circular base (the number of cubes in the base layer) and multiply by the height.

You can extend the Rectangular Prism Volume Conjecture to all right prisms and right cylinders. Complete the conjecture below.



Right Prism-Cylinder Volume Conjecture If B is the area of the base of a right prism (or cylinder) and H is the height of the solid, then the formula for the volume is $V =$ _____.

C-86b

(continued)

Lesson 10.2 • Volume of Prisms and Cylinders (continued)

What about the volume of an *oblique* prism or cylinder? Page 532 of your book shows that you can model an oblique rectangular prism with a slanted stack of paper. You can then “straighten” the stack to form a right rectangular prism with the same volume. The right prism has the same base area and the same height as the oblique prism. So, you find the volume of the oblique prism by multiplying the area of its base by its height. A similar argument works for other oblique prisms and cylinders.

Now you can extend the Right Prism-Cylinder Volume Conjecture to oblique prisms and cylinders.

Oblique Prism-Cylinder Volume Conjecture The volume of an oblique prism (or cylinder) is the same as the volume of a right prism (or cylinder) that has the same _____ and the same _____.

C-86c

Be careful when calculating the volume of an oblique prism or cylinder. Because the lateral edges are not perpendicular to the bases, the height of the prism or cylinder is *not* the length of a lateral edge.

You can combine the last three conjectures into one conjecture.

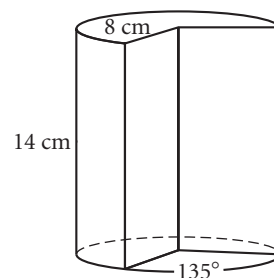
Prism-Cylinder Volume Conjecture The volume of a prism or a cylinder is the _____ multiplied by the _____.

C-86

Examples A and B in your book show you how to find the volumes of a trapezoidal prism and an oblique cylinder. Read both examples. Try to find the volumes yourself before reading the solutions. Then read the example below.

EXAMPLE

The solid at right is a right cylinder with a 135° slice removed. Find the volume of the solid. Round your answer to the nearest cm.



► Solution

The base is $\frac{225}{360}$, or $\frac{5}{8}$, of a circle with radius 8 cm. The whole circle has area $64\pi \text{ cm}^2$, so the base has area $\frac{5}{8}(64\pi)$, or $40\pi \text{ cm}^2$. Now use the volume formula.

$$V = BH \quad \text{The volume formula.}$$

$$= 40\pi(14) \quad \text{Substitute the area of the base and the height.}$$

$$= 560\pi \quad \text{Multiply.}$$

$$\approx 1759 \quad \text{Use the } \pi \text{ key on your calculator to get an approximate answer.}$$

The volume is about 1759 cm^3 .

10.3

Volume of Pyramids and Cones

In this lesson you will

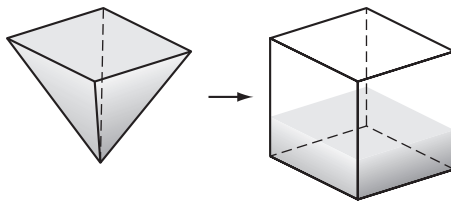
- Discover the volume formula for **pyramids** and **cones**

There is a simple relationship between the volumes of prisms and pyramids with congruent bases and the same height, and between cylinders and cones with congruent bases and the same height.

Investigation: The Volume Formula for Pyramids and Cones

If you have the materials listed at the beginning of the investigation, follow the steps in your book before continuing. The text below summarizes the results of the investigation.

Suppose you start with a prism and a pyramid with congruent bases and the same height. If you fill the pyramid with sand or water and then pour the contents into the prism three times, you will exactly fill the prism. In other words, the volume of the pyramid is $\frac{1}{3}$ the volume of the prism.



You will get the same result no matter what shape the bases have, as long as the base of the pyramid is congruent to the base of the prism and the heights are the same.

If you repeat the experiment with a cone and a cylinder with congruent bases and the same height, you will get the same result. That is, you can empty the contents of the cone into the cylinder exactly three times.

The results can be summarized in a conjecture.

Pyramid-Cone Volume Conjecture If B is the area of the base of a pyramid or a cone and H is the height of the solid, then the formula for the volume is $V = \frac{1}{3}BH$.

C-87

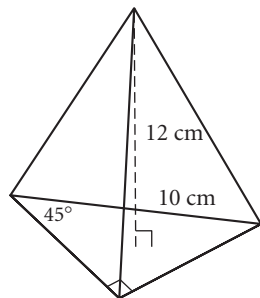
Example A in your book shows how to find the volume of a regular hexagonal pyramid. In the example, you need to use the 30° - 60° - 90° Triangle Conjecture to find the apothem of the base. Example B involves the volume of a cone. Work through both examples. Then read the example on the next page.

(continued)

Lesson 10.3 • Volume of Pyramids and Cones (continued)

EXAMPLE

Find the volume of this triangular pyramid.



► Solution

The base is an isosceles right triangle. To find the area of the base, you need to know the lengths of the legs. Let l be the length of a leg, and use the Isosceles Right Triangle Conjecture.

$$l(\sqrt{2}) = 10 \quad \text{The length of the hypotenuse is the length of a leg times } \sqrt{2}.$$

$$l = \frac{10}{\sqrt{2}} \quad \text{Solve for } l.$$

The length of each leg is $\frac{10}{\sqrt{2}}$. Now find the area of the triangle.

$$A = \frac{1}{2}bh \quad \text{Area formula for triangles.}$$

$$= \frac{1}{2}\left(\frac{10}{\sqrt{2}}\right)\left(\frac{10}{\sqrt{2}}\right) \quad \text{Substitute the known values.}$$

$$= 25 \quad \text{Multiply.}$$

So, the base has area 25 cm^2 . Now find the volume of the pyramid.

$$V = \frac{1}{3}BH \quad \text{Volume formula for pyramids and cones.}$$

$$= \frac{1}{3}(25)(12) \quad \text{Substitute the known values.}$$

$$= 100 \quad \text{Multiply.}$$

The volume of the pyramid is 100 cm^3 .

10.4

Volume Problems

In this lesson you will

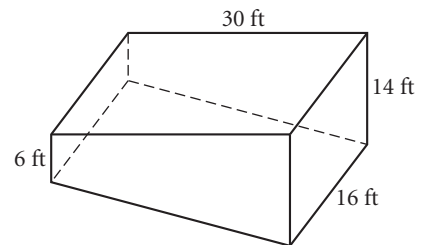
- Use the volume formulas you have learned to **solve problems**

You have learned volume formulas for prisms, cylinders, pyramids, and cones. In this lesson you will use these formulas to solve problems.

In Example A in your book, the volume of a right triangular prism is given and you must find the height. In Example B, the volume of a sector of a right cylinder is given and you must find the radius of the base. Try to solve the problems yourself before reading the solutions. Below are some more examples.

EXAMPLE A

A swimming pool is in the shape of the prism shown at right. How many gallons of water can the pool hold? (A cubic foot of water is about 7.5 gallons.)

► **Solution**

First, find the volume of the pool in cubic feet. The pool is in the shape of a trapezoidal prism. The trapezoid has bases of length 6 feet and 14 feet and a height of 30 feet. The height of the prism is 16 feet.

$$V = BH$$

Volume formula for prisms.

$$= \frac{1}{2}(30)(6 + 14) \cdot 16$$

Use the formula $\frac{1}{2}h(b_1 + b_2)$ for the area of a trapezoid.

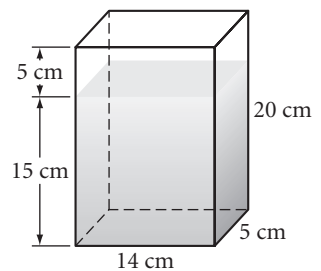
$$= 4800$$

Solve.

The pool has volume 4800 ft³. A cubic foot is about 7.5 gallons, so the pool holds 4800(7.5), or 36,000 gallons of water.

EXAMPLE B

A sealed rectangular container 5 cm by 14 cm by 20 cm is sitting on its smallest face. It is filled with water up to 5 cm from the top. How many centimeters from the bottom will the water level reach if the container is placed on its largest face?

► **Solution**

The smallest face is a 5-by-14-centimeter rectangle. When the prism is resting on its smallest face, the water is in the shape of a rectangular prism with base area 70 cm² and height 15 cm. So, the volume of the water is 1050 cm³.

(continued)

Lesson 10.4 • Volume Problems (continued)

If the container is placed on its largest face, the volume will still be 1050 cm^3 , but the base area and height will change. The area of the new base will be $14(20)$, or 280 cm^2 . You can use the volume formula to find the height.

$$V = BH \quad \text{Volume formula for prisms.}$$

$$1050 = 280H \quad \text{Substitute the known values.}$$

$$3.75 = H \quad \text{Solve for } H.$$

The height of the water will be 3.75 cm. So, the water level will be 3.75 cm from the bottom of the container.

EXAMPLE C

Find the volume of a rectangular prism with dimensions that are twice those of another rectangular prism with volume 120 cm^3 .

► Solution

For the rectangular prism with volume 120 cm^3 , let the dimensions of the rectangular base be x and y and the height be z . The volume of this prism is xyz , so $xyz = 120$.

The dimensions of the base of the other prism are $2x$ and $2y$, and the height is $2z$. Let V be the volume of this prism. Then

$$V = BH \quad \text{Volume formula for prisms.}$$

$$= (2x)(2y)(2z) \quad \text{Substitute the known values.}$$

$$= 8xyz \quad \text{Multiply.}$$

$$= 8(120) \quad \text{Substitute the known value.}$$

$$= 960 \quad \text{Multiply.}$$

The volume of the larger prism is 960 cm^3 .

10.5

Displacement and Density

In this lesson you will

- Learn how the idea of **displacement** can be used to find the volume of an object
- Learn how to calculate the **density** of an object

To find the volumes of geometric solids, such as prisms and cones, you can use a volume formula. But what if you want to find the volume of an irregularly shaped object like a rock? As Example A in your book illustrates, you can submerge the object in a regularly shaped container filled with water. The volume of water that is **displaced** will be the same as the volume of the object. Read Example A in your book. Then read the example below.

EXAMPLE A When Tom puts a rock into a cylindrical container with diameter 7 cm, the water level rises 3 cm. What is the volume of the rock to the nearest tenth of a cubic centimeter?

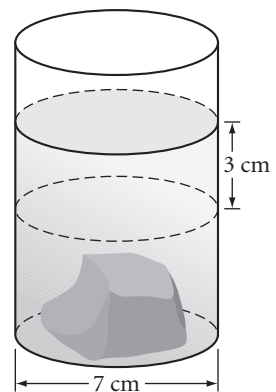
► **Solution**

The “slice” of water that is displaced is a cylinder with diameter 7 cm and height 3 cm.

Use the volume formula to find the volume of the displaced water.

$$\begin{aligned} V &= BH \\ &= \pi(3.5)^2 \cdot 3 \\ &\approx 115.5 \end{aligned}$$

The volume of the rock is about 115.5 cm^3 , the same as the volume of the displaced water.



An important property of a material is its density. **Density** is the mass of matter in a given volume. It is calculated by dividing mass by volume.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

The table on page 551 of your book gives the densities of ten metals. In Example B, the mass of a clump of metal and information about the amount of water it displaces are used to identify the type of metal. Read this example carefully. Then read the following example.

(continued)

Lesson 10.5 • Displacement and Density (continued)

EXAMPLE B

Chemist Preethi Bhatt is given a clump of metal and is told that it is sodium. She finds that the metal has a mass of 184.3 g. She places it into a nonreactive liquid in a cylindrical beaker with base diameter 10 cm. If the metal is indeed sodium, how high should the liquid level rise?

► Solution

The table on page 551 of your book indicates that the density of sodium is 0.97 g/cm^3 . Use the density formula to find what the volume of the clump of metal should be if it is sodium.

$$\text{density} = \frac{\text{mass}}{\text{volume}} \quad \text{Density formula.}$$

$$0.97 = \frac{184.3}{\text{volume}} \quad \text{Substitute the known information.}$$

$$\text{volume} \cdot 0.97 = 184.3 \quad \text{Multiply both sides by } \text{volume}.$$

$$\text{volume} = \frac{184.3}{0.97} \quad \text{Divide both sides by } 0.97.$$

$$\text{volume} = 190 \quad \text{Simplify.}$$

So, if the metal is sodium, it should displace 190 cm^3 of water. Use the volume formula to find the height of the liquid that should be displaced.

$$V = BH \quad \text{Volume formula for cylinders.}$$

$$190 = (5^2\pi)H \quad \text{Substitute the known information. (The base is a circle with radius 5 cm.)}$$

$$190 = 25\pi H \quad \text{Multiply.}$$

$$2.4 \approx H \quad \text{Solve for } H.$$

The liquid should rise about 2.4 cm.

10.6

Volume of a Sphere

In this lesson you will

- Discover the volume formula for a **sphere**

You can find the volume of a sphere by comparing it to the volume of a cylinder. In the investigation you will see how.

Investigation: The Formula for the Volume of a Sphere

If you have the materials listed at the beginning of the investigation, follow the steps in your book before continuing. The text below summarizes the results of the investigation.

Suppose you have a hemisphere and a cylinder. The radius of the cylinder equals the radius of the hemisphere, and the height of the cylinder is twice the radius. Note that the cylinder is the smallest one that would enclose a sphere made from two of the hemispheres.

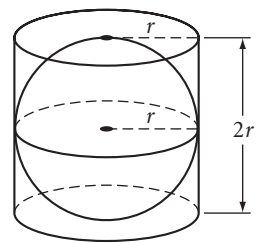
The volume of the cylinder is $\pi r^2(2r)$, or $2\pi r^3$.

If you fill the hemisphere with sand or water and empty the contents into the cylinder, the cylinder will be $\frac{1}{3}$ full.

If you fill the hemisphere again and empty the contents into the cylinder, the cylinder will be $\frac{2}{3}$ full. So, the volume of the sphere (two hemispheres) is equal to $\frac{2}{3}$ the volume of the cylinder.

The volume of the cylinder is $2\pi r^3$. So, the volume of the sphere is $\frac{2}{3}(2\pi r^3)$, or $\frac{4}{3}\pi r^3$.

The results can be summarized as a conjecture.



Sphere Volume Conjecture The volume of a sphere with radius r is given by the formula $V = \frac{4}{3}\pi r^3$.

C-88

Read Example A in your book, which involves finding the percentage of plaster cut away when the largest possible sphere is carved from a cube. The solution involves four steps:

Step 1 Find the volume of the sphere.

Step 2 Find the volume of the cube.

Step 3 Subtract the volume of the sphere from the volume of the cube to find the volume of the plaster cut away.

Step 4 Divide the volume cut away by the volume of the original cube and convert the answer to a percent to find the percentage cut away.

Read Example B in your book. Solve the problem yourself and then check your work by reading the given solution.

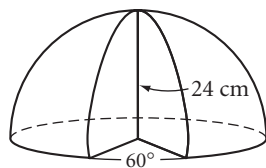
The following two examples give you more practice working with the volume of a sphere.

(continued)

Lesson 10.6 • Volume of a Sphere (continued)

EXAMPLE A

Find the volume of this solid.



► Solution

The solid is a hemisphere with a 60° sector cut away. First, find the volume of the entire hemisphere. Because the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, the formula for the volume of a hemisphere is $V = \frac{2}{3}\pi r^3$.

$$V = \frac{2}{3}\pi r^3 \quad \text{Volume formula for a hemisphere.}$$

$$= \frac{2}{3}\pi(24)^3 \quad \text{The radius is 24 cm.}$$

$$= 9216\pi \quad \text{Simplify.}$$

The volume of the entire hemisphere is $9216\pi \text{ cm}^3$. A 60° sector has been cut away, so the fraction of the hemisphere that remains is $\frac{300}{360}$, or $\frac{5}{6}$. So, the volume of the solid is $\frac{5}{6}(9216\pi) = 7680\pi \text{ cm}^3$, or about $24,127 \text{ cm}^3$.

EXAMPLE B

A marble is submerged in water in a rectangular prism with a 2 cm-by-2 cm base. The water in the prism rises 0.9 cm when the marble is submerged. What is the diameter of the marble?

► Solution

First find the volume of the water displaced by the marble.

$$V = BH \quad \text{Volume formula for a prism.}$$

$$= (2)(2)(0.9) \quad \text{Substitute the known information.}$$

$$= 3.6 \quad \text{Simplify.}$$

So, the volume of the displaced water, and thus of the marble, is 3.6 cm^3 .

Next use the volume of the marble to find its radius. Substitute 3.6 for V in the formula for the volume of a sphere and solve for r .

$$V = \frac{4}{3}\pi r^3 \quad \text{Volume formula for a sphere.}$$

$$3.6 = \frac{4}{3}\pi r^3 \quad \text{Substitute the known information.}$$

$$\frac{\frac{3}{4}(3.6)}{\pi} = r^3 \quad \text{Multiply both sides by } \frac{3}{4} \text{ and divide by } \pi.$$

$$0.86 \approx r^3 \quad \text{Simplify.}$$

$$r \approx \sqrt[3]{0.86} \approx 0.95 \quad \text{Take the cube root of both sides.}$$

The radius of the marble is about 0.95 cm, so the diameter is about 1.9 cm.

10.7

Surface Area of a Sphere

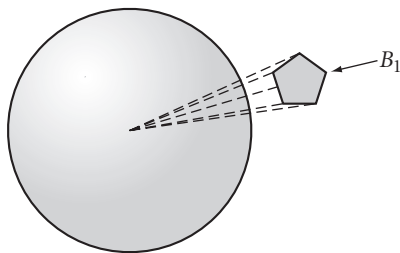
In this lesson you will

- Discover the formula for the **surface area of a sphere**

You can use the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, to find the formula for the surface area of a sphere.

Investigation: The Formula for the Surface Area of a Sphere

Imagine a sphere's surface divided into tiny shapes that are nearly flat. The surface area of the sphere is equal to the sum of the areas of these “near polygons.” If you imagine radii connecting each of the vertices of the “near polygons” to the center of the sphere, you are mentally dividing the volume of the sphere into many “near pyramids.” Each of the “near polygons” is a base for one of the pyramids, and the radius, r , of the sphere is the height of the pyramid. The volume, V , of the sphere is the sum of the volumes of all the pyramids.



Step 1 Imagine that the surface of a sphere is divided into 1000 “near polygons” with areas $B_1, B_2, B_3, \dots, B_{1000}$. The surface area, S , of the sphere is the sum of the areas of these “near polygons”:

$$S = B_1 + B_2 + B_3 + \dots + B_{1000}$$

Step 2 The pyramid with base area B_1 has volume $\frac{1}{3}(B_1)r$, the pyramid with base area B_2 has volume $\frac{1}{3}(B_2)r$, and so on. The volume of the sphere, V , is the sum of these volumes:

$$V = \frac{1}{3}(B_1)r + \frac{1}{3}(B_2)r + \frac{1}{3}(B_3)r + \dots + \frac{1}{3}(B_{1000})r$$

Factor $\frac{1}{3}r$ from each term on the right side of the equation:

$$V = \frac{1}{3}r(B_1 + B_2 + B_3 + \dots + B_{1000})$$

Step 3 Because $V = \frac{4}{3}\pi r^3$, you can substitute $\frac{4}{3}\pi r^3$ for V :

$$\frac{4}{3}\pi r^3 = \frac{1}{3}r(B_1 + B_2 + B_3 + \dots + B_{1000})$$

Now substitute S for $B_1 + B_2 + B_3 + \dots + B_{1000}$:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}rS$$

(continued)

Lesson 10.7 • Surface Area of a Sphere (continued)

Step 4 Solve the last equation for the surface area, S .

$$\frac{4}{3}\pi r^3 = \frac{1}{3}rS \quad \text{The equation from Step 3.}$$

$$4\pi r^3 = rS \quad \text{Multiply both sides by 3.}$$

$$4\pi r^2 = S \quad \text{Divide both sides by } r.$$

You now have a formula for finding the surface area of a sphere in terms of the radius. You can state the result as a conjecture.

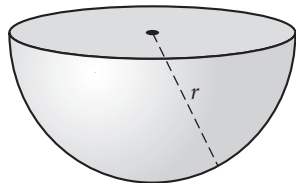
Sphere Surface Area Conjecture The surface area, S , of a sphere with radius r is given by the formula $S = 4\pi r^2$.

C-89

The example in your book shows how to find the surface area of a sphere if you know its volume. Try to find the surface area on your own before reading the solution. Then solve the problem in the example below.

EXAMPLE

The base of this hemisphere has circumference 32π cm. Find the surface area of the hemisphere (including the base).



► Solution

Because the base of the hemisphere has circumference 32π cm, the radius is 16 cm.

The area of the base of the hemisphere is $\pi(16)^2$, or 256π cm².

The area of the curved surface is half the surface area of a sphere with radius 16 cm.

$$\begin{aligned} S &= \frac{1}{2} \cdot 4\pi r^2 \\ &= \frac{1}{2} \cdot 4\pi(16)^2 \\ &= 512\pi \end{aligned}$$

So, the total surface area is $256\pi + 512\pi = 768\pi$ cm², or about 2413 cm².

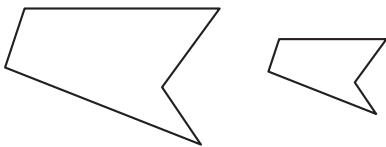
11.1

Similar Polygons

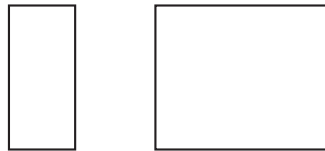
In this lesson you will

- Learn what it means for two figures to be **similar**
- Use the definition of similarity to **find missing measures** in similar polygons
- Explore **dilations** of figures on a coordinate plane

You know that figures that have the same size and shape are *congruent figures*. Figures that have the same shape but not necessarily the same size are **similar figures**. You can think of similar figures as enlargements or reductions of one another with no distortion. The pentagons below are similar. The rectangles are *not* similar. You could not enlarge or reduce one rectangle to fit exactly over the other.



These pentagons are similar.



These rectangles are *not* similar.

In the investigation you will explore what makes polygons similar.

Investigation 1: What Makes Polygons Similar?

Hexagons $ABCDEF$ and $PQRSTU$ in your book are similar. Use patty paper to compare the corresponding angles. You should find that the corresponding angles are congruent.

Now carefully measure the corresponding sides and find the ratio of each side length of $ABCDEF$ to the corresponding side length of $PQRSTU$. You should find that each ratio is approximately equal to $\frac{5}{9}$. So, the corresponding sides are proportional.

Finally, calculate and compare these side length ratios within each polygon: $\frac{AB}{BC}$ with $\frac{PQ}{QR}$ and $\frac{EF}{CD}$ with $\frac{TU}{RS}$. You should find that $\frac{AB}{BC} \approx \frac{PQ}{QR}$ and $\frac{EF}{CD} \approx \frac{TU}{RS}$. (The ratios may not be exactly equal because the measurements are not exact.) So, the ratio between two sides of polygon $ABCDEF$ is the same as the ratio between the corresponding sides of $PQRSTU$.

The relationships you discovered in the investigation illustrate the mathematical definition of similar polygons.

Two polygons are **similar** if and only if the corresponding angles are congruent and the corresponding sides are proportional.

The statement $CORN \sim PEAS$ says that quadrilateral $CORN$ is similar to quadrilateral $PEAS$. The order of the letters tells you which sides and angles correspond.

Look at quadrilaterals $SQUE$ and $RCTL$ on page 583 of your book. These figures have congruent corresponding angles, but they are *not* similar because the corresponding sides are *not* proportional.

(continued)

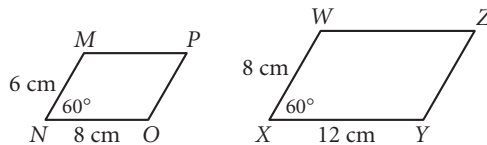
Lesson 11.1 • Similar Polygons (continued)

Now look at quadrilaterals *SQUE* and *RHOM*. These figures have corresponding sides that are proportional, but they are *not* similar because the corresponding angles are *not* congruent.

These examples illustrate that for two figures to be similar, both conditions—proportional sides and congruent angles—must hold. Here is another example.

EXAMPLE

Determine whether parallelogram *MNOP* is similar to parallelogram *WXYZ*.



► Solution

$m\angle N = m\angle X$. Using angle properties of parallelograms, $m\angle M = m\angle W = 120^\circ$, $m\angle P = m\angle Z = 60^\circ$, and $m\angle O = m\angle Y = 120^\circ$. So, the corresponding angles are congruent.

However, because $\frac{MN}{WX} = \frac{6}{8} = \frac{3}{4}$ and $\frac{NO}{XY} = \frac{8}{12} = \frac{2}{3}$, the corresponding sides are not proportional. Therefore, the parallelograms are not similar.

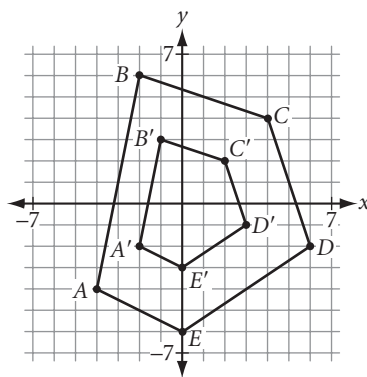
If you know that two polygons are similar, you can use the definition of similar polygons to find missing measures. The example in your book shows you how. Read this example carefully and make sure you understand it.

In Chapter 7, you saw that rigid transformations—translations, rotations, and reflections—preserve the size and shape of a figure, resulting in an image that is congruent to the original. In the next investigation you will look at a nonrigid transformation called a **dilation**.

Investigation 2: Dilations on the Coordinate Plane

To *dilate* a figure in the coordinate plane about the origin, you multiply the coordinates of all its vertices by the same number, called a **scale factor**.

Pentagon *ABCDE* in your book has vertices with coordinates $A(-4, -4)$, $B(-2, 6)$, $C(4, 4)$, $D(6, -2)$, $E(0, -6)$. If you multiply each coordinate by $\frac{1}{2}$, you get $A'(-2, -2)$, $B'(-1, 3)$, $C'(2, 2)$, $D'(3, -1)$, $E'(0, -3)$. The figure at right shows the original pentagon and the image.



If you compare the corresponding sides and angles, you will find that the corresponding angles are congruent and that each side of the image pentagon is half the length of its corresponding original side. So, the pentagons are similar.

Choose one of the other scale factors listed in your book— $\frac{3}{4}$, 2, or 3—and multiply each coordinate of *ABCDE* by the factor you choose. Graph the image pentagon on the same axes as the original pentagon. Compare the corresponding angle measures and side lengths. Are the two pentagons similar? State your findings as a conjecture.

Dilation Similarity Conjecture If one polygon is a dilated image of another polygon, then _____.

C-90

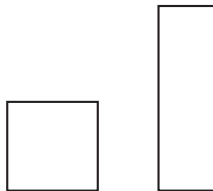
11.2

Similar Triangles

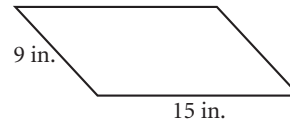
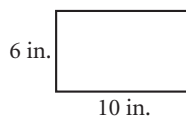
In this lesson you will

- Learn **shortcuts** for determining whether two triangles are similar

In Lesson 11.1, you saw that to determine whether two quadrilaterals are congruent, you must check *both* that their corresponding sides are proportional *and* that their corresponding angles are congruent.



Angles are congruent, but quadrilaterals are *not* similar.



Sides are proportional, but quadrilaterals are *not* similar.

However, triangles are different. In Chapter 4, you discovered that you don't have to check every pair of sides and angles to determine whether two triangles are congruent. You found that SSS, SAS, ASA, and SAA are congruence shortcuts. In this lesson you will find that there are also similarity shortcuts.

Page 589 of your book shows two triangles in which only one pair of angles is congruent. The triangles are clearly not similar. So, knowing only that one pair of angles is congruent is not enough to conclude that two triangles are similar. What if two pairs of angles are congruent?

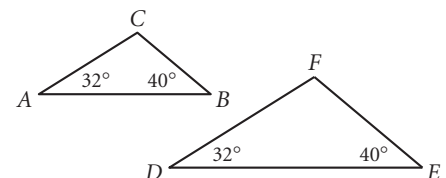
Investigation 1: Is AA a Similarity Shortcut?

In the triangles at right, $\angle A \cong \angle D$ and $\angle B \cong \angle E$.
What must be true about $\angle C$ and $\angle F$? Why?

Measure the sides and compare the ratios of corresponding side lengths. Is $\frac{AB}{DE} \approx \frac{AC}{DF} \approx \frac{BC}{EF}$?

Now draw your own triangle ABC . Use a compass and straightedge to construct triangle DEF , with $\angle A \cong \angle D$ and $\angle B \cong \angle E$. Are your triangles similar? Explain.

Your findings should support this conjecture.



AA Similarity Conjecture If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

C-91

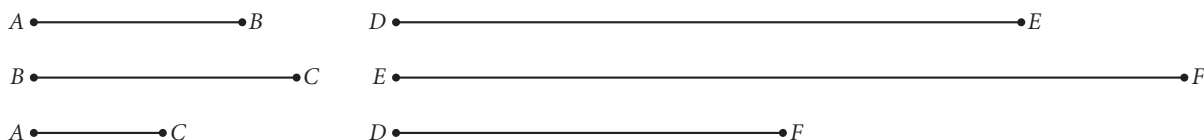
Now consider similarity shortcuts that compare only corresponding sides. The illustration on page 590 of your book shows that you cannot conclude that two triangles are similar given that two pairs of corresponding sides are proportional. What if all three pairs of corresponding sides are proportional?

(continued)

Lesson 11.2 • Similar Triangles (continued)

Investigation 2: Is SSS a Similarity Shortcut?

Use the segments on the left to construct $\triangle ABC$. Each segment on the right is three times the length of the corresponding segment on the left. Use the segments on the right to construct $\triangle DEF$.



The side lengths of $\triangle DEF$ are proportional to the side lengths of $\triangle ABC$. Measure the angles and see how they compare. Are the triangles similar?

Construct another pair of triangles so that the side lengths of one triangle are a multiple of the side lengths of the other. Compare the corresponding angles of your triangles.

You can now complete this conjecture.

SSS Similarity Conjecture If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are _____.

C-92

If AA is a similarity shortcut, then so are ASA, SAA, and AAA, because each of those shortcuts contains two angles. That leaves SAS and SSA as possible shortcuts. In the next investigation you will look at SAS.

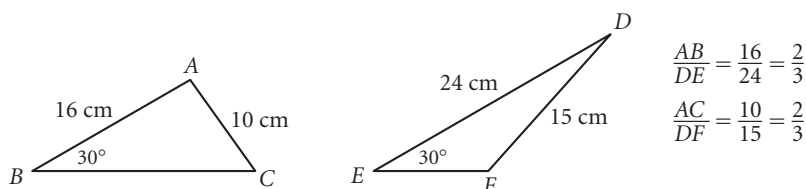
Investigation 3: Is SAS a Similarity Shortcut?

Try to construct two different triangles that are *not* similar, but that have two pairs of sides proportional and the pair of included angles equal in measure. Can you do it? Your findings should support this conjecture.

SAS Similarity Conjecture If two sides of one triangle are proportional to two sides of another triangle and the included angles are congruent, then the triangles are similar.

C-93

In the triangles below, two pairs of corresponding sides are proportional and one pair of non-included angles is congruent. However, the triangles are clearly not similar. This shows that SSA is *not* a similarity shortcut.



$\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle B \cong \angle E$, but the triangles are not similar.

Indirect Measurement with Similar Triangles

In this lesson you will

- Learn how to use similar triangles to measure tall objects and large distances **indirectly**

Suppose you want to find the height of a tall object such as a flagpole. It would be difficult to measure the flagpole directly—you would need a very tall ladder and a very long tape measure! In this lesson you will learn how you can use similar triangles to find the heights of tall objects indirectly.

Investigation: Mirror, Mirror

You will need another person to help you with this investigation. You will also need a small mirror, a meterstick, and masking tape.

Mark crosshairs on your mirror with masking tape or a soluble pen. Label the intersection point X .

Choose a tall object, such as a flagpole, a tall tree, a basketball hoop, or a tall building.

Set the mirror, faceup, on the ground several yards from the object you wish to measure.

Step back from the mirror, staying in line with the mirror and the object, until you see the reflection of the top of the object at point X on the mirror.

Have another person measure and record the distances from you to X and from X to the base of the object. Also, have the person measure and record your height at eye level.

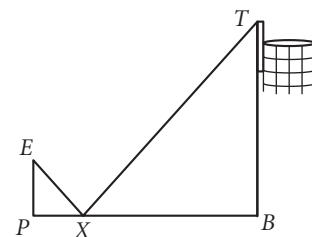
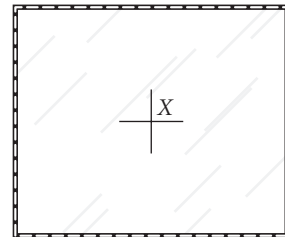
Sketch a diagram of your setup, like this one. Label the top of the object T , the base of the object B , the point where you stood P , and your eye level E . Label \overline{PX} , \overline{BX} , and \overline{EP} with the measurements your helper found.

Think of \overline{TX} as the path of a light ray that bounced back to your eye along \overline{XE} . Because the incoming angle must be congruent to the outgoing angle, $\angle EXP \cong \angle TXB$. Also, because \overline{EP} and \overline{TB} are perpendicular to the ground, $\angle P \cong \angle B$. By the AA Similarity Conjecture, $\triangle EPX \sim \triangle TBX$.

Because the triangles are similar, you can set up a proportion to find TB , the height of the tall object.

$$\frac{EP}{PX} = \frac{TB}{BX}$$

Find the height of your object. Then write a paragraph summarizing your work. Discuss possible causes of error.



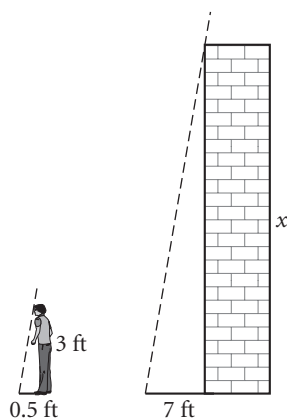
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Lesson 11.3 • Indirect Measurement with Similar Triangles (continued)

The example in your book illustrates a method of indirect measurement that involves shadows. Read the example and make sure you can explain why the two triangles are similar. Here is another example.

EXAMPLE | A brick building casts a shadow 7 feet long. At the same time, a 3-foot-tall child casts a shadow 6 inches long. How tall is the building?

► **Solution** | The drawing below shows the similar triangles formed. Find x by setting up and solving a proportion.



$$\frac{\text{Height of child}}{\text{Length of child's shadow}} = \frac{\text{Height of building}}{\text{Length of building's shadow}}$$

$$\frac{3}{0.5} = \frac{x}{7}$$

$$6 = \frac{x}{7}$$

$$7 \cdot 6 = x$$

$$42 = x$$

The building is 42 feet tall.

Corresponding Parts of Similar Triangles

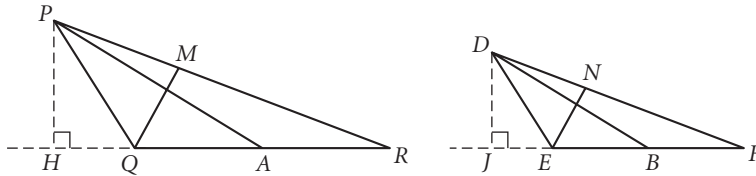
In this lesson you will

- Investigate the relationship between **corresponding altitudes**, **corresponding medians**, and **corresponding angle bisectors** of similar triangles
- **Prove** that the lengths of corresponding medians of similar triangles are proportional to the lengths of corresponding sides
- Discover a **proportional relationship involving angle bisectors**

If two triangles are similar, then their side lengths are proportional. In the next investigation you will see if there is a relationship between the lengths of corresponding altitudes, corresponding medians, or corresponding angle bisectors.

Investigation 1: Corresponding Parts

$\triangle PQR \sim \triangle DEF$. The scale factor from $\triangle PQR$ to $\triangle DEF$ is $\frac{3}{4}$.



\overline{PH} and \overline{DJ} are corresponding altitudes. How do the lengths of these altitudes compare? How does the comparison relate to the scale factor?

\overline{PA} and \overline{DB} are corresponding medians. How do the lengths of these medians compare?

\overline{QM} and \overline{EN} are corresponding angle bisectors. How do the lengths of these angle bisectors compare?

Now draw your own triangle and then construct a similar triangle of a different size. Tell what scale factor you used. Follow Steps 2–4 in your book to construct and compare the lengths of corresponding altitudes, medians, and angle bisectors.

Summarize your findings in this investigation by completing the conjecture below.

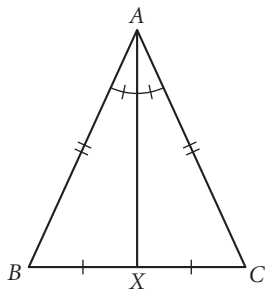
Proportional Parts Conjecture If two triangles are similar, then the lengths of the corresponding _____, _____, and _____ are _____ to the lengths of the corresponding sides.

C-94

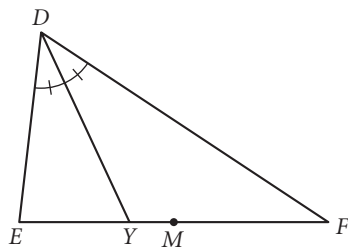
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Lesson 11.4 • Corresponding Parts of Similar Triangles (continued)

If a triangle is isosceles, the bisector of the vertex angle divides the opposite sides into equal parts. (That is, the angle bisector is also a median.) However, as the triangle on the right below shows, this is not true for all triangles.



\overline{AX} is an angle bisector.
Point X is the midpoint of \overline{BC} .



\overline{DY} is an angle bisector.
Point M is the midpoint of \overline{EF} .

The angle bisector does, however, divide the opposite side in a particular way.

Investigation 2: Opposite Side Ratios

Follow Steps 1–5 in your book.

You should find that both ratios are equal to $\frac{1}{2}$.

Repeat Steps 1–5 in your book with $AC = 6$ units and $AB = 18$ units.

This time you should find that $\frac{CA}{BA}$ and $\frac{CD}{BD}$ are both equal to $\frac{1}{3}$.

You can state your findings as a conjecture.

Angle Bisector/Opposite Side Conjecture A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as the lengths of the two sides forming the angle.

C-95

The example in your book proves that in similar triangles, the lengths of the corresponding medians are proportional to the lengths of corresponding sides. Read the example, following along with each step of the proof.

11.5

Proportions with Area

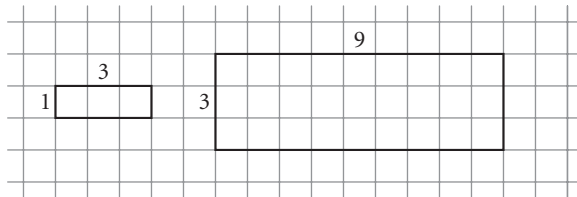
In this lesson you will

- Discover the **relationship between the areas** of similar figures

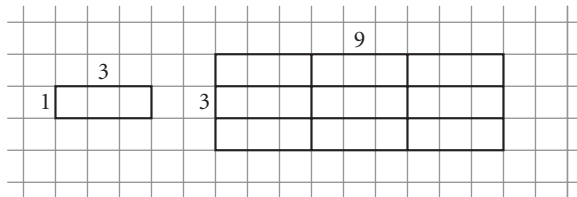
How does multiplying the dimensions of a two-dimensional figure by a scale factor affect its area? In this lesson you will explore this question.

Investigation 1: Area Ratios

The rectangle on the right was created by multiplying the side lengths of the rectangle on the left by 3.



The area of the small rectangle is 3 square units. The area of the large rectangle is 27 square units. The ratio of side lengths of the larger rectangle to side lengths of the smaller rectangle is $\frac{3}{1}$, and the ratio of areas is $\frac{9}{1}$. Notice that nine copies of the small rectangle fit inside the large rectangle.



Now draw your own rectangle on graph paper. Then create a larger or smaller rectangle by multiplying the sides by a scale factor other than 3. What is the ratio of side lengths of the larger rectangle to side lengths of the smaller rectangle? What is the ratio of the areas? How do the two ratios compare?

Draw a triangle on graph paper (your work will be easiest if you draw a right triangle). Draw a similar triangle by multiplying the side lengths by a scale factor. Find the ratio of side lengths and the ratio of areas. How do the ratios compare?

Do you think your findings would be the same for any pair of polygons?

Would your findings be the same for a circle? Consider a circle with radius 5 cm and a circle with radius 20 cm. How does the ratio of radii compare to the ratio of areas?

(continued)

Lesson 11.5 • Proportions with Area (continued)

The following conjecture summarizes the relationship between the areas of similar figures.

Proportional Areas Conjecture If corresponding side lengths of two similar polygons or the radii of two circles compare in the ratio $\frac{m}{n}$, then their areas compare in the ratio $\frac{m^2}{n^2}$.

C-96

The reason behind the Proportional Areas Conjecture is that area is a two-dimensional measure. Calculating area involves multiplying two linear measures, such as base and height. So, if a rectangle has base b and height h , then its area is bh . If the base and height are each multiplied by 2, then the area of the new rectangle is $2b \cdot 2h$, or $4bh$. This is four times the area of the original rectangle. Similarly, if the base and height are each multiplied by 3, then the area of the new rectangle is $3b \cdot 3h$, or $9bh$. This is nine times the area of the original rectangle.

Investigation 2: Surface Area Ratios

In this investigation you will explore whether the Proportional Areas Conjecture is true for surface areas of similar figures. For this investigation you will need interlocking cubes and isometric dot paper.

Follow Steps 1–3 in your book.

You should find that the surface area for the Step 1 figure is 22 square units, while the surface area for a similar prism enlarged by the scale factor 2 is 88 square units. So, lengths of corresponding edges have ratio 2 to 1, while the ratio of the surface areas is 88 to 22, or 4 to 1.

Follow Step 4 in your book. Make sure you include the area of every face, including the face that the figure is resting on.

You should find that the surface area of the figure is 28 square units.

Follow Steps 5 and 6 in your book.

If your answers for Steps 5 and 6 are correct, you can correctly conclude that the Proportional Areas Conjecture also applies to surface area.

The example in your book shows how to use the Proportional Areas Conjecture to solve a real-life problem. Solve the problem yourself before reading the solution.

11.6

Proportions with Volume

In this lesson you will

- Discover the **relationship between the volumes** of similar solids

How does multiplying every dimension of a three-dimensional solid by the same scale factor affect its volume? In this lesson you will explore this question.

Suppose you are going to create a statue that is 2 feet high. You first create a smaller version of the statue that is 4 inches tall and weighs 8 ounces. How much clay should you buy for the larger statue? Would you believe you'll need 108 pounds of clay? When you finish this lesson, you'll understand why.

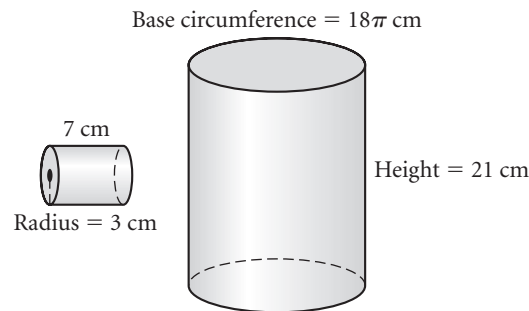
Similar solids are solids that have the same shape, but not necessarily the same size. All cubes are similar, but not all prisms are similar. All spheres are similar, but not all cylinders are similar. Two polyhedrons are similar if all their corresponding faces are similar and the lengths of their corresponding edges are proportional. Two right cylinders or right cones are similar if their radii and heights are proportional.

Examples A and B in your book involve determining whether two given solids are similar. Try to answer the problems yourself before reading the solutions.

Here is another example.

EXAMPLE

Are these two right cylinders similar?

► **Solution**

Find the radius of the larger cylinder.

$$C = 2\pi r$$

$$18\pi = 2\pi r$$

$$r = 9$$

The radius is 9 cm.

Comparing the lengths of corresponding parts we see:

The ratio of the radii is $\frac{3}{9} = \frac{1}{3}$.

The ratio of the heights is $\frac{7}{21} = \frac{1}{3}$.

The radii and heights are proportional, so the right cylinders are similar.

(continued)

Lesson 11.6 • Proportions with Volume (continued)

Investigation: Volume Ratios

In this investigation you'll explore how the ratio of the edge lengths of similar solids compares to the ratio of the volumes. You will need interlocking cubes and isometric dot paper.

Follow Steps 1 and 2 in your book. In Step 2, make sure you multiply all three dimensions—length, width, and height—by 2.

What is the ratio of the side lengths (larger to smaller) for the two “snakes”?
What is the ratio of the volumes? How do the ratios compare?

Follow Steps 4–6 in your book. How would the volume change if you multiplied each dimension by 5? By $\frac{1}{2}$?

Your findings can be stated as a conjecture.

Proportional Volumes Conjecture If corresponding edge lengths (or radii, or heights) of two similar solids compare in the ratio $\frac{m}{n}$, then their volumes compare in the ratio $\frac{m^3}{n^3}$.

C-97

The example in your book shows how to apply both the Proportional Areas Conjecture and the Proportional Volumes Conjecture. Work through the example to understand how each conjecture is used. Then read the example below. Solve the problem yourself before reading the solution.

EXAMPLE

A “square can” is a right cylinder that has a height equal to its diameter. One square can has height 5 cm and another has height 12 cm. About how many full cans of water from the smaller can are needed to fill the larger can?

► Solution

The ratio of the radius of the larger can to the radius of the smaller can is $\frac{6}{2.5} = \frac{12}{5}$, which is the same as the ratio of the heights. So, the two square cans are similar.

The amount of water needed to fill each can is determined by the volume of the can, so find the ratio of the volumes. The ratio of the heights is $\frac{12}{5}$. Therefore, the ratio of the volumes is $\frac{12^3}{5^3}$, or $\frac{1728}{125}$.

$\frac{1728}{125} \approx 13.8$, so it takes almost 14 cans of water from the smaller can to fill the larger can.

Proportional Segments Between Parallel Lines

In this lesson you will

- Explore the relationships in the lengths of segments formed when one or more lines parallel to one side of a triangle intersect the other two sides
- Learn how you can use the relationship you discover to divide a given segment into any number of equal parts

The top of page 623 in your book shows $\triangle LUV$ and line MT , with $\overline{MT} \parallel \overline{LU}$. It appears that $\triangle LUV \sim \triangle MTV$. The paragraph proof given uses the AA Similarity Conjecture to prove this is true. Example A in your book uses the similarity of two triangles to solve a problem. Read the example, and follow along with each step in the solution.

Look at the figure from Example A. Notice that $\frac{LE}{EM} = \frac{45}{60} = \frac{3}{4}$ and $\frac{NO}{OM} = \frac{36}{48} = \frac{3}{4}$, so there are more relationships in the figure than the ones found by using similar triangles. You will explore these relationships in the next investigation.

Investigation 1: Parallels and Proportionality

Step 1 of the investigation gives three triangles, each with a line parallel to one side that intersects the other two sides. For each triangle, find x and then find the values of the specified ratios. Here is the solution to part a.

- a. Use the fact that $\triangle CDE \sim \triangle BDA$ to write and solve a proportion.

$$\frac{DE}{DA} = \frac{DC}{DB}$$

Corresponding parts of similar triangles are proportional.

$$\frac{8}{24} = \frac{12}{12 + x}$$

Substitute the lengths from the figure.

$$\frac{1}{3} = \frac{12}{12 + x}$$

Simplify the left side.

$$\frac{1(3)(12 + x)}{3} = \frac{12(3)(12 + x)}{12 + x}$$

Multiply both sides by $3(12 + x)$.

$$12 + x = 36$$

Simplify.

$$x = 24$$

Subtract 12 from both sides.

$$\text{So, } \frac{DE}{AE} = \frac{8}{16} = \frac{1}{2} \text{ and } \frac{DC}{BC} = \frac{12}{24} = \frac{1}{2}.$$

In each part of Step 1, you should find that the ratios of the lengths of the segments cut by the parallel line are equal. In other words: If a line parallel to one side of a triangle passes through the other two sides, then it divides the other two sides proportionally.

Do you think the converse of this statement is also true? That is, if a line divides two sides of a triangle proportionally, is it parallel to the third side?

Follow Steps 3–7 in your book. You should find that $\angle PAC \cong \angle PBD$, so \overline{AC} and \overline{BD} are parallel.

Repeat Steps 3–7, but this time mark off your own lengths, such that $\frac{PA}{AB} = \frac{PC}{CD}$. Again, you should find that \overline{AC} is parallel to \overline{BD} . You can use your findings from this investigation to state a conjecture.

(continued)

Parallel/Proportionality Conjecture If a line parallel to one side of a triangle passes through the other two sides, then it divides the other two sides proportionally. Conversely, if a line cuts two sides of a triangle proportionally, then it is parallel to the third side.

C-98

Example B proves the first part of the Parallel/Proportionality Conjecture. Write a proof yourself before reading the one in the book. Use the fact that $\triangle AXY \sim \triangle ABC$ to set up a proportion. Then write a series of algebraic steps until you get $\frac{a}{c} = \frac{b}{d}$.

Investigation 2: Extended Parallel/Proportionality

In the triangles in Step 1, more than one segment is drawn parallel to one side of a triangle. Find the missing lengths. Here is the solution to part a. To find x and y , apply the Parallel/Proportionality Conjecture to the appropriate triangles and lines.

- a. To find x , use $\triangle AEL$ and \overline{FT} .

$$\frac{EF}{FL} = \frac{ET}{TA}$$

$$\frac{21}{35} = \frac{42}{x}$$

$$x = 70$$

To find y , use $\triangle REG$ and \overline{LA} .

$$\frac{EL}{LG} = \frac{EA}{AR}$$

$$\frac{56}{28} = \frac{112}{y}$$

$$y = 56$$

Using the values of x and y , you can see that $\frac{FL}{LG} = \frac{TA}{AR} = \frac{5}{4}$.

The results of Step 1 lead to the following conjecture.

Extended Parallel/Proportionality Conjecture If two or more lines pass through two sides of a triangle parallel to the third side, then they divide the two sides proportionally.

C-99

You can use the Extended Parallel/Proportionality Conjecture to divide a segment into any number of equal parts. Example C in your book shows you how to divide a segment, \overline{AB} , into three equal parts. Read the example carefully. To make sure you understand the process, divide \overline{XY} below into three equal parts using a compass and straightedge.



12.1

Trigonometric Ratios

In this lesson you will

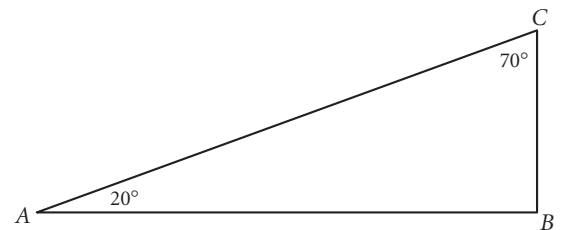
- Learn about the trigonometric ratios **sine**, **cosine**, and **tangent**
- Use trigonometric ratios to **find unknown side lengths** in right triangles
- Use **inverse trigonometric functions** to **find unknown angle measures** in right triangles

Read up to Example A in your book. Your book explains that, in any right triangle with an acute angle of a given measure, the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle is the same. The ratio is known as the **tangent** of the angle. Example A uses the fact that $\tan 31^\circ \approx \frac{3}{5}$ to solve a problem. Read the example carefully.

In addition to tangent, mathematicians have named five other ratios relating the side lengths of right triangles. In this book, you will work with three ratios: **sine**, **cosine**, and **tangent**, abbreviated sin, cos, and tan. These ratios are defined on pages 641–642 of your book.

Investigation: Trigonometric Tables

Measure the side lengths of $\triangle ABC$ to the nearest millimeter. Then use the side lengths and the definitions of sine, cosine, and tangent to fill in the “First \triangle ” row of the table. Express the ratios as decimals to the nearest thousandth.



	$m\angle A$	$\sin A$	$\cos A$	$\tan A$	$m\angle C$	$\sin C$	$\cos C$	$\tan C$
First \triangle	20°				70°			
Second \triangle	20°				70°			
Average	—				—			

Now use your protractor to draw a different right triangle ABC , with $m\angle A = 20^\circ$ and $m\angle C = 70^\circ$. Measure the sides to the nearest millimeter and fill in the “Second \triangle ” row of the table.

Calculate the average for each ratio and put the results in the last row of the table. Look for patterns in your table. You should find that $\sin 20^\circ = \cos 70^\circ$ and $\sin 70^\circ = \cos 20^\circ$. Notice also that $\tan 20^\circ = \frac{1}{\tan 70^\circ}$ and $\tan 70^\circ = \frac{1}{\tan 20^\circ}$. Use the definitions of sine, cosine, and tangent to explain why these relationships exist.

You can use your calculator to find the sine, cosine, or tangent of any angle. Experiment with your calculator until you figure out how to do this. Then use your calculator to find $\sin 20^\circ$, $\cos 20^\circ$, $\tan 20^\circ$, $\sin 70^\circ$, $\cos 70^\circ$, and $\tan 70^\circ$. Compare the results to the ratios you found by measuring sides.

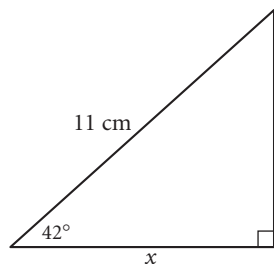
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Lesson 12.1 • Trigonometric Ratios (continued)

You can use trigonometric ratios to find unknown side lengths of a right triangle given the measures of any side and any acute angle. Read Example B in your book and then read Example A below.

EXAMPLE A

Find the value of x .



► Solution

You need to find the length of the leg adjacent to the 42° angle. You are given the length of the hypotenuse. The trigonometric ratio that relates the adjacent leg and the hypotenuse is the cosine ratio.

$$\cos 42^\circ = \frac{x}{11}$$

$$11 \cdot \cos 42^\circ = x \quad \text{Multiply both sides by 11.}$$

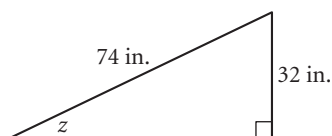
$$8.17 \approx x \quad \text{Use your calculator to find } \cos 42^\circ \text{ and multiply the result by 11.}$$

The value of x is about 8.2 cm.

If you know the lengths of any two sides of a right triangle, you can use *inverse trigonometric functions* to find the angle measures. Example C in your book shows how to use the inverse tangent, or \tan^{-1} , function. The example below uses the inverse sine, or \sin^{-1} , function.

EXAMPLE B

Find the measure of the angle opposite the 32-inch leg.



► Solution

You are given the lengths of the leg opposite the angle and the hypotenuse. The ratio that relates these lengths is the sine ratio.

$$\sin z = \frac{32}{74}$$

$$\sin^{-1}(\sin z) = \sin^{-1}\left(\frac{32}{74}\right) \quad \text{Take the inverse sine of both sides.}$$

$$z = \sin^{-1}\left(\frac{32}{74}\right) \quad \text{The inverse sine function undoes the sine function.}$$

$$z \approx 25.6^\circ \quad \text{Use your calculator to find } \sin^{-1}\left(\frac{32}{74}\right).$$

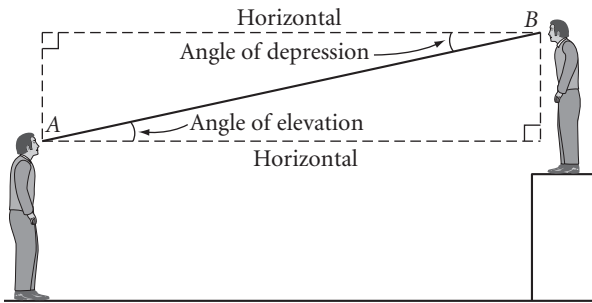
The measure of the angle opposite the 32-inch side is about 26° .

Problem Solving with Right Triangles

In this lesson you will

- Use trigonometry to solve problems involving right triangles

Right triangle trigonometry is often used to find the height of a tall object indirectly. To solve a problem of this type, measure the angle from the horizontal to your line of sight when you look at the top or bottom of the object.



If you look up, you measure the **angle of elevation**. If you look down, you measure the **angle of depression**.

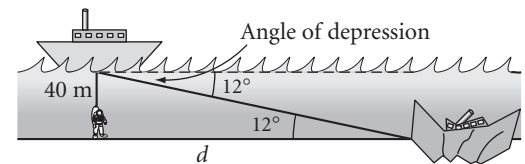
The example in your book uses the angle of elevation to find a distance indirectly. Read the example carefully. Try to solve the problem on your own before reading the solution. Then try to solve the problems in the examples below. Example A is Exercise 13 in your book. It involves an angle of depression.

EXAMPLE A

A salvage ship's sonar locates wreckage at a 12° angle of depression. A diver is lowered 40 meters to the ocean floor. How far does the diver need to walk along the ocean floor to the wreckage?

► Solution

Make a sketch to illustrate the situation. Notice that because the ocean floor is parallel to the surface of the water, the angle of elevation from the wreckage to the ship is equal to the angle of depression from the ship to the wreckage (by the AIA Conjecture).



The distance the diver is lowered (40 m) is the length of the leg opposite the 12° angle. The distance the diver must walk is the length of the leg adjacent to the 12° angle. Set up the tangent ratio.

$$\tan 12^\circ = \frac{40}{d}$$

$$d \tan 12^\circ = 40$$

$$d = \frac{40}{\tan 12^\circ}$$

$$d \approx 188.19$$

The diver must walk approximately 188 meters to reach the wreckage.

(continued)

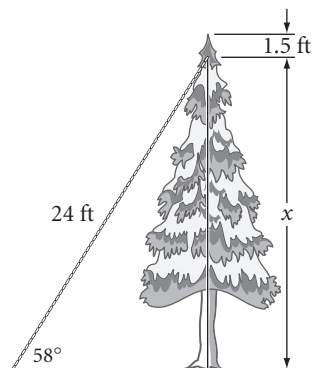
Lesson 12.2 • Problem Solving with Right Triangles (continued)

EXAMPLE B

An evergreen tree is supported by a wire extending from 1.5 feet below the top of the tree to a stake in the ground. The wire is 24 feet long and forms a 58° angle with the ground. How tall is the tree?

► Solution

Make a sketch to illustrate the situation.



The length of the hypotenuse is given, and the unknown distance is the length of the side opposite the 58° angle. Set up the sine ratio.

$$\sin 58^\circ = \frac{x}{24}$$

$$24 \cdot \sin 58^\circ = x$$

$$20.4 \approx x$$

The distance from the ground to the point where the wire is attached to the tree is about 20.4 feet. Because the wire is attached 1.5 feet from the top of the tree, the tree's height is about $20.4 + 1.5$, or 21.9 feet.

12.3

The Law of Sines

In this lesson you will

- Find the **area of a triangle** when you know two side lengths and the measure of the included angle
- Derive the **Law of Sines**, which relates the side lengths of a triangle to the sines of the angle measures
- Use the **Law of Sines to find an unknown side length** of a triangle when you know the measures of two angles and one side **or to find an unknown acute angle measure** when you know the measures of two sides and one angle

You have used trigonometry to solve problems involving right triangles. In the next two lessons you will see that you can use trigonometry with *any* triangle.

Example A in your book gives the lengths of two sides of a triangle and the measure of the included angle and shows you how to find the area. Read the example carefully. In the next investigation you will generalize the method used in the example.

Investigation 1: Area of a Triangle

Step 1 gives three triangles with the lengths of two sides and the measure of the included angle labeled. Use Example A as a guide to find the area of each triangle. Here is a solution to part b.

b. First find h .

$$\sin 72^\circ = \frac{h}{21}$$

$$21 \cdot \sin 72^\circ = h$$

Now find the area.

$$A = 0.5bh$$

$$A = 0.5(38.5)(21 \cdot \sin 72^\circ)$$

$$A \approx 384.46$$

The area is about 384 cm^2 .

Then use the triangle shown in Step 2 to derive a general formula. The conjecture below summarizes the results.

SAS Triangle Area Conjecture The area of a triangle is given by the formula $A = \frac{1}{2}ab \sin C$, where a and b are the lengths of two sides and C is the angle between them.

C-100

(continued)

Lesson 12.3 • The Law of Sines (continued)

You can use what you've learned to derive the property called the Law of Sines.

Investigation 2: The Law of Sines

Complete Steps 1–3 in your book. Below are the results you should find.

Step 1 $\sin B = \frac{h}{a}$, so $h = a \sin B$

Step 2 $\sin A = \frac{h}{b}$, so $h = b \sin A$

Step 3 Because both $b \sin A$ and $a \sin B$ are equal to h , you can set them equal to one another.

$$b \sin A = a \sin B$$

$$\frac{b \sin A}{ab} = \frac{a \sin B}{ab} \quad \text{Divide both sides by } ab.$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{Simplify.}$$

Now complete Steps 4–6. Combine Steps 3 and 6 to get this conjecture.

Law of Sines For a triangle with angles A , B , and C and sides of lengths a , b , and c (a opposite A , b opposite B , and c opposite C),
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

C-101

Example B in your book shows you how to use the Law of Sines to find the lengths of a triangle's sides when you know one side length and two angle measures. Try to solve the problem yourself before reading the solution.

Read the text before Example C, which explains that you can use the Law of Sines to find the measure of a missing angle *only if* you know whether the angle is acute or obtuse. You will only be asked to find acute angle measures. Example C shows you how to do this. Here is another example.

EXAMPLE | Find the measure of acute angle C .

► **Solution**

Use the Law of Sines.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

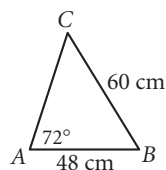
$$\sin C = \frac{c \sin A}{a}$$

$$\sin C = \frac{48 \sin 72^\circ}{60}$$

$$C = \sin^{-1}\left(\frac{48 \sin 72^\circ}{60}\right)$$

$$C \approx 49.54$$

The measure of $\angle C$ is approximately 50° .



12.4

The Law of Cosines

In this lesson you will

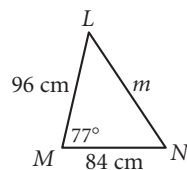
- Use the **Law of Cosines** to find side lengths and angle measures in a triangle

You have solved many problems by using the Pythagorean Theorem. The Pythagorean Theorem is a very powerful problem-solving tool, but it is limited to right triangles. There is a more general relationship that applies to all triangles.

Think of a right angle that is made by a hinge with two legs of fixed length as its sides. What happens to the length of the third side (the hypotenuse when the angle measures 90°) and to the Pythagorean relationship as the hinge closes to less than a right angle or opens to more than a right angle? To explore this question, look at the triangles pictured at the top of page 661 and read the paragraphs that follow, including the Law of Cosines. Add the Law of Cosines to your conjecture list.

The Law of Cosines works for both acute and obtuse triangles. In your book, read the derivation of the Law of Cosines for acute triangles on page 662. In Example A, the Law of Cosines is used to find the length of the third side of a triangle when you are given the lengths of two sides and the measure of their included angle. Read Example A in your book. Then work through Example A below.

EXAMPLE A Find m , the length of side \overline{NL} in acute $\triangle LMN$.



► **Solution** Use the Law of Cosines and solve for m .

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The Law of Cosines.

$$m^2 = 96^2 + 84^2 - 2(96)(84)(\cos 77^\circ)$$

Substitute m for c , 96 for a , 84 for b , and 77° for C .

$$m = \sqrt{96^2 + 84^2 - 2(96)(84)(\cos 77^\circ)}$$

Take the positive square root of both sides.

$$m \approx 112.45$$

Evaluate.

The length of side \overline{NL} is about 112 cm.

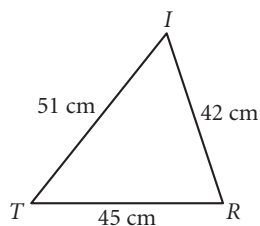
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Lesson 12.4 • The Law of Cosines (continued)

Example B in your book uses the Law of Cosines to find an angle measure. Here is another example. Solve the problem yourself before reading the solution.

EXAMPLE B

Find the measure of $\angle I$ in $\triangle TRI$.



► Solution

Use the Law of Cosines and solve for I .

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The Law of Cosines.

$$45^2 = 51^2 + 42^2 - 2(51)(42)(\cos I)$$

Substitute 45 for c , 51 for a , 42 for b , and I for C .

$$\cos I = \frac{45^2 - 51^2 - 42^2}{-2(51)(42)}$$

Solve for $\cos I$.

$$I = \cos^{-1}\left(\frac{45^2 - 51^2 - 42^2}{-2(51)(42)}\right)$$

Take the inverse cosine of both sides.

$$I \approx 56.89$$

Evaluate.

The measure of $\angle I$ is about 57° .

Problem Solving with Trigonometry

In this lesson you will

- Use trigonometry to solve problems, including problems that involve **vectors**

Some of the practical applications of trigonometry involve vectors. In earlier vector activities, you used a ruler and a protractor to measure the size of the resulting vector and the angle between vectors. Now you will be able to calculate resulting vectors by using the Law of Sines and the Law of Cosines.

In the example in your book, the Law of Cosines is used to find the length of a resultant vector and the Law of Sines is used to find its direction. Read the example and make sure you understand each step.

The example below is Exercise 5 in your book. Try to solve the problem on your own before reading the solution.

EXAMPLE

Annie and Sashi are backpacking in the Sierra Nevada. They walk 8 km from their base camp at a bearing of 42° . After lunch, they change direction to a bearing of 137° and walk another 5 km.

- How far are Annie and Sashi from their base camp?
- At what bearing must Sashi and Annie travel to return to their base camp?

► Solution

- Draw a diagram to illustrate the situation. (Remember, a bearing is measured clockwise from north.) Here, the distance from base camp is r . To find r , you can find the value of θ and then use the Law of Cosines.

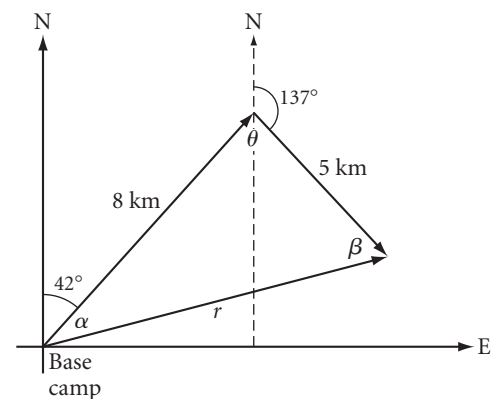
Think of θ as being made up of two parts, the part to the left of the vertical and the part to the right. Using the AIA Conjecture, the part to the left has measure 42° . Because the part to the right and the 137° angle are a linear pair, the part to the right has measure 43° . So, the measure of θ is $42^\circ + 43^\circ$, or 85° . Now use the Law of Cosines.

$$r^2 = 8^2 + 5^2 - 2(8)(5)(\cos 85^\circ)$$

$$r = \sqrt{8^2 + 5^2 - 2(8)(5)(\cos 85^\circ)}$$

$$r \approx 9.06$$

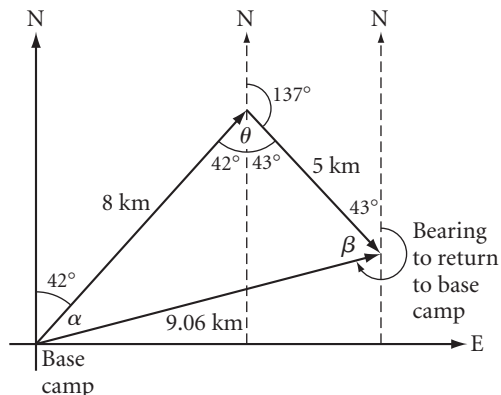
Sashi and Annie are about 9.1 km from their base camp.



(continued)

Lesson 12.5 • Problem Solving with Trigonometry (continued)

- b. Add the information you found in part a to the diagram.



The diagram indicates that the bearing Sashi and Annie must travel to return to the base camp is $360^\circ - (43^\circ + \beta)$. To find β , use the Law of Sines.

$$\frac{\sin \beta}{8} \approx \frac{\sin 85^\circ}{9.06}$$

$$\sin \beta \approx \frac{8 \sin 85^\circ}{9.06}$$

$$\beta \approx \sin^{-1}\left(\frac{8 \sin 85^\circ}{9.06}\right)$$

$$\beta \approx 61.6$$

β is about 62° , so the bearing is about $360^\circ - (43^\circ + 62^\circ)$, or 255° .

13.1

The Premises of Geometry

In this lesson you will

- Learn about Euclid's **deductive system** for organizing geometry properties
- Become familiar with the four types of premises for geometry

Beginning in about 600 B.C.E., mathematicians began to use logical reasoning to deduce mathematical ideas. The Greek mathematician Euclid (ca. 325–265 B.C.E.) created a **deductive system** to organize geometry properties. He started with a simple collection of statements called **postulates**. He considered these postulates to be obvious truths that did not need to be proved. Euclid then systematically demonstrated how each geometry discovery followed logically from his postulates and his previously proved conjectures, or **theorems**.

Up to now, you have used informal proofs to explain why certain conjectures are true. However, you often relied on unproved conjectures in your proofs. A conclusion in a proof is true if and only if your premises are true and all of your arguments are valid.

In this chapter you will look at geometry as Euclid did. You will start with premises and systematically prove your earlier conjectures. Once you have proved a conjecture, it becomes a theorem that you can use to prove other conjectures. Read the four types of premises on page 693 of your book.

You are already familiar with the first type of premise. You have learned the undefined terms—point, line, and plane—and you have a list of definitions in your notebook.

The second type of premise is the properties of arithmetic, equality, and congruence. Read through the properties of arithmetic and equality in your book. You have used these properties many times to solve algebraic equations. The example in your book shows the solution to an algebraic equation, along with the reason for each step. This type of step-by-step solution is actually an algebraic proof. Here is another example.

EXAMPLE | Prove that if $(a + b)(a - b) = (a + b)(a + b)$ and $a + b \neq 0$, then $b = 0$.

► Solution	$(a + b)(a - b) = (a + b)(a + b)$	Given
	$a - b = a + b$	Division property of equality ($a + b \neq 0$)
	$a = a + 2b$	Addition property of equality
	$0 = 2b$	Subtraction property of equality
	$0 = b$	Division property of equality

Just as you use equality to express a relationship between numbers, you use congruence to express a relationship between geometric figures. Read the definition of congruence on page 695 of your book. Here are the properties of congruence.

(continued)

Properties of Congruence

In the statements below, “figure” refers to a segment, an angle, or a geometric shape.

Reflexive Property of Congruence

Any figure is congruent to itself.

Transitive Property of Congruence

If Figure A \cong Figure B and Figure B \cong Figure C, then Figure A \cong Figure C.

Symmetric Property of Congruence

If Figure A \cong Figure B, then Figure B \cong Figure A.

The third type of premise is the postulates of geometry. Postulates are very basic statements that are useful and easy for everyone to agree on. Read the postulates of geometry on pages 696–697 of your book.

Some of the postulates allow you to add auxiliary lines, segments, and points to a diagram. For example, you can use the Line Postulate to construct a diagonal of a polygon, and you can use the Perpendicular Postulate to construct an altitude in a triangle.

Notice that the Corresponding Angles Conjecture and its converse are stated as a postulate, but the Alternate Interior Angles Conjecture is not. This means that you will need to prove the Alternate Interior Angles Conjecture before you can use it to prove other conjectures. Similarly, the SSS, SAS, and ASA Congruence Conjectures are stated as postulates, but SAA is not, so you will need to prove it.

The fourth type of premise is previously proved geometry conjectures, or theorems. Each time you prove a conjecture, you may rename it as a theorem and add it to your theorem list. You can use the theorems on your list to prove conjectures.

13.2

Planning a Geometry Proof

In this lesson you will

- Learn the **five tasks** involved in **writing a proof**
- Prove several conjectures about angles
- Learn how to create a **logical family tree** for a theorem

A proof in geometry is a sequence of statements, starting with a given set of premises and leading to a valid conclusion. Each statement must follow from previous statements and must be supported by a reason. The reason must come from the set of premises you learned about in Lesson 13.1.

Writing a proof involves five tasks.

Task 1 is to identify what is given and what is to be proved. This is easiest if the conjecture is a *conditional*, or “if-then,” statement. The “if” part is what you are given, and the “then” part is what you must show. If a conjecture is not given this way, you can often restate it. For example, the conjecture “Vertical angles are congruent” can be rewritten as “If two angles are vertical angles, then they are congruent.”

Task 2 is to make a diagram and to mark it so it illustrates the given information. Task 3 is to restate the “given” and “show” information in terms of the diagram. Task 4 is to make a plan for your proof, organizing your reasoning either mentally or on paper. Task 5 is to use your plan to write the proof. Page 703 of your book summarizes the five tasks involved in writing a proof.

A flowchart proof of the Vertical Angles Conjecture is given on page 704 of your book. Notice that the proof uses only postulates, definitions, and properties of equality. Thus, it is a valid proof. You can now call the conjecture the Vertical Angles (VA) Theorem and add it to your theorem list.

In Lesson 13.1, the Corresponding Angles (CA) Conjecture was stated as a postulate, but the Alternate Interior Angles (AIA) Conjecture was not. Example A in your book goes through the five-task process for proving the AIA Conjecture. Read the example carefully, and then add the AIA Theorem to your theorem list.

Example B proves the Triangle Sum Conjecture. The proof requires using the Parallel Postulate to construct a line parallel to one side of the triangle. After you read and understand the proof, add the Triangle Sum Theorem to your theorem list.

Pages 706–707 of your book give a proof of the Third Angle Conjecture. Read the proof, and then add the Third Angle Theorem to your theorem list.

A *logical family tree* for a theorem traces the theorem back to all the postulates on which the theorem relies. On page 707 of your book, you’ll see how to construct the logical family tree for the Third Angle Theorem. Make sure you understand how this theorem is rooted in four postulates: the Parallel Postulate, the CA Postulate, the Linear Pair Postulate, and the Angle Addition Postulate.

On page 707 you can see that the Third Angle Theorem follows from the Triangle Sum Theorem without relying on any other postulate or theorem. A theorem that is an immediate consequence of another proven theorem is called a **corollary**. So, the Third Angle Theorem is a corollary of the Triangle Sum Theorem.

(continued)

Lesson 13.2 • Planning a Geometry Proof (continued)

The next example is Exercise 8 in your book. It illustrates the five-task process of proving the Converse of the Alternate Interior Angles Conjecture.

EXAMPLE

Prove the Converse of the AIA Conjecture: If two lines are cut by a transversal forming congruent alternate interior angles, then the lines are parallel. Then create a family tree for the Converse of the AIA Theorem.

► Solution

Task 1: Identify what is given and what you must show.

Given: Two lines cut by a transversal to form congruent alternate interior angles

Show: The lines are parallel

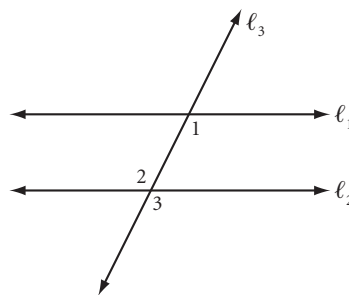
Task 2: Draw and label a diagram.

(Note: You may not realize that labeling $\angle 3$ is useful until you make your plan.)

Task 3: Restate the given and show information in terms of your diagram.

Given: ℓ_1 and ℓ_2 cut by transversal ℓ_3 ;
 $\angle 1 \cong \angle 2$

Show: $\ell_1 \parallel \ell_2$

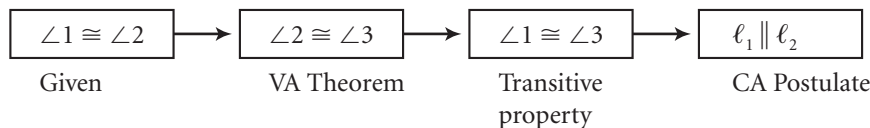


Task 4: Make a plan.

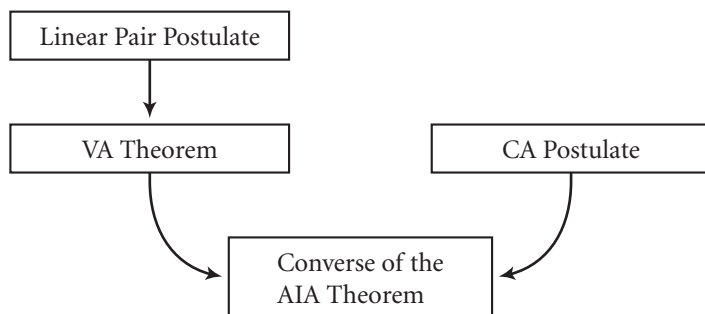
Plan: I need to prove that $\ell_1 \parallel \ell_2$. The only theorem or postulate I have for proving lines are parallel is the CA Postulate. If I can show that $\angle 1 \cong \angle 3$, I can use the CA Postulate to conclude that $\ell_1 \parallel \ell_2$. I know that $\angle 1 \cong \angle 2$. By the VA Theorem, $\angle 2 \cong \angle 3$. So, by the transitive property of congruence, $\angle 1 \cong \angle 3$.

Task 5: Create a proof.

Flowchart Proof



Here is a logical family tree for the Converse of the AIA Theorem.



13.3

Triangle Proofs

In this lesson you will

- Prove conjectures involving **properties of triangles**
- Learn how to write a **two-column proof**

In this lesson you will focus on triangle proofs. Read the lesson in your book. It takes you through the five-task process for proving the Angle Bisector Conjecture and explains how to write a **two-column proof**. The examples below are Exercises 1 and 2 in your book. For each example, write each proof yourself before reading the solution.

EXAMPLE A Write a flowchart proof of the Perpendicular Bisector Conjecture: If a point is on the perpendicular bisector of a segment, then it is equally distant from the endpoints of the segment.

► **Solution**

Task 1: Identify what is given and what you must show.

Given: A point on the perpendicular bisector of a segment

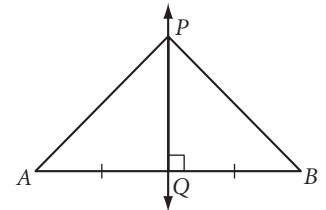
Show: The point is equally distant from the endpoints of the segment

Task 2: Draw and label a diagram to illustrate the given information.

Task 3: Restate the given and show information in terms of the diagram.

Given: \overline{PQ} is the perpendicular bisector of \overline{AB}

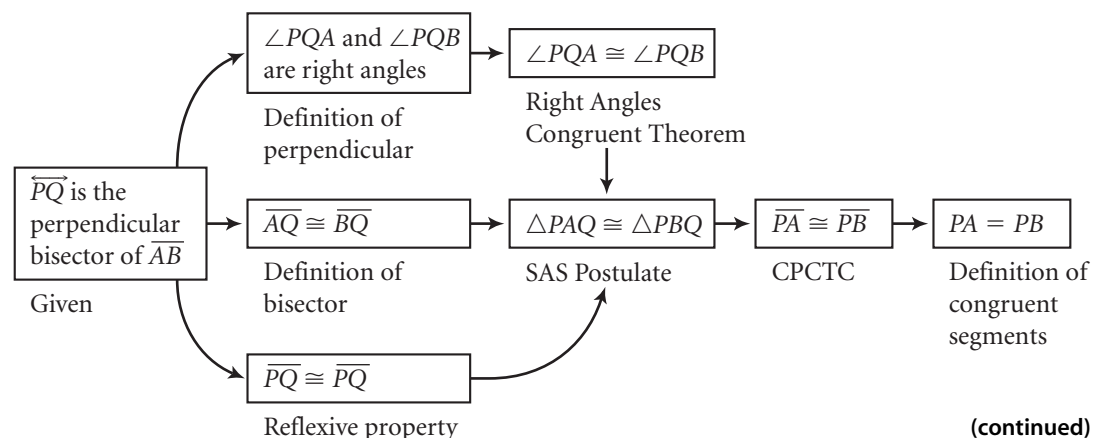
Show: $PA = PB$



Task 4: Plan a proof.

Plan: I can show that $PA = PB$ if \overline{PA} and \overline{PB} are corresponding parts of congruent triangles. I know that $\overline{AQ} \cong \overline{BQ}$ and $\angle PQA \cong \angle PQB$. I also know that $\overline{PQ} \cong \overline{PQ}$. Therefore, $\triangle PAQ \cong \triangle PBQ$ by the SAS Postulate. Thus, $\overline{PA} \cong \overline{PB}$ by CPCTC, so $PA = PB$.

Task 5: Write a proof based on your plan.

Flowchart Proof

(continued)

Lesson 13.3 • Triangle Proofs (continued)

EXAMPLE B

Write a two-column proof of the Converse of the Perpendicular Bisector Conjecture: If a point is equally distant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

► Solution

Task 1: Identify what is given and what you must show.

Given: A point that is equally distant from the endpoints of a segment

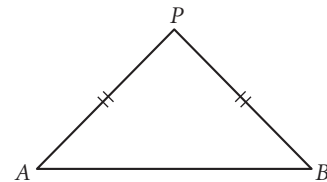
Show: The point is on the perpendicular bisector of the segment

Task 2: Draw and label a diagram to illustrate the given information.

Task 3: Restate the given and show information in terms of the diagram.

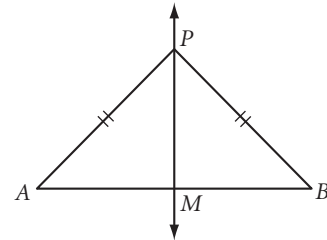
Given: $PA = PB$

Show: P is on the perpendicular bisector of \overline{AB}



Task 4: Plan a proof.

I can start by constructing the midpoint M of \overline{AB} and \overrightarrow{PM} . I know that \overrightarrow{PM} is a bisector of \overline{AB} , so I need only show that it is perpendicular to \overline{AB} . I can show that $\triangle PAM \cong \triangle PBM$ by SSS. Therefore, $\angle PMA \cong \angle PMB$. Because the angles form a linear pair, they are supplementary, so each has measure 90° . So, $\overrightarrow{PM} \perp \overline{AB}$.



Task 5: Write a proof based on your plan.

Statement	Reason
1. Construct the midpoint M of \overline{AB}	1. Midpoint Postulate
2. Construct \overrightarrow{PM}	2. Line Postulate
3. $PA = PB$	3. Given
4. $\overline{PA} \cong \overline{PB}$	4. Definition of congruence
5. $\overline{AM} \cong \overline{BM}$	5. Definition of midpoint
6. $\overline{PM} \cong \overline{PM}$	6. Reflexive property of congruence
7. $\triangle PAM \cong \triangle PBM$	7. SSS Postulate
8. $\angle PMA \cong \angle PMB$	8. CPCTC
9. $\angle PMA$ and $\angle PMB$ are supplementary	9. Linear Pair Postulate
10. $\angle PMA$ and $\angle PMB$ are right angles	10. Congruent and Supplementary Theorem
11. $\overrightarrow{PM} \perp \overline{AB}$	11. Definition of perpendicular
12. \overrightarrow{PM} is the perpendicular bisector of \overline{AB}	12. Definition of perpendicular bisector

Quadrilateral Proofs

In this lesson you will

- Prove conjectures involving **properties of quadrilaterals**
- Learn the term **lemma**

You can prove many quadrilateral theorems by using triangle theorems. For example, you can prove some parallelogram properties by using the fact that a diagonal divides a parallelogram into two congruent triangles. This fact is an example of a **lemma**. A lemma is an auxiliary theorem used specifically to prove other theorems. The proof of the lemma is given as an example in your book. Write your own proof and then compare it to the one in the book. Call the lemma the Parallelogram Diagonal Lemma and add it to your list of theorems.

Developing Proof: Proving Parallelogram Conjectures

In this activity you will prove three of your previous conjectures about parallelograms. Remember to draw a diagram, restate what is given and what you must show in terms of your diagram, and then make a plan before you write your proof.

Complete Step 1 in your book. (*Hint:* The proof will be a snap if you use the Parallelogram Diagonal Lemma.)

Now complete Step 2. (Don't forget the lemma!)

Step 3 asks you to state and prove the converse of the Opposite Sides Conjecture. The five-task proof process is started below.

Task 1: Identify what is given and what you must show.

Given: A quadrilateral with opposite sides that are congruent

Show: The quadrilateral is a parallelogram

Task 2: Draw and label a diagram to illustrate the given information.

Task 3: Restate the given and show information in terms of the diagram.

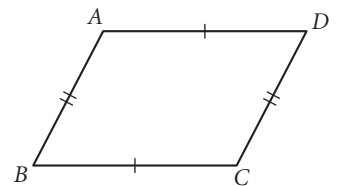
Given: Quadrilateral $ABCD$ with $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$

Show: $ABCD$ is a parallelogram

Task 4: Make a plan.

Try to make a plan yourself. Here are some hints to get you started:

- So far all the quadrilateral proofs have involved drawing a diagonal to form triangles. Consider using that approach here.
- You need to show that the opposite sides of $ABCD$ are parallel. Look back and find theorems and postulates that can be used to prove that two lines are parallel. Which one do you think would be most useful in this situation?
- How can the triangle congruence postulates help you in your proof?



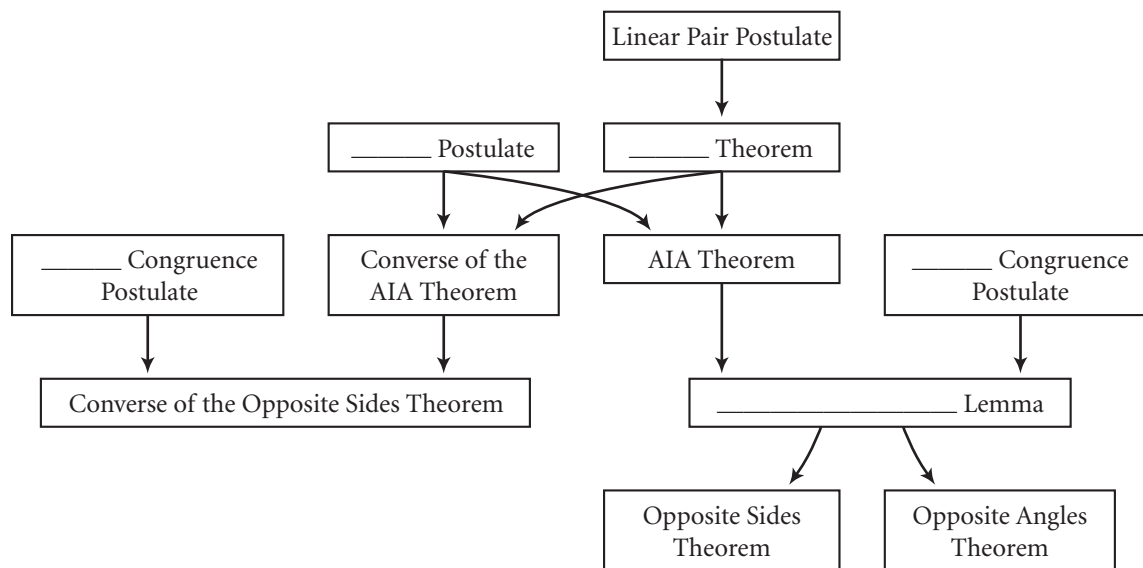
(continued)

Lesson 13.4 • Quadrilateral Proofs (continued)

Task 5: Write a proof.

You're on your own for this one!

Step 4 asks you to create a family tree for the theorems you proved in this investigation. See if you can fill in the blanks in the partially finished tree below.



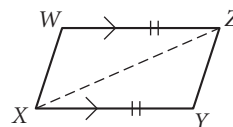
The following example is Exercise 2 in your book.

EXAMPLE

Prove the Opposite Sides Parallel and Congruent Conjecture: If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

► Solution

Given: $\overline{WZ} \parallel \overline{XY}$; $\overline{WZ} \cong \overline{XY}$
Show: $WXYZ$ is a parallelogram



Statement

1. Construct \overline{XZ}
2. $\overline{WZ} \parallel \overline{XY}$
3. $\angle WZX \cong \angle YXZ$
4. $\overline{WZ} \cong \overline{XY}$
5. $\overline{XZ} \cong \overline{XZ}$
6. $\triangle WXZ \cong \triangle YZX$
7. $\angle WXZ \cong \angle YZX$
8. $\overline{WX} \parallel \overline{ZY}$
9. $WXYZ$ is a parallelogram

Reason

1. Line Postulate
2. Given
3. AIA Theorem
4. Given
5. Reflexive property of congruence
6. SAS Congruence Postulate
7. CPCTC
8. Converse of the AIA Theorem
9. Definition of parallelogram

13.5

Indirect Proof

In this lesson you will

- Learn to how to prove mathematical statements **indirectly**

Consider this multiple-choice question:

Which person twice won a Nobel prize?

- A. Sherlock Holmes
- B. Leonardo da Vinci
- C. Marie Curie
- D. Tiger Woods

You may not know the answer off the top of your head, but you can try to eliminate choices until only one possibility remains. Sherlock Holmes cannot be the correct answer because he is a fictional character. Leonardo da Vinci died long before Nobel prizes were awarded. Because there is no Nobel prize for golf, you can also eliminate Tiger Woods. That leaves one possibility, Marie Curie. Choice C must be the answer.

The type of thinking you used to answer the multiple-choice question is known as *indirect reasoning*. You can use this same type of reasoning to write an **indirect proof** of a mathematical statement.

For a given mathematical statement, there are two possibilities: Either the statement is true or it is not true. To prove indirectly that a statement is true, you start by assuming it is not true. You then use logical reasoning to show that this assumption leads to a contradiction. If an assumption leads to a contradiction, it must be false. Therefore, you can eliminate the possibility that the statement is not true. This leaves only one possibility—namely, that the statement is true!

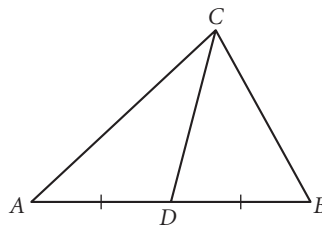
Examples A and B in your book illustrate how an indirect proof works. Read these examples carefully. The example below is Exercise 7 in your book.

EXAMPLE | Prove that in a scalene triangle, the median cannot be the altitude.

► **Solution**

Given: Scalene triangle ABC with median \overline{CD}

Show: \overline{CD} is not the altitude to \overline{AB}



(continued)

Lesson 13.5 • Indirect Proof (continued)

Statement	Reason
1. Assume \overline{CD} is the altitude to \overline{AB}	1. Assume the statement is <i>not</i> true
2. $\angle CDA$ and $\angle CDB$ are right angles	2. Definition of altitude
3. $\angle CDA \cong \angle CDB$	3. Right Angles Congruent Theorem
4. \overline{CD} is a median	4. Given
5. $\overline{AD} \cong \overline{BD}$	5. Definition of median
6. $\overline{CD} \cong \overline{CD}$	6. Reflexive property of congruence
7. $\triangle CDA \cong \triangle CDB$	7. SAS Congruence Postulate
8. $\overline{CA} \cong \overline{CB}$	8. CPCTC
But the statement $\overline{CA} \cong \overline{CB}$ contradicts the fact that $\triangle ABC$ is scalene. Thus, the assumption that \overline{CD} is the altitude to \overline{AB} is false. Therefore, \overline{CD} is <i>not</i> the altitude to \overline{AB} .	

In Chapter 6, you discovered the Tangent Conjecture, which states that a tangent of a circle is perpendicular to the radius drawn to the point of tangency. In the developing proof activity you will prove this conjecture and its converse indirectly.

Developing Proof: Proving the Tangent Conjecture

The activity in your book leads you through the steps of an indirect proof of the Tangent Conjecture. Complete the investigation on your own and then compare your answers to those below. Steps 9 and 10 are left for you to complete on your own.

Step 1 Perpendicular Postulate

Step 2 Segment Duplication Postulate

Step 3 Line Postulate

Step 4 Two reasons: $\angle ABO$ and $\angle CBO$ are right angles because of the definition of perpendicular. $\angle ABO \cong \angle CBO$ because of the Right Angles Congruent Theorem.

Step 5 Reflexive property of congruence

Step 6 SAS Congruence Postulate

Step 7 CPCTC

Step 8 It is given that \overleftrightarrow{AT} is a tangent.

After you have completed the investigation, add the Tangent Theorem and its converse to your list of theorems.

13.6

Circle Proofs

In this lesson you will

- Learn the **Arc Addition Postulate**
- Prove conjectures involving properties of circles

Read Lesson 13.6 in your book. It introduces the Arc Addition Postulate and verifies that the Inscribed Angle Conjecture can now be called a theorem. The examples below are Exercises 1 and 2 in your book. For each example, write the proofs yourself before reading the solutions.

EXAMPLE A | Prove the Inscribed Angles Intercepting Arcs Conjecture: Inscribed angles that intercept the same or congruent arcs are congruent.

► **Solution**

Break the statement into two cases.

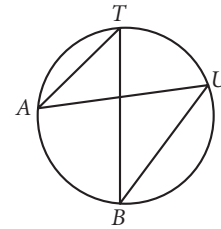
Case 1: The angles intercept the same arc.

Given: $\angle A$ and $\angle B$ intercept \widehat{TU}

Show: $\angle A \cong \angle B$

Paragraph Proof

By the Inscribed Angle Theorem, $m\angle A = \frac{1}{2}m\widehat{TU}$ and $m\angle B = \frac{1}{2}m\widehat{TU}$. By the transitive property of equality, $m\angle A = m\angle B$. By the definition of congruent angles, $\angle A \cong \angle B$.



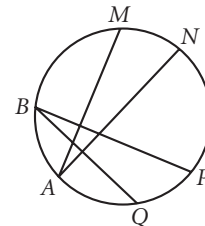
Case 2: The angles intercept congruent arcs.

Given: $\angle A$ intercepts \widehat{MN} ; $\angle B$ intercepts \widehat{PQ} ; $\widehat{MN} \cong \widehat{PQ}$

Show: $\angle A \cong \angle B$

Paragraph Proof

Because $\widehat{MN} \cong \widehat{PQ}$, $m\widehat{MN} = m\widehat{PQ}$ by the definition of congruent arcs. By the multiplication property, $\frac{1}{2}m\widehat{MN} = \frac{1}{2}m\widehat{PQ}$. By the Inscribed Angle Theorem, $m\angle A = \frac{1}{2}m\widehat{MN}$ and $m\angle B = \frac{1}{2}m\widehat{PQ}$. Therefore, by the transitive property, $m\angle A = m\angle B$. By the definition of congruent angles, $\angle A \cong \angle B$.



(continued)

Lesson 13.6 • Circle Proofs (continued)

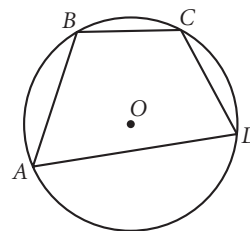
EXAMPLE B

Prove the Cyclic Quadrilateral Conjecture: The opposite angles of an inscribed quadrilateral are supplementary.

► Solution

Given: $ABCD$ is inscribed in circle O

Show: $\angle A$ and $\angle C$ are supplementary;
 $\angle B$ and $\angle D$ are supplementary



Statement

Reason

- | | |
|--|--|
| 1. $m\angle A = \frac{1}{2}m\widehat{BD}$;
$m\angle C = \frac{1}{2}m\widehat{BAD}$ | 1. Inscribed Angle Theorem |
| 2. $m\angle A + m\angle C$
$= \frac{1}{2}m\widehat{BD} + \frac{1}{2}m\widehat{BAD}$ | 2. Addition property |
| 3. $m\angle A + m\angle C$
$= \frac{1}{2}(m\widehat{BD} + m\widehat{BAD})$ | 3. Distributive property |
| 4. $m\angle A + m\angle C$
$= \frac{1}{2}(\text{arc measure of circle } O)$ | 4. Arc Addition Postulate |
| 5. $m\angle A + m\angle C$
$= \frac{1}{2}(360^\circ) = 180^\circ$ | 5. Definition of arc measure of circle |
| 6. $m\angle A$ and $m\angle C$ are supplementary | 6. Definition of supplementary |

The steps above can be repeated for $\angle B$ and $\angle D$. Therefore, the opposite angles of $ABCD$ are supplementary.

13.7

Similarity Proofs

In this lesson you will

- Learn **properties of similarity**
- Prove conjectures involving similarity

The properties of equality and congruence can be extended to similarity. Read the properties of similarity on page 730 of your book. To prove similarity conjectures, you need to add the AA Similarity Postulate (formerly, the AA Similarity Conjecture) to the list of postulates. This postulate is stated in your book.

The example in your book shows how to use the AA Similarity Postulate to prove the SAS Similarity Conjecture. The proof is rather tricky, so read it carefully, following along with pencil and paper. Note that to get from Step 6 to Step 7, DE is substituted for PB in the denominator of the left ratio. This can be done because P was located so that $PB = DE$.

Below are the algebraic operations needed to get from Step 9 to Step 10.

$$\frac{BC}{BQ} = \frac{BC}{EF} \quad \text{Step 9.}$$

$$EF \cdot BC = BQ \cdot BC \quad \text{Multiply both sides by } BQ \cdot EF.$$

$$EF = BQ \quad \text{Divide both sides by } BC.$$

Once you have worked through the example and understand it, add the SAS Similarity Theorem to your theorem list.

Developing Proof: Proving the SSS Similarity Conjecture

In this activity you will prove the SSS Similarity Conjecture: If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.

Given: Two triangles with corresponding sides proportional

Show: The two triangles are similar

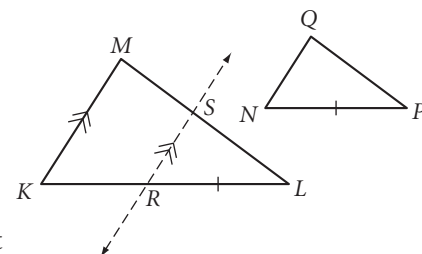
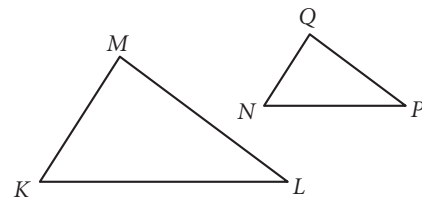
What you are given and what you must show are stated in terms of the diagram shown at right.

Given: $\triangle KLM$ and $\triangle NPQ$ with $\frac{KL}{NP} = \frac{LM}{PQ} = \frac{MK}{QN}$

Show: $\triangle KLM \sim \triangle NPQ$

Write a plan for the proof. (*Hint:* Use an auxiliary line like the one in the example in your book.) After you have written your plan, compare it to the one below.

Plan: To show that $\triangle KLM \sim \triangle NPQ$, you need to show that one pair of corresponding angles is congruent. (We will show that $\angle L \cong \angle P$.) Then you can use the SAS Similarity Theorem to prove the triangles are similar. Use the same approach used in the example. Locate a point R on KL such that $RL = NP$. Then, through R , construct a line \overleftrightarrow{RS} parallel to \overleftrightarrow{KM} .



(continued)

Lesson 13.7 • Similarity Proofs (continued)

From the CA Postulate, $\angle SRL \cong \angle K$ and $\angle RSL \cong \angle M$. This means that $\triangle KLM \sim \triangle RLS$ by the AA Similarity Postulate. Now, if you can show that $\triangle RLS \cong \triangle NPQ$, then $\angle L \cong \angle P$ by CPCTC. Because $\triangle KLM \sim \triangle RLS$, then $\frac{KL}{RL} = \frac{LM}{LS} = \frac{MK}{SR}$ by the definition of similar triangles (CSSTP). Substituting NP for RL gives $\frac{KL}{NP} = \frac{LM}{LS} = \frac{MK}{SR}$.

Combining this with the given fact that $\frac{KL}{NP} = \frac{LM}{PQ} = \frac{MK}{QN}$, you can get the proportions $\frac{LM}{PQ} = \frac{LM}{LS}$ and $\frac{MK}{QN} = \frac{MK}{SR}$. Using some algebra gives $LS = PQ$ and $SR = QN$. So, $\triangle RLS \cong \triangle NPQ$ by the SSS Congruence Postulate. Thus, $\angle L \cong \angle P$ by CPCTC, so $\triangle KLM \sim \triangle NPQ$ by the SAS Similarity Theorem.

Step 4 gives part of a two-column proof. Fill in the missing reasons, and then write the steps and reasons needed to complete the proof. Compare your completed proof to the one below.

Statement	Reason
1. Locate R such that $RL = NP$	1. Segment Duplication Postulate
2. Construct $\overleftrightarrow{RS} \parallel \overleftrightarrow{KM}$	2. Parallel Postulate
3. $\angle SRL \cong \angle K$	3. CA Postulate
4. $\angle RSL \cong \angle M$	4. CA Postulate
5. $\triangle KLM \sim \triangle RLS$	5. AA Similarity Postulate
6. $\frac{KL}{RL} = \frac{LM}{LS} = \frac{MK}{SR}$	6. CSSTP (Definition of similar triangles)
7. $\frac{KL}{NP} = \frac{LM}{LS}$	7. Substitution property of equality
8. $\frac{KL}{NP} = \frac{LM}{PQ}$	8. Given
9. $\frac{KL}{NP} = \frac{MK}{SR}$	9. Substitution property of equality
10. $\frac{KL}{NP} = \frac{MK}{QN}$	10. Given
11. $\frac{LM}{LS} = \frac{LM}{PQ}$	11. Transitive property of equality
12. $LS = PQ$	12. Multiplication and division properties of equality
13. $\frac{MK}{SR} = \frac{MK}{QN}$	13. Transitive property of equality
14. $SR = QN$	14. Multiplication and division properties of equality
15. $\overline{LS} \cong \overline{PQ}$, $\overline{SR} \cong \overline{QN}$, $\overline{RL} \cong \overline{NP}$	15. Definition of congruence
16. $\triangle RLS \cong \triangle NPQ$	16. SSS Congruence Postulate
17. $\angle L \cong \angle P$	17. CPCTC
18. $\triangle KLM \sim \triangle NPQ$	18. SAS Similarity Theorem

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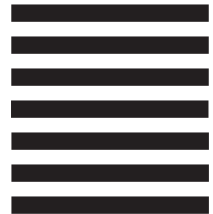


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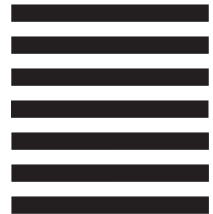


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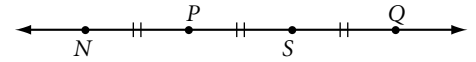


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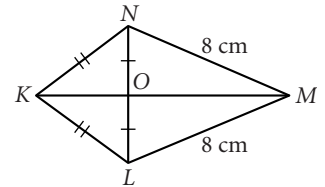
Lesson 1.1 • Building Blocks of Geometry

Name _____ Period _____ Date _____

For Exercises 1–7, complete each statement. $\overline{PS} = 3$ cm.

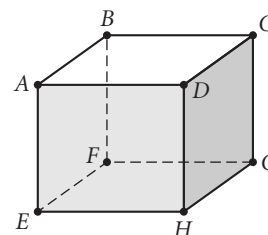


1. The midpoint of \overline{PQ} is _____.
2. $NQ =$ _____.
3. Another name for \overline{NS} is _____.
4. S is the _____ of \overrightarrow{SQ} .
5. P is the midpoint of _____.
6. $\overline{NS} \cong$ _____.
7. Another name for \overrightarrow{SN} is _____.
8. Name all pairs of congruent segments in $KLMN$. Use the congruence symbol to write your answer.
9. $M(-4, 8)$ is the midpoint of \overline{DE} . D has coordinates $(6, 1)$. Find the coordinates of E .



For Exercises 10 and 11, use a ruler to draw each figure. Label the figure and mark the congruent parts.

10. \overline{AB} and \overline{CD} with M as the midpoint of both \overline{AB} and \overline{CD} . $AB = 6.4$ cm and $CD = 4.0$ cm. A , B , and C are not collinear.
11. \overline{AB} and \overline{CD} . C is the midpoint of \overline{AB} , with $AC = 1.5$ cm. D , not on \overline{AB} , is the midpoint of \overline{AE} , with $AD = 2BC$.
12. Sketch six points A , B , C , D , E , and F , no three of which are collinear. Name the lines defined by these points. How many lines are there?
13. In the figure below, $\{B, C, H, E\}$ is a set of four coplanar points. Name two other sets of four coplanar points. How many sets of four coplanar points are there?

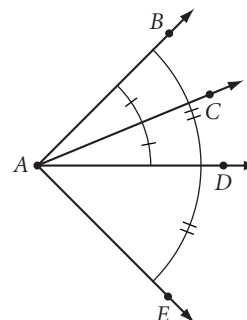


Cube

Lesson 1.2 • Poolroom Math

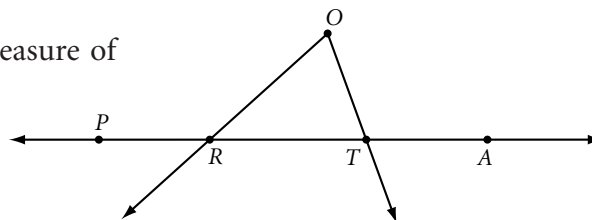
Name _____ Period _____ Date _____

For Exercises 1–5, use the figure at right to complete each statement.



1. A is the _____ of $\angle BAE$.
2. \overrightarrow{AD} is the _____ of $\angle BAE$.
3. \overrightarrow{AD} is a _____ of $\angle DAE$.
4. If $m\angle BAC = 42^\circ$, then $m\angle CAE =$ _____.
5. $\angle DAB \cong$ _____.

For Exercises 6–9, use your protractor to find the measure of each angle to the nearest degree.



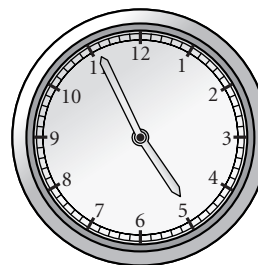
6. $m\angle PRO$
7. $m\angle ORT$
8. $m\angle O$
9. $m\angle RTO$

For Exercises 10–12, use your protractor to draw and then label each angle with the given measure.

10. $m\angle MNO = 15^\circ$
11. $m\angle RIG = 90^\circ$
12. $m\angle z = 160^\circ$

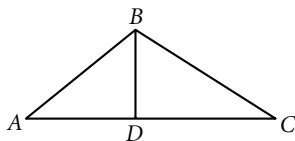
For Exercises 13–15, find the measure of the angle formed by the hands at each time.

13. 3:00
14. 4:00
15. 3:30

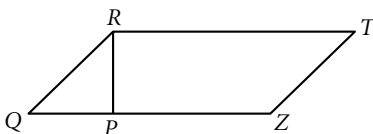


For Exercises 16 and 17, mark each figure with all the given information.

16. $m\angle ADB = 90^\circ$, $AD = BD$, $\angle DAB \cong \angle DBA$



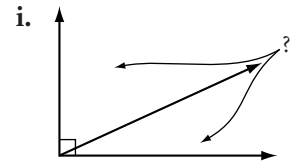
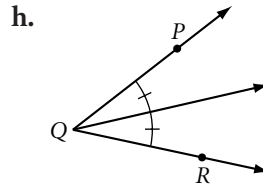
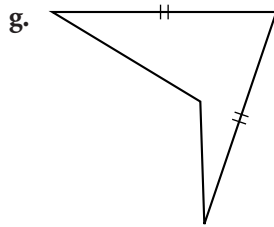
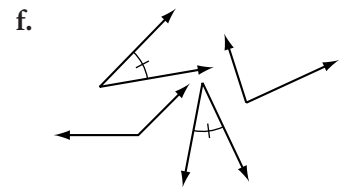
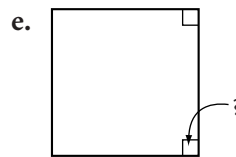
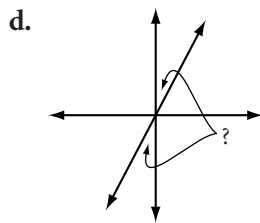
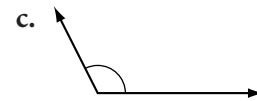
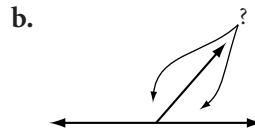
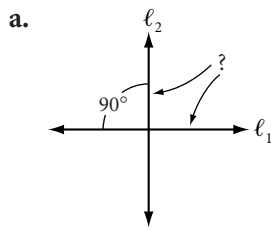
17. $m\angle RPQ = 90^\circ$, $QR = TZ$, $RT = QZ$, $\angle Q \cong \angle T$



Lesson 1.3 • What's a Widget?

Name _____ Period _____ Date _____

For Exercises 1–9, match each term with one of the items (a to i) below.



- | | |
|-----------------------------|--------------------------------|
| 1. _____ Vertical angles | 2. _____ Obtuse angle |
| 3. _____ Right angle | 4. _____ Complementary angles |
| 5. _____ Congruent angles | 6. _____ Linear pair of angles |
| 7. _____ Bisected angle | 8. _____ Perpendicular lines |
| 9. _____ Congruent segments | |
10. If $m\angle P = 13^\circ$, $m\angle Q = 77^\circ$, and $\angle Q$ and $\angle R$ are complementary, what can you conclude about $\angle P$ and $\angle R$? Explain your reasoning.

For Exercises 11–13, sketch, label, and mark a figure showing each property.

11. $\ell_1 \parallel \ell_2$, $\ell_2 \perp \ell_3$

12. $\overline{PQ} \perp \overline{PR}$

13. $\angle BAC \cong \angle XAY$, $CX = BC$

Lesson 1.4 • Polygons

Name _____ Period _____ Date _____

For Exercises 1–8, complete the table.

Polygon name	Number of sides	Number of diagonals
1. Triangle		
2.		2
3.	5	
4. Hexagon		
5. Heptagon		
6.	8	
7.		35
8.	12	

For Exercises 9 and 10, sketch and label each figure. Mark the congruences.

9. Concave pentagon *PENTA*, with external diagonal \overline{ET} , and $\overline{TA} \cong \overline{PE}$.
10. Equilateral quadrilateral *QUAD*, with $\angle Q \not\cong \angle U$.

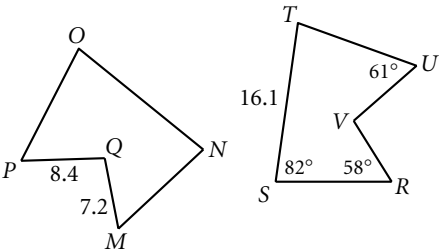
For Exercises 11–14, sketch and use hexagon *ABCDEF*.

11. Name the diagonals from *A*.
12. Name a pair of consecutive sides.
13. Name a pair of consecutive angles.
14. Name a pair of non-intersecting diagonals.

For Exercises 15–18, use the figures at right.

$MNOPQ \cong RSTUV$

15. $m\angle N =$ _____
16. $VR =$ _____
17. $m\angle P =$ _____
18. $ON =$ _____



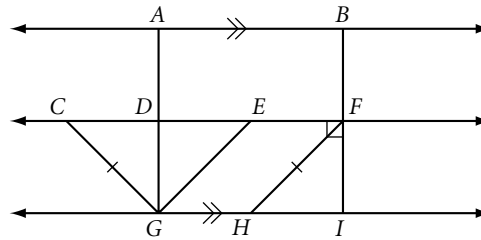
19. The perimeter of a regular pentagon is 31 cm. Find the length of each side.

Lesson 1.5 • Triangles

Name _____ Period _____ Date _____

For Exercises 1–5, use the figure at right. Name a pair of

1. Parallel segments
2. Perpendicular segments
3. Congruent segments
4. Supplementary angles
5. Linear angles



For Exercises 6 and 7, sketch, label, and mark each figure.

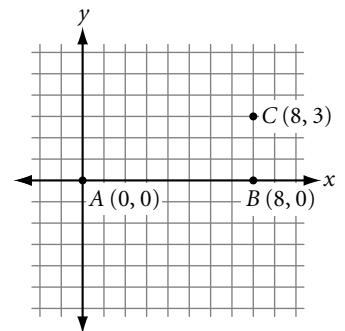
6. Isosceles obtuse triangle TRI with vertex angle T .
7. Scalene right triangle SCA with midpoints L , M , and N on \overline{SC} , \overline{CA} , and \overline{SA} , respectively.

For Exercises 8 and 9, use your geometry tools to draw each figure.

8. Acute isosceles triangle ACD with vertex angle A measuring 40° .
9. Scalene right triangle RGH .

For Exercises 10–12, use the graph at right.

10. Locate F so that $\triangle ABF$ is a right triangle.
11. Locate D so that $\triangle ABD$ is an isosceles triangle.
12. Locate G so that $\triangle ABG$ is scalene and not a right triangle.



Lesson 1.6 • Special Quadrilaterals

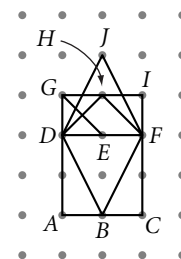
Name _____ Period _____ Date _____

For Exercises 1–6, sketch, label, and mark each figure.

1. Parallelogram $PGRA$
2. Square $SQRE$
3. Rhombus $RHOM$ with acute $\angle H$.
4. Trapezoid $TRAP$ with $\overline{TR} \parallel \overline{AP}$, $\overline{RE} \perp \overline{PA}$, and P , E , and A collinear.
5. Kite $KITE$ with $EK = KI$ and obtuse $\angle K$.
6. Rectangle $RANG$ with perimeter $2a + 4b$

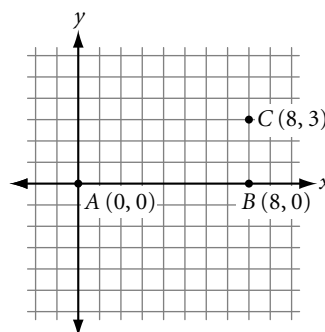
For Exercises 7–10, name each polygon in the figure. Assume that the grid is square.

7. Square
8. Parallelogram
9. Rhombus
10. Kite



For Exercises 11–13, use the graph at right.

11. Locate D so that $ABCD$ is a rectangle.
12. Locate E so that $ABCE$ is a trapezoid.
13. Locate G so that points A , B , C , and G determine a parallelogram that is not a rectangle.

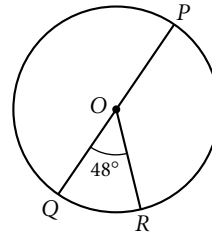


Lesson 1.7 • Circles

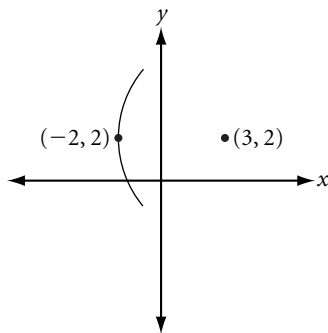
Name _____ Period _____ Date _____

For Exercises 1–4, use the figure at right.

1. $m\widehat{QR} = \underline{\hspace{2cm}}$
2. $m\widehat{PR} = \underline{\hspace{2cm}}$
3. $m\widehat{PQR} = \underline{\hspace{2cm}}$
4. $m\widehat{QPR} = \underline{\hspace{2cm}}$



5. Sketch a circle with an inscribed pentagon.
6. Sketch a circle with a circumscribed quadrilateral.
7. A circle with center $(3, 2)$ goes through $(-2, 2)$. Give the coordinates of three other points on the circle.
8. Use a compass, protractor, and straightedge to draw circle O with diameter \overline{AB} ; radius \overline{OC} with $\overline{OC} \perp \overline{AB}$; \overline{OD} , the angle bisector of $\angle AOC$, with D on the circle; chords \overline{AC} and \overline{BC} ; and a tangent at D .



9. Use a compass to construct a circle. Label the center P . Sketch two parallel tangents. Connect the points of tangency. What do you notice about the chord?
10. Use your compass and protractor to make an arc with measure 50° , an arc with measure 180° , and an arc with measure 290° . Label each arc with its measure.
11. Use your compass to construct two circles with different radii that intersect in two points. Label the centers P and Q and the points of intersection A and B . Construct quadrilateral $PAQB$. What type of quadrilateral is it?

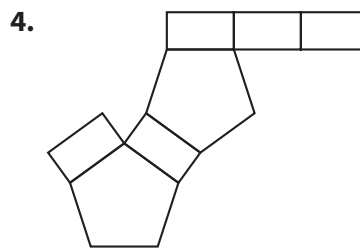
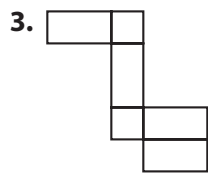
Lesson 1.8 • Space Geometry

Name _____ Period _____ Date _____

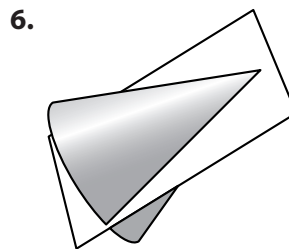
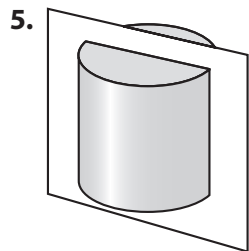
For Exercises 1 and 2, draw each figure.

1. A prism with a rectangular base.
2. A cylinder with base diameter greater than height.

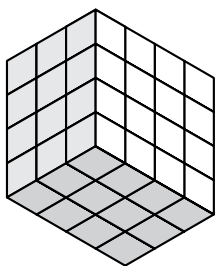
For Exercises 3 and 4, sketch the three-dimensional figure formed by folding each net into a solid. Name the solid.



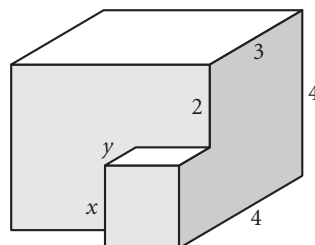
For Exercises 5 and 6, sketch the section formed when each solid is sliced by the plane as shown.



7. The prism below is built with 1-cm cubes. How many cubes are completely hidden from sight, as seen from this angle?



8. Find the lengths of x and y .

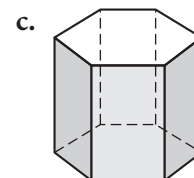
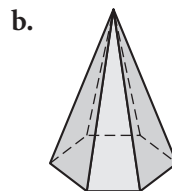
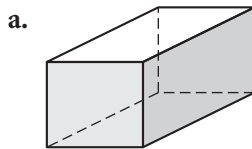


Lesson 1.9 • A Picture Is Worth a Thousand Words

Name _____ Period _____ Date _____

Read and reread each problem carefully, determining what information you are given and what it is that you trying to find.

1. A pair of parallel interstate gas and power lines run 10 meters apart and are equally distant from relay station A. The power company needs to locate a gas-monitoring point on one of the lines exactly 12 meters from relay station A. Draw a diagram showing the locus of possible locations.
2. The six members of the Senica High School math club are having a group photo taken for the yearbook. The photographer has asked the club to submit the height of each member so that he can quickly arrange them in order. The math club sent him the following information. Anica is 4 inches taller than Bruce. Charles is the same height as Ellen but an inch taller than Anica. Fred is midway between Bruce and Dora. Dora is 2 inches taller than Anica. Help out the photographer by arranging the club members in order from tallest to shortest.
3. Create a Venn diagram showing the relationships among triangles, acute triangles, isosceles triangles, and scalene triangles.
4. Sketch a possible net for each solid.



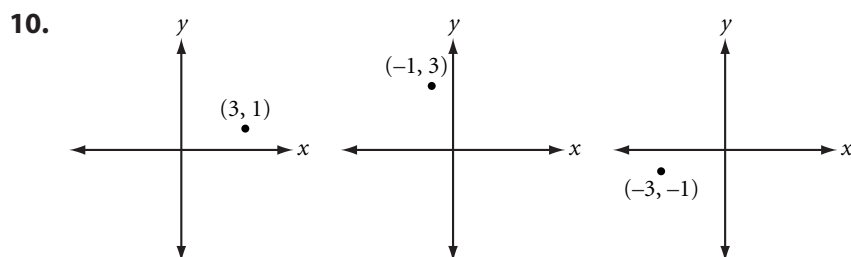
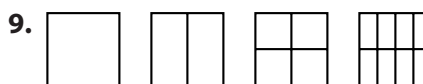
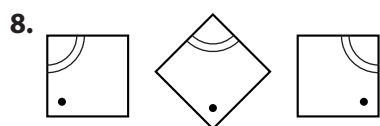
Lesson 2.1 • Inductive Reasoning

Name _____ Period _____ Date _____

For Exercises 1–7, use inductive reasoning to find the next two terms in each sequence.

1. 4, 8, 12, 16, _____, _____
2. 400, 200, 100, 50, 25, _____, _____
3. $\frac{1}{8}, \frac{2}{7}, \frac{1}{2}, \frac{4}{5},$ _____, _____
4. $-5, 3, -2, 1, -1, 0,$ _____, _____
5. 360, 180, 120, 90, _____, _____
6. 1, 3, 9, 27, 81, _____, _____
7. 1, 5, 14, 30, 55, _____, _____

For Exercises 8–10, use inductive reasoning to draw the next two shapes in each picture pattern.



For Exercises 11–13, use inductive reasoning to test each conjecture. Decide if the conjecture seems true or false. If it seems false, give a counterexample.

11. The square of a number is larger than the number.
12. Every multiple of 11 is a “palindrome,” that is, a number that reads the same forward and backward.
13. The difference of two consecutive square numbers is an odd number.

Lesson 2.2 • Finding the n th Term

Name _____ Period _____ Date _____

For Exercises 1–4, tell whether the rule is a linear function.

1.

n	1	2	3	4	5
$f(n)$	8	15	22	29	36

2.

n	1	2	3	4	5
$g(n)$	14	11	8	5	2

3.

n	1	2	3	4	5
$h(n)$	-9	-6	-2	3	9

4.

n	1	2	3	4	5
$j(n)$	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$

For Exercises 5 and 6, complete each table.

5.

n		1	2	3	4	5
$f(n) = 7n - 12$						

6.

n		1	2	3	4	5
$g(n) = -8n - 2$						

For Exercises 7–9, find the function rule for each sequence. Then find the 50th term in the sequence.

7.

n	1	2	3	4	5	6	...	n	...	50
$f(n)$	9	13	17	21	25	29	

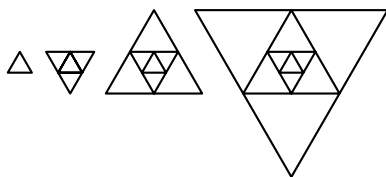
8.

n	1	2	3	4	5	6	...	n	...	50
$g(n)$	6	1	-4	-9	-14	-19	

9.

n	1	2	3	4	5	6	...	n	...	50
$h(n)$	6.5	7	7.5	8	8.5	9	

10. Use the figures to complete the table.



n	1	2	3	4	5	...	n	...	50
Number of triangles	1	5	9			

11. Use the figures above to complete the table. Assume that the area of the first figure is 1 square unit.

n	1	2	3	4	5	...	n	...	50
Area of figure	1	4	16			

Lesson 2.3 • Mathematical Modeling

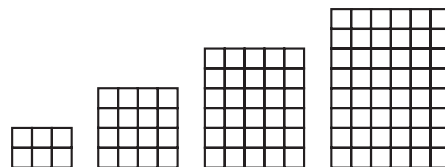
Name _____ Period _____ Date _____

1. Draw the next figure in this pattern.

a. How many small squares will there be in the 10th figure?

b. How many in the 25th figure?

c. What is the general function rule for this pattern?

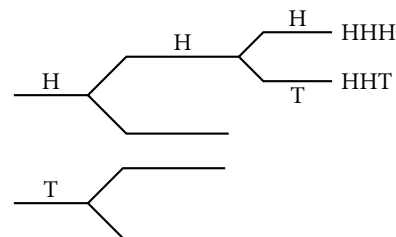


2. If you toss a coin, you will get a head or a tail. Copy and complete the geometric model to show all possible results of three consecutive tosses.

a. How many sequences of results are possible?

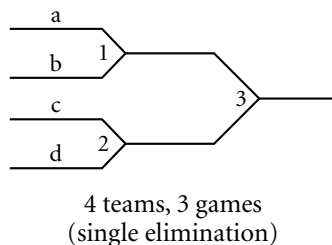
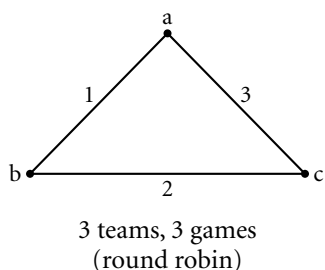
b. How many sequences have exactly one tail?

c. Assuming a head or a tail is equally likely, what is the probability of getting exactly one tail in three tosses?



3. If there are 12 people sitting at a round table, how many different pairs of people can have conversations during dinner, assuming they can all talk to each other? What geometric figure can you use to model this situation?

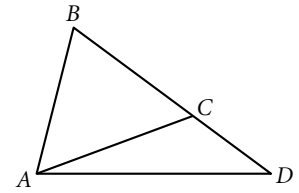
4. Tournament games and results are often displayed using a geometric model. Two examples are shown below. Sketch a geometric model for a tournament involving 5 teams and a tournament involving 6 teams. Each team must have the same chance to win. Try to have as few games as possible in each tournament. Show the total number of games in each tournament. Name the teams a, b, c . . . and number the games 1, 2, 3



Lesson 2.4 • Deductive Reasoning

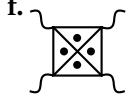
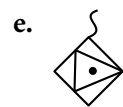
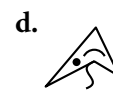
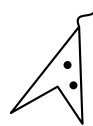
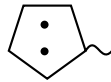
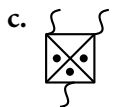
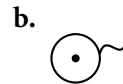
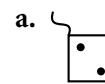
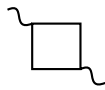
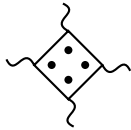
Name _____ Period _____ Date _____

1. $\triangle ABC$ is equilateral. Is $\triangle ABD$ equilateral? Explain your answer. What type of reasoning, inductive or deductive, do you use when solving this problem?



2. $\angle A$ and $\angle D$ are complementary. $\angle A$ and $\angle E$ are supplementary. What can you conclude about $\angle D$ and $\angle E$? Explain your answer. What type of reasoning, inductive or deductive, do you use when solving this problem?

3. Which figures in the last group are whatnots? What type of reasoning, inductive or deductive, do you use when solving this problem?



Whatnots

Not whatnots

Which are whatnots?

4. Solve each equation for x . Give a reason for each step in the process. What type of reasoning, inductive or deductive, do you use when solving these problems?

a. $4x + 3(2 - x) = 8 - 2x$

b. $\frac{19 - 2(3x - 1)}{5} = x + 2$

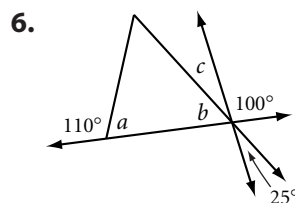
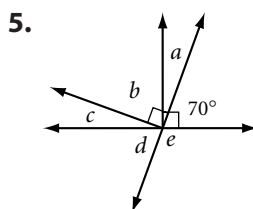
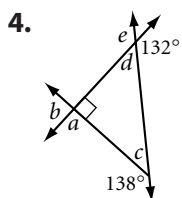
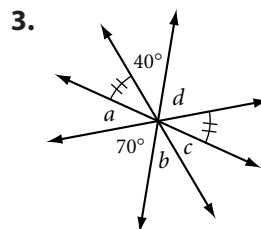
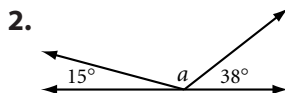
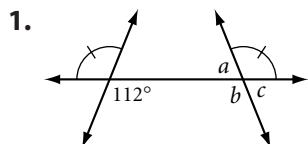
5. A sequence begins $-4, 1, 6, 11 \dots$

- Give the next two terms in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?
- Find a rule that generates the sequence. Then give the 50th term in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?

Lesson 2.5 • Angle Relationships

Name _____ Period _____ Date _____

For Exercises 1–6, find each lettered angle measure without using a protractor.



For Exercises 7–10, tell whether each statement is always (A), sometimes (S), or never (N) true.

7. _____ The sum of the measures of two acute angles equals the measure of an obtuse angle.
8. _____ If $\angle XAY$ and $\angle PAQ$ are vertical angles, then either X , A , and P or X , A , and Q are collinear.
9. _____ If two angles form a linear pair, then they are complementary.
10. _____ If a statement is true, then its converse is true.

For Exercises 11–15, fill in each blank to make a true statement.

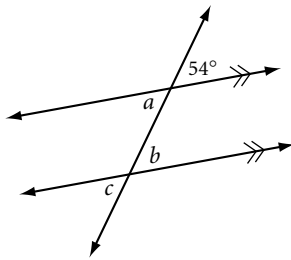
11. If one angle of a linear pair is obtuse, then the other is _____.
12. If $\angle A \cong \angle B$ and the supplement of $\angle B$ has measure 22° , then $m\angle A =$ _____.
13. If $\angle P$ is a right angle and $\angle P$ and $\angle Q$ form a linear pair, then $m\angle Q$ is _____.
14. If $\angle S$ and $\angle T$ are complementary and $\angle T$ and $\angle U$ are supplementary, then $\angle U$ is a(n) _____ angle.
15. Switching the “if” and “then” parts of a statement changes the statement to its _____.

Lesson 2.6 • Special Angles on Parallel Lines

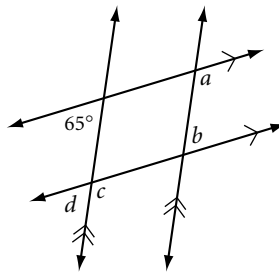
Name _____ Period _____ Date _____

For Exercises 1–3, use your conjectures to find each angle measure.

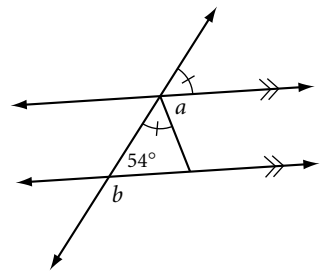
1.



2.

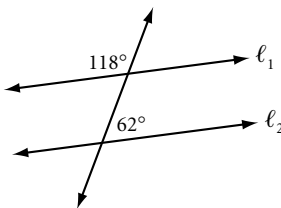


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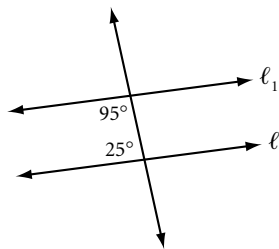


For Exercises 4–6, use your conjectures to determine whether $\ell_1 \parallel \ell_2$, and explain why. If not enough information is given, write “cannot be determined.”

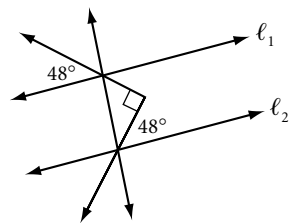
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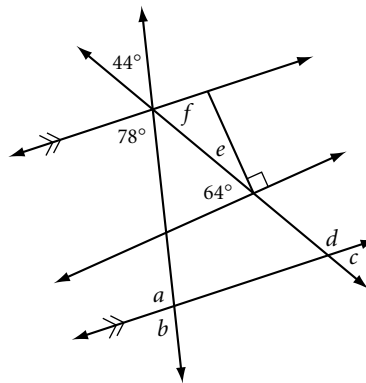
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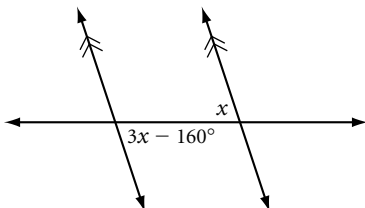
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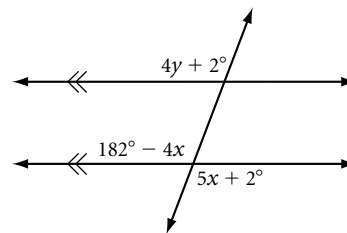
7. Find each angle measure.



8. Find x .



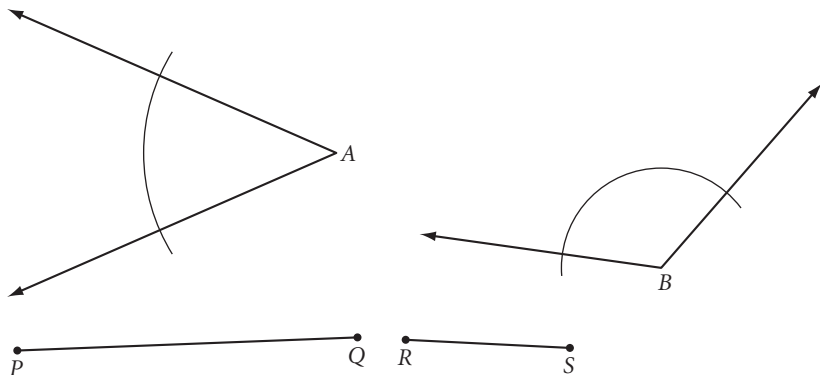
9. Find x and y .



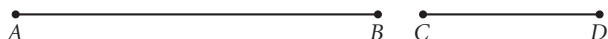
Lesson 3.1 • Duplicating Segments and Angles

Name _____ Period _____ Date _____

In Exercises 1–3, use the segments and angles below. Complete the constructions on a separate piece of paper.



1. Using only a compass and straightedge, duplicate each segment and angle. There is an arc in each angle to help you.
2. Construct a line segment with length $3PQ - 2RS$.
3. Duplicate the two angles so that the angles have the same vertex and share a common side, and the nonshared side of one angle falls inside the other angle. Then use a protractor to measure the three angles you created. Write an equation relating their measures.
4. Use a compass and straightedge to construct an isosceles triangle with two sides congruent to \overline{AB} and base congruent to \overline{CD} .



5. Repeat Exercise 4 with patty paper and a straightedge.
6. Construct an equilateral triangle with sides congruent to \overline{CD} .



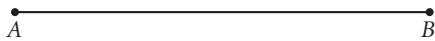
Lesson 3.2 • Constructing Perpendicular Bisectors

Name _____ Period _____ Date _____

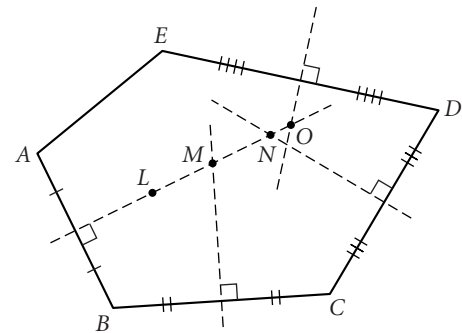
For Exercises 1–6, construct the figures on a separate sheet of paper using only a compass and a straightedge.

1. Draw a segment and construct its perpendicular bisector.
2. Construct two congruent segments that are the perpendicular bisectors of each other. Form a quadrilateral by connecting the four endpoints. What type of quadrilateral does this seem to be?

3. Duplicate \overline{AB} . Then construct a segment with length $\frac{5}{4}AB$.



4. Draw a segment; label it \overline{CM} . \overline{CM} is a median of $\triangle ABC$. Construct $\triangle ABC$. Is $\triangle ABC$ unique? If not, construct a different triangle, $\triangle A'B'C'$, also having \overline{CM} as a median.
5. Draw a segment; label it \overline{PQ} . \overline{PQ} is a midsegment of $\triangle ABC$. Construct $\triangle ABC$. Is $\triangle ABC$ unique? If not, construct a different triangle, $\triangle A'B'C'$, also having \overline{PQ} as a midsegment.
6. Construct a right triangle. Label it $\triangle ABC$ with right angle B . Construct median \overline{BD} . Compare BD , AD , and CD .
7. Complete each statement as fully as possible.
 - a. L is equidistant from _____.
 - b. M is equidistant from _____.
 - c. N is equidistant from _____.
 - d. O is equidistant from _____.



Lesson 3.3 • Constructing Perpendiculars to a Line

Name _____ Period _____ Date _____

For Exercises 1–5, decide whether each statement is true or false. If the statement is false, explain why or give a counterexample.

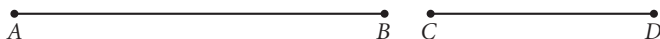
1. In a triangle, an altitude is shorter than either side from the same vertex.
2. In a triangle, an altitude is shorter than the median from the same vertex.
3. In a triangle, if a perpendicular bisector of a side and an altitude coincide, then the triangle is isosceles.
4. Exactly one altitude lies outside a triangle.
5. The intersection of the perpendicular bisectors of the sides lies inside the triangle.

For Exercises 6 and 7, use patty paper. Attach your patty paper to your worksheet.

6. Construct a right triangle. Construct the altitude from the right angle to the opposite side.
7. Mark two points, P and Q . Fold the paper to construct square $PQRS$.

Use your compass and straightedge and the definition of distance to complete Exercises 8 and 9 on a separate sheet of paper.

8. Construct a rectangle with sides equal in length to \overline{AB} and \overline{CD} .



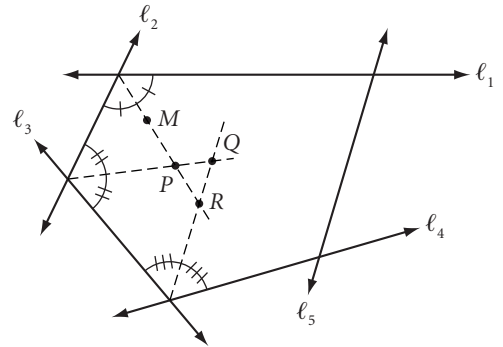
9. Construct a large equilateral triangle. Let P be any point inside the triangle. Construct \overline{WX} equal in length to the sum of the distances from P to each of the sides. Let Q be any other point inside the triangle. Construct \overline{YZ} equal in length to the sum of the distances from Q to each side. Compare WX and YZ .

Lesson 3.4 • Constructing Angle Bisectors

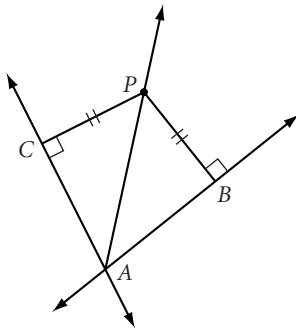
Name _____ Period _____ Date _____

1. Complete each statement as fully as possible.

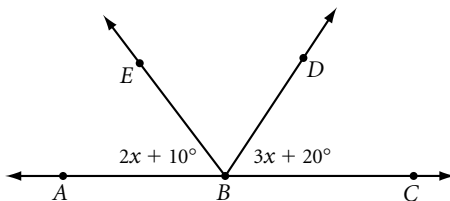
- M is equidistant from _____.
- P is equidistant from _____.
- Q is equidistant from _____.
- R is equidistant from _____.



2. If the converse of the Angle Bisector Conjecture is true, what can you conclude about this figure?



3. If BE bisects $\angle ABD$, find x and $m\angle ABE$.



- Draw an obtuse angle. Use a compass and straightedge to construct the angle bisector. Draw another obtuse angle and fold to construct the angle bisector.
- Draw a large triangle on patty paper. Fold to construct the three angle bisectors. What do you notice?

For Exercises 6 and 7, construct a figure with the given specifications using a straightedge and compass or patty paper. Use additional sheets of paper to show your work.

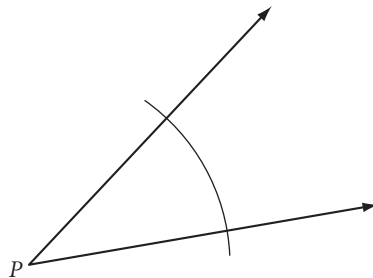
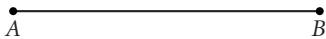
- Using only your compass and straightedge, construct an isosceles right triangle.
- Construct right triangle RGH with right angle R . Construct median \overline{RM} , perpendicular \overline{MN} from M to \overline{RG} , and perpendicular \overline{MO} from M to \overline{RH} . Compare RN and GN , and compare RO and HO .

Lesson 3.5 • Constructing Parallel Lines

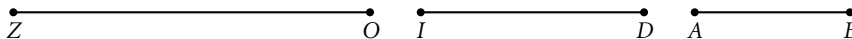
Name _____ Period _____ Date _____

For Exercises 1–6, construct a figure with the given specifications using a straightedge and compass or patty paper. Use additional sheets of paper to show your work.

1. Draw a line and a point not on the line. Use a compass and straightedge to construct a line through the given point parallel to the given line.
2. Repeat Exercise 1, but draw the line and point on patty paper and fold to construct the parallel line.
3. Use a compass and straightedge to construct a parallelogram.
4. Use patty paper and a straightedge to construct an isosceles trapezoid.
5. Construct a rhombus with sides equal in length to \overline{AB} and having an angle congruent to $\angle P$.



6. Construct trapezoid $ZOID$ with \overline{ZO} and \overline{ID} as nonparallel sides and \overline{AB} as the distance between the parallel sides.

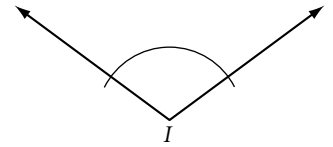
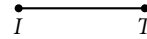
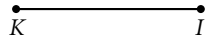


Lesson 3.6 • Construction Problems

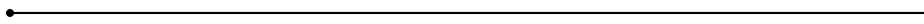
Name _____ Period _____ Date _____

For Exercises 1–5, construct a figure with the given specifications using either a compass and straightedge or patty paper. Use additional sheets of paper to show your work.

1. Construct kite $KITE$ using these parts.



2. Construct a rectangle with perimeter the length of this segment.

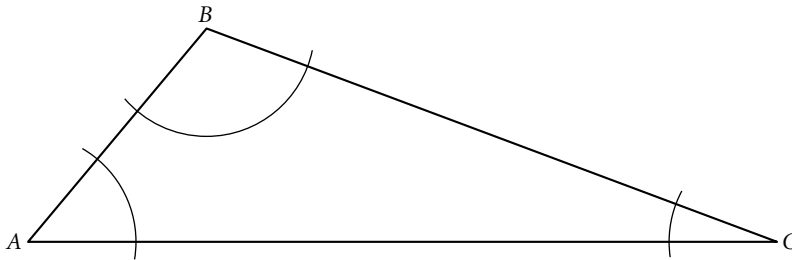


3. Construct a rectangle with this segment as its diagonal.



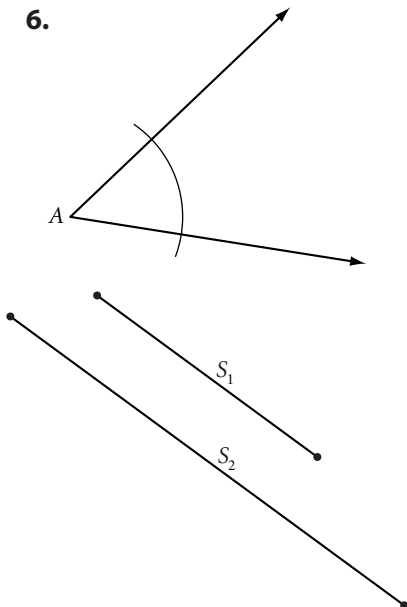
4. Draw obtuse $\triangle OBT$. Construct and label the three altitudes \overline{OU} , \overline{BS} , and \overline{TE} .

5. Construct a triangle congruent to $\triangle ABC$. Describe your steps.

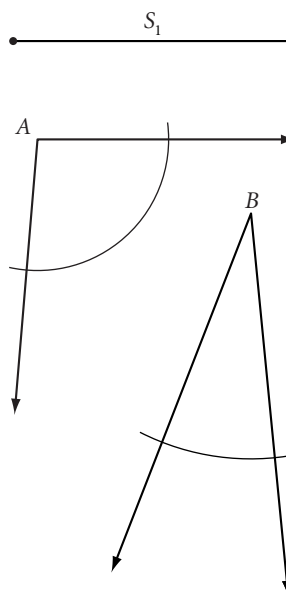


In Exercises 6–8, construct a triangle using the given parts. Then, if possible, construct a different (noncongruent) triangle using the same parts.

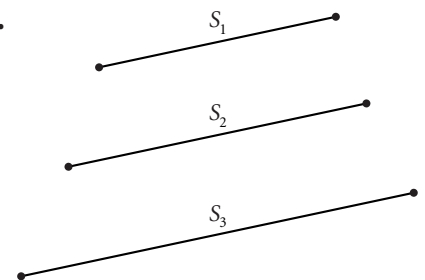
6.



7.



8.



Lesson 3.7 • Constructing Points of Concurrency

Name _____ Period _____ Date _____

For Exercises 1 and 2, make a sketch and explain how to find the answer.

1. A circular revolving sprinkler needs to be set up to water every part of a triangular garden. Where should the sprinkler be located so that it reaches all of the garden, but doesn't spray farther than necessary?
2. You need to supply electric power to three transformers, one on each of three roads enclosing a large triangular tract of land. Each transformer should be the same distance from the power-generation plant and as close to the plant as possible. Where should you build the power plant, and where should you locate each transformer?

For Exercises 3–5, construct a figure with the given specifications using a compass and straightedge. Use additional sheets of paper to show your work.

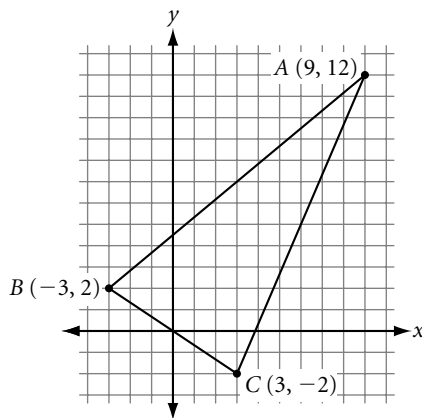
3. Draw an obtuse triangle. Construct the inscribed and the circumscribed circles.
4. Construct an equilateral triangle. Construct the inscribed and the circumscribed circles. How does this construction differ from Exercise 3?
5. Construct two obtuse, two acute, and two right triangles. Locate the circumcenter of each triangle. Make a conjecture about the relationship between the location of the circumcenter and the measure of the angles.

Lesson 3.8 • The Centroid

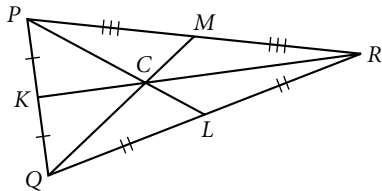
Name _____ Period _____ Date _____

For Exercises 1–3, use additional sheets of paper to show your work.

1. Draw a large acute triangle. Construct the centroid.
2. Construct a regular hexagon and locate its center of gravity.
3. Use a ruler and compass to find the center of gravity of a sheet-metal triangle with sides measuring 6 cm, 8 cm, and 10 cm. How far is the center from each vertex, to the nearest tenth of a centimeter?
4. $\triangle ABC$ has vertices $A(9, 12)$, $B(-3, 2)$, and $C(3, -2)$. Find the coordinates of the centroid.



5. $PL = 24$, $QC = 10$, and $KC = 7$. Find PC , CL , QM , and CR .



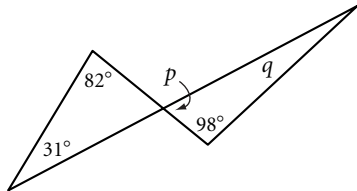
6. Identify each statement as describing the incenter, circumcenter, orthocenter, or centroid.
 - a. _____ The point equally distant from the three sides of a triangle.
 - b. _____ The center of gravity of a thin metal triangle.
 - c. _____ The point equidistant from the three vertices.
 - d. _____ The intersection of the perpendicular bisectors of the sides of a triangle.
 - e. _____ The intersection of the altitudes of a triangle.
 - f. _____ The intersection of the angle bisectors of a triangle.
 - g. _____ The intersection of the medians of a triangle.

Lesson 4.1 • Triangle Sum Conjecture

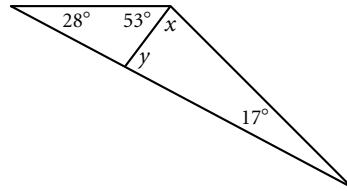
Name _____ Period _____ Date _____

In Exercises 1–9, determine the angle measures.

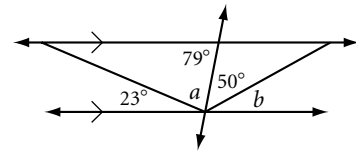
1. $p =$ _____, $q =$ _____



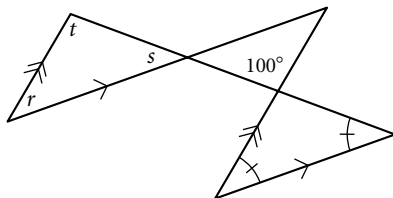
2. $x =$ _____, $y =$ _____



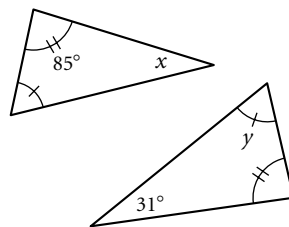
3. $a =$ _____, $b =$ _____



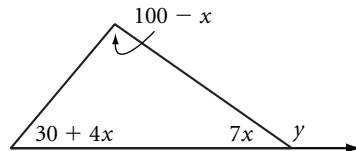
4. $r =$ _____, $s =$ _____,
 $t =$ _____



5. $x =$ _____, $y =$ _____



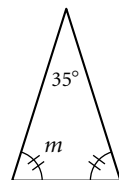
6. $y =$ _____



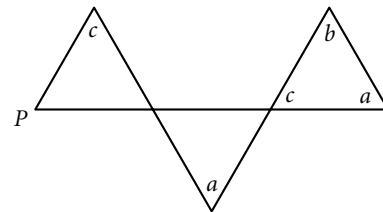
7. $s =$ _____



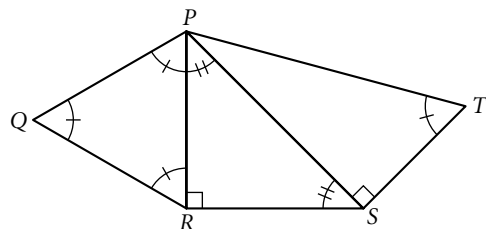
8. $m =$ _____



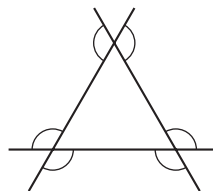
9. $m\angle P =$ _____



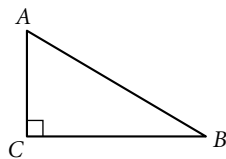
10. Find the measure of $\angle QPT$.



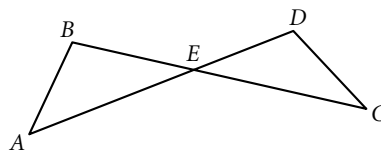
11. Find the sum of the measures of the marked angles.



12. Use the diagram to explain why $\angle A$ and $\angle B$ are complementary.



13. Use the diagram to explain why $m\angle A + m\angle B = m\angle C + m\angle D$.

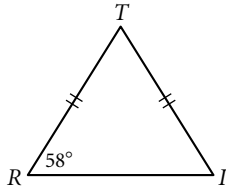


Lesson 4.2 • Properties of Isosceles Triangles

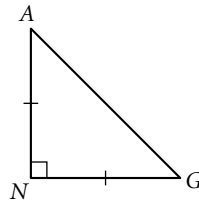
Name _____ Period _____ Date _____

In Exercises 1–3, find the angle measures.

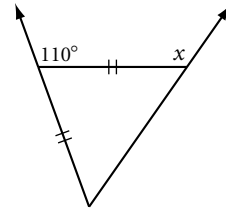
1. $m\angle T =$ _____



2. $m\angle G =$ _____

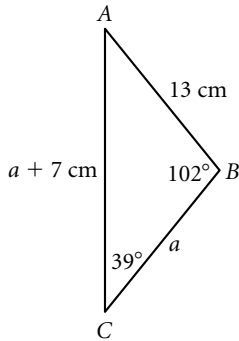


3. $x =$ _____

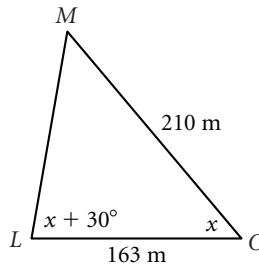


In Exercises 4–6, find the measures.

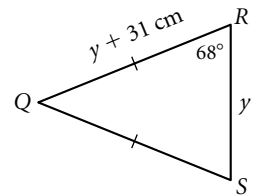
4. $m\angle A =$ _____, perimeter of $\triangle ABC =$ _____



5. The perimeter of $\triangle LMO$ is 536 m. $LM =$ _____, $m\angle M =$ _____



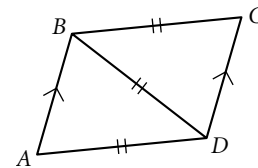
6. The perimeter of $\triangle QRS$ is 344 cm. $m\angle Q =$ _____, $QR =$ _____



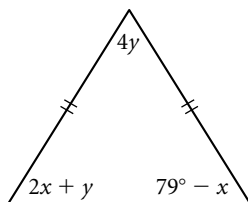
7. a. Name the angle(s) congruent to $\angle DAB$.

b. Name the angle(s) congruent to $\angle ADB$.

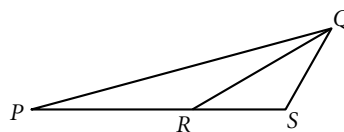
c. What can you conclude about \overline{AD} and \overline{BC} ? Why?



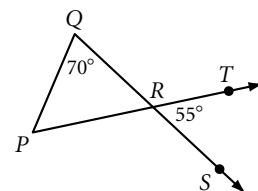
8. $x =$ _____, $y =$ _____



9. $PR = QR$ and $QS = RS$. If $m\angle RSQ = 120^\circ$, what is $m\angle QPR$?



10. Use the diagram to explain why $\triangle PQR$ is isosceles.



Lesson 4.3 • Triangle Inequalities

Name _____ Period _____ Date _____

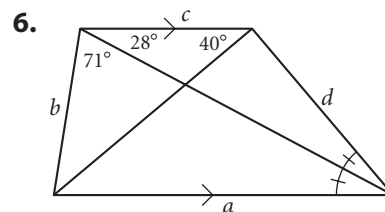
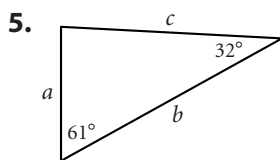
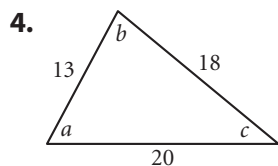
In Exercises 1 and 2, determine whether it is possible to draw a triangle with sides of the given measures. If it is possible, write yes. If it is not possible, write no and make a sketch demonstrating why it is not possible.

1. 16 cm, 30 cm, 45 cm

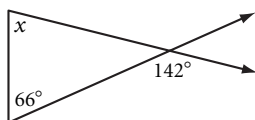
2. 9 km, 17 km, 28 km

3. If 17 and 36 are the lengths of two sides of a triangle, what is the range of possible values for the length of the third side?

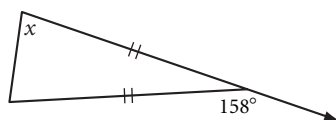
In Exercises 4–6, arrange the unknown measures in order from greatest to least.



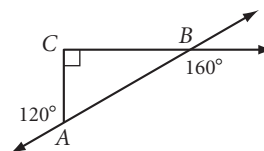
7. $x =$ _____



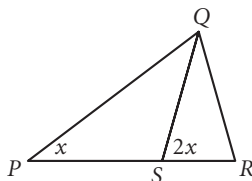
8. $x =$ _____



9. What's wrong with this picture?



10. Explain why $\triangle PQS$ is isosceles.



In Exercises 11 and 12, use a compass and straightedge to construct a triangle with the given sides. If it is not possible, explain why not.

11. $A \bullet \text{---} \bullet B$

$B \bullet \text{---} \bullet C$

$A \bullet \text{---} \bullet C$

12. $P \bullet \text{---} \bullet Q$

$Q \bullet \text{---} \bullet R$

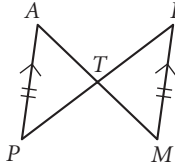
$P \bullet \text{---} \bullet R$

Lesson 4.4 • Are There Congruence Shortcuts?

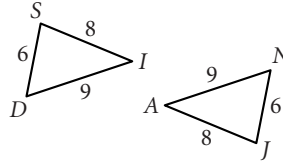
Name _____ Period _____ Date _____

In Exercises 1–3, name the conjecture that leads to each congruence.

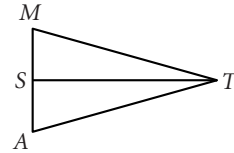
1. $\triangle PAT \cong \triangle IMT$



2. $\triangle SID \cong \triangle JAN$



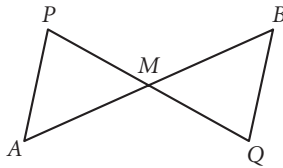
3. \overline{TS} bisects \overline{MA} , $\overline{MT} \cong \overline{AT}$, and $\triangle MST \cong \triangle AST$



In Exercises 4–9, name a triangle congruent to the given triangle and state the congruence conjecture. If you cannot show any triangles to be congruent from the information given, write “cannot be determined” and redraw the triangles so that they are clearly not congruent.

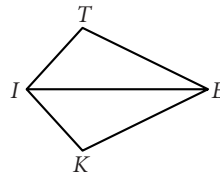
4. M is the midpoint of \overline{AB} and \overline{PQ} .

$\triangle APM \cong \triangle$ _____

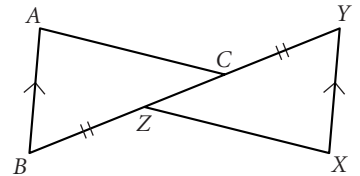


5. $KITE$ is a kite with $KI = TI$.

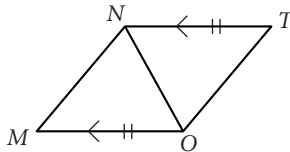
$\triangle KIE \cong \triangle$ _____



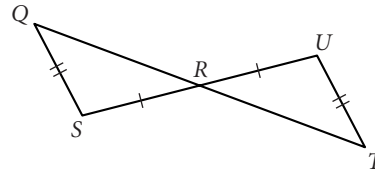
6. $\triangle ABC \cong$ _____



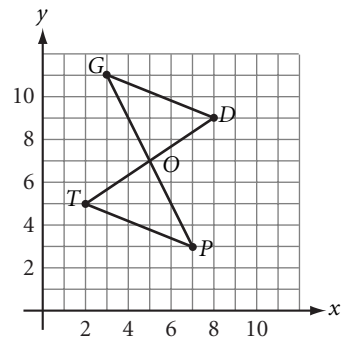
7. $\triangle MON \cong$ _____



8. $\triangle SQR \cong$ _____

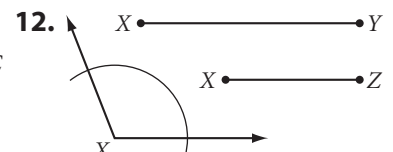
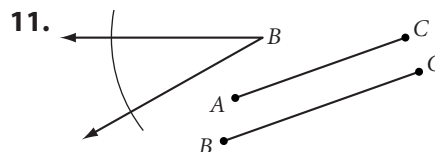


9. $\triangle TOP \cong$ _____



In Exercises 10–12, use a compass and a straightedge or patty paper and a straightedge to construct a triangle with the given parts. Then, if possible, construct a different (noncongruent) triangle with the same parts. If it is not possible, explain why not.

10. S • ————— • T
 T • ————— • U
 U • ————— • S

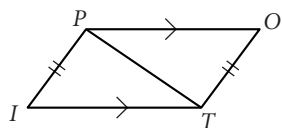


Lesson 4.5 • Are There Other Congruence Shortcuts?

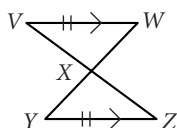
Name _____ Period _____ Date _____

In Exercises 1–6, name a triangle congruent to the given triangle and state the congruence conjecture. If you cannot show any triangles to be congruent from the information given, write “cannot be determined” and explain why.

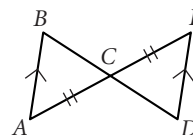
1. $\triangle PIT \cong \triangle$ _____



2. $\triangle XVW \cong \triangle$ _____

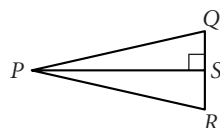


3. $\triangle ECD \cong \triangle$ _____

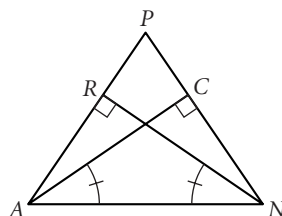


4. \overline{PS} is the angle bisector of $\angle QPR$.

$\triangle PQS \cong \triangle$ _____

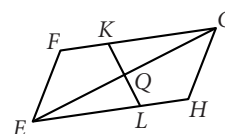


5. $\triangle ACN \cong \triangle$ _____

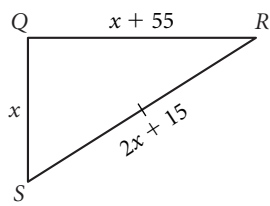
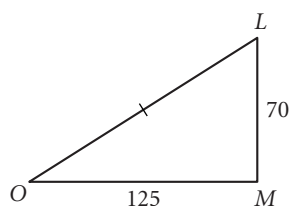


6. $EFGH$ is a parallelogram.
 $GQ = EQ$.

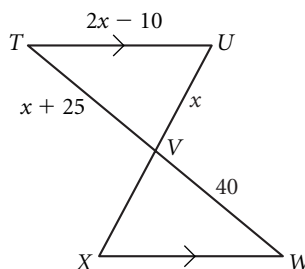
$\triangle EQL \cong \triangle$ _____



7. The perimeter of $\triangle QRS$ is 350 cm.
Is $\triangle QRS \cong \triangle MOL$? Explain.

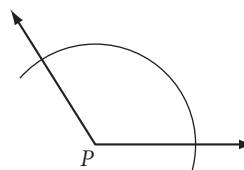
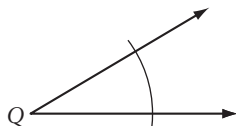


8. The perimeter of $\triangle TUV$ is 95 cm.
Is $\triangle TUV \cong \triangle WXV$? Explain.

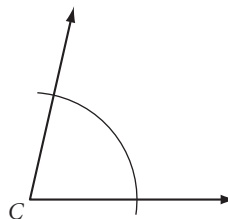
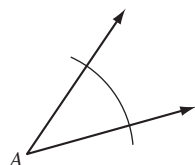


In Exercises 9 and 10, construct a triangle with the given parts. Then, if possible, construct a different (noncongruent) triangle with the same parts. If it is not possible, explain why not.

9. P • ————— • Q



10. A • ————— • B

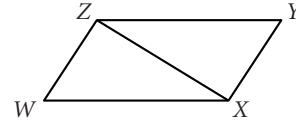


Lesson 4.6 • Corresponding Parts of Congruent Triangles

Name _____ Period _____ Date _____

1. Give the shorthand name for each of the four triangle congruence conjectures.

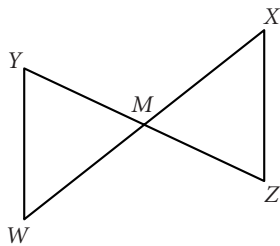
In Exercises 2–5, use the figure at right to explain why each congruence is true. $WXYZ$ is a parallelogram.



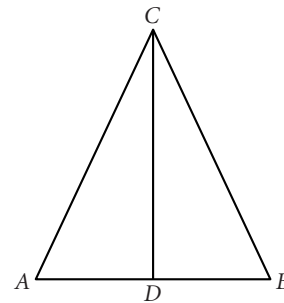
2. $\angle WXZ \cong \angle YZX$
3. $\angle WZX \cong \angle YXZ$
4. $\triangle WZX \cong \triangle YXZ$
5. $\angle W \cong \angle Y$

For Exercises 6 and 7, mark the figures with the given information. To demonstrate whether the segments or the angles indicated are congruent, determine that two triangles are congruent. Then state which conjecture proves them congruent.

6. M is the midpoint of \overline{WX} and \overline{YZ} . Is $\overline{YW} \cong \overline{ZX}$? Why?

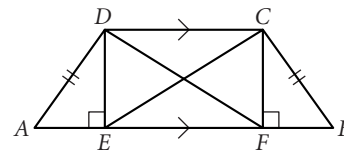


7. $\triangle ABC$ is isosceles and \overline{CD} is the bisector of the vertex angle. Is $\overline{AD} \cong \overline{BD}$? Why?

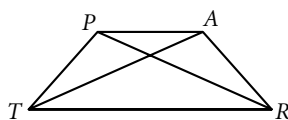


In Exercises 8 and 9, use the figure at right to write a paragraph proof for each statement.

8. $\overline{DE} \cong \overline{CF}$
9. $\overline{EC} \cong \overline{FD}$



10. $TRAP$ is an isosceles trapezoid with $TP = RA$ and $\angle PTR \cong \angle ART$. Write a paragraph proof explaining why $\overline{TA} \cong \overline{RP}$.



Lesson 4.7 • Flowchart Thinking

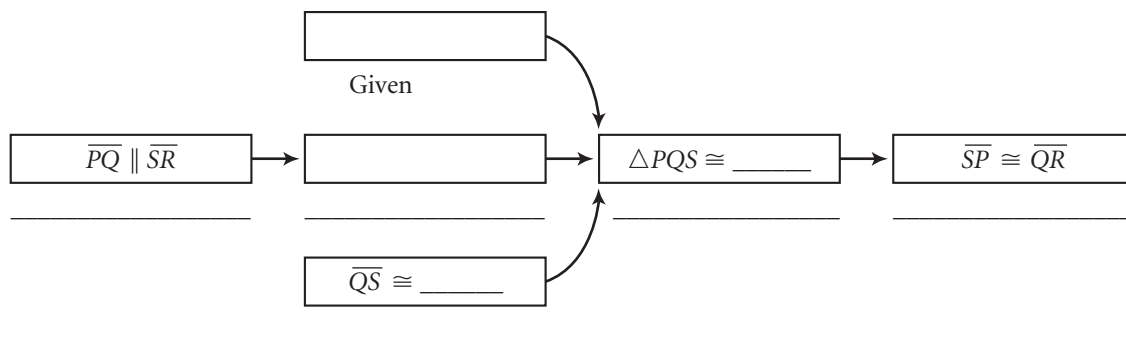
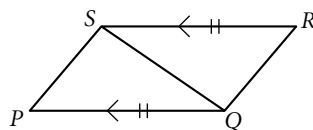
Name _____ Period _____ Date _____

Complete the flowchart for each proof.

1. **Given:** $\overline{PQ} \parallel \overline{SR}$ and $\overline{PQ} \cong \overline{SR}$

Show: $\overline{SP} \cong \overline{QR}$

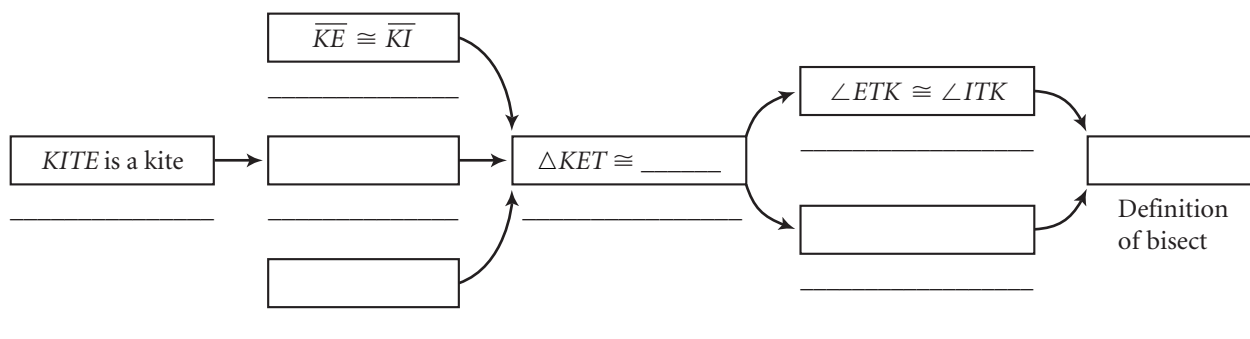
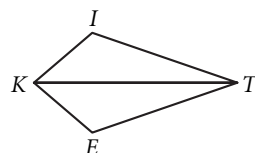
Flowchart Proof



2. **Given:** Kite $KITE$ with $\overline{KE} \cong \overline{KI}$

Show: \overline{KT} bisects $\angle EKI$ and $\angle ETI$

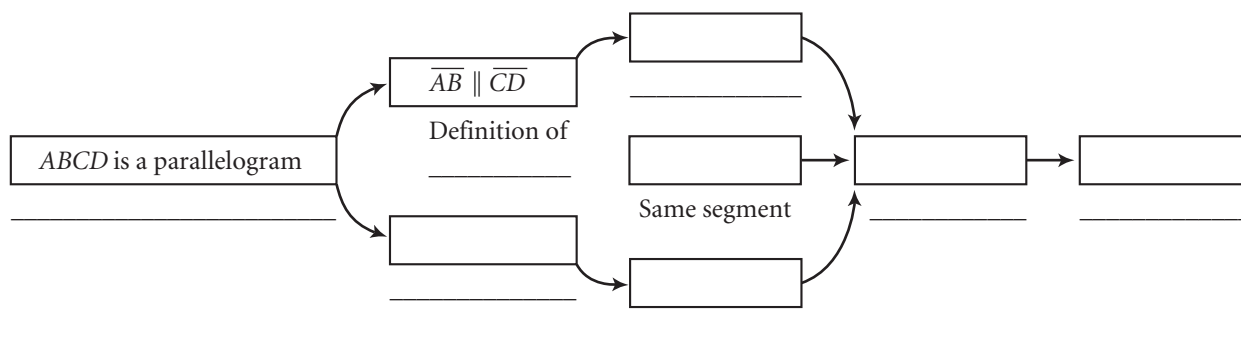
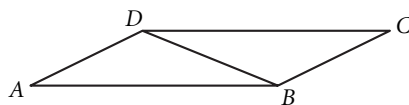
Flowchart Proof



3. **Given:** $ABCD$ is a parallelogram

Show: $\angle A \cong \angle C$

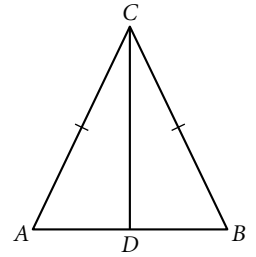
Flowchart Proof



Lesson 4.8 • Proving Special Triangle Conjectures

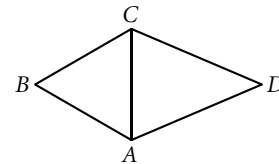
Name _____ Period _____ Date _____

In Exercises 1–3, use the figure at right.



1. \overline{CD} is a median, perimeter $\triangle ABC = 60$, and $AC = 22$. $AD =$ _____
2. \overline{CD} is an angle bisector, and $m\angle A = 54^\circ$. $m\angle ACD =$ _____
3. \overline{CD} is an altitude, perimeter $\triangle ABC = 42$, $m\angle ACD = 38^\circ$, and $AD = 8$.
 $m\angle B =$ _____, $CB =$ _____
4. $\triangle EQU$ is equilateral.
 $m\angle E =$ _____
5. $\triangle ANG$ is equiangular
 and perimeter $\triangle ANG = 51$.
 $AN =$ _____

6. $\triangle ABC$ is equilateral, $\triangle ACD$ is isosceles with base \overline{AC} ,
 perimeter $\triangle ABC = 66$, and perimeter $\triangle ACD = 82$.
 Perimeter $ABCD =$ _____

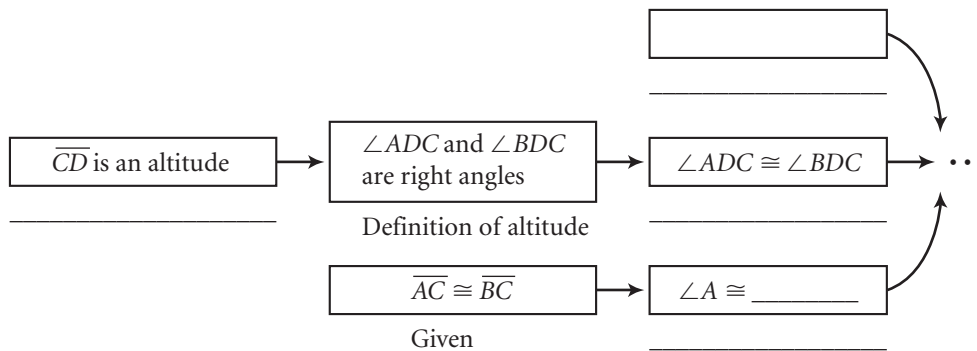
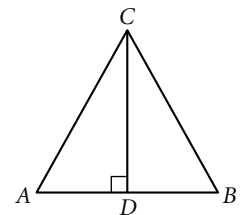


7. Complete a flowchart proof for this conjecture: In an isosceles triangle, the altitude from the vertex angle is the median to the base.

Given: Isosceles $\triangle ABC$ with $\overline{AC} \cong \overline{BC}$ and altitude \overline{CD}

Show: \overline{CD} is a median

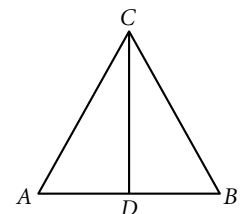
Flowchart Proof



8. Write a flowchart proof for this conjecture: In an isosceles triangle, the median to the base is also the angle bisector of the vertex angle.

Given: Isosceles $\triangle ABC$ with $\overline{AC} \cong \overline{BC}$ and median \overline{CD}

Show: \overline{CD} bisects $\angle ACB$

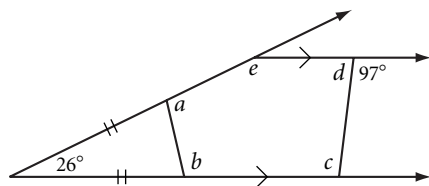


Lesson 5.1 • Polygon Sum Conjecture

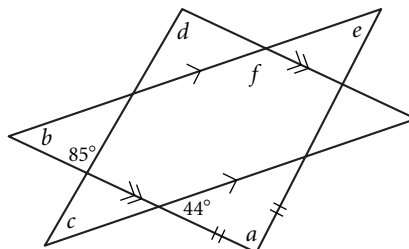
Name _____ Period _____ Date _____

In Exercises 1 and 2, find each lettered angle measure.

1. $a = \underline{\hspace{1cm}}$, $b = \underline{\hspace{1cm}}$, $c = \underline{\hspace{1cm}}$,
 $d = \underline{\hspace{1cm}}$, $e = \underline{\hspace{1cm}}$



2. $a = \underline{\hspace{1cm}}$, $b = \underline{\hspace{1cm}}$, $c = \underline{\hspace{1cm}}$,
 $d = \underline{\hspace{1cm}}$, $e = \underline{\hspace{1cm}}$, $f = \underline{\hspace{1cm}}$

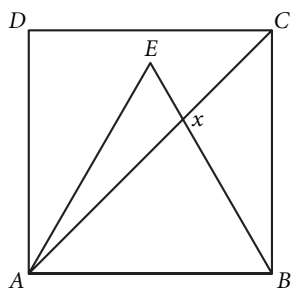


3. One exterior angle of a regular polygon measures 10° . What is the measure of each interior angle? How many sides does the polygon have?

4. The sum of the measures of the interior angles of a regular polygon is 2340° . How many sides does the polygon have?

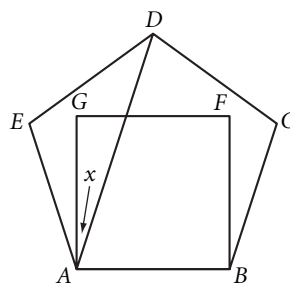
5. $ABCD$ is a square. ABE is an equilateral triangle.

$x = \underline{\hspace{1cm}}$



6. $ABCDE$ is a regular pentagon. $ABFG$ is a square.

$x = \underline{\hspace{1cm}}$



7. Use a protractor to draw pentagon $ABCDE$ with $m\angle A = 85^\circ$, $m\angle B = 125^\circ$, $m\angle C = 110^\circ$, and $m\angle D = 70^\circ$. What is $m\angle E$? Measure it, and check your work by calculating.

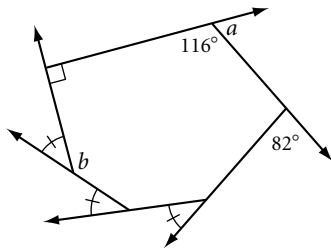
Lesson 5.2 • Exterior Angles of a Polygon

Name _____ Period _____ Date _____

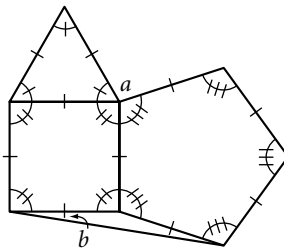
- How many sides does a regular polygon have if each exterior angle measures 30° ?
- How many sides does a polygon have if the sum of the measures of the interior angles is 3960° ?
- If the sum of the measures of the interior angles of a polygon equals the sum of the measures of its exterior angles, how many sides does it have?
- If the sum of the measures of the interior angles of a polygon is twice the sum of its exterior angles, how many sides does it have?

In Exercises 5–7, find each lettered angle measure.

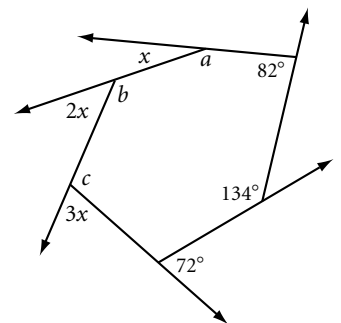
5. $a = \underline{\hspace{2cm}}$, $b = \underline{\hspace{2cm}}$



6. $a = \underline{\hspace{2cm}}$, $b = \underline{\hspace{2cm}}$



7. $a = \underline{\hspace{2cm}}$, $b = \underline{\hspace{2cm}}$,
 $c = \underline{\hspace{2cm}}$



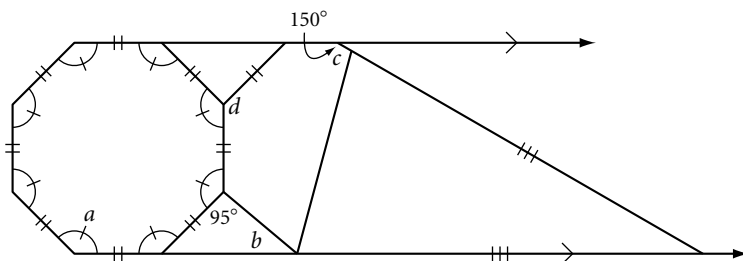
8. Find each lettered angle measure.

$a = \underline{\hspace{2cm}}$

$b = \underline{\hspace{2cm}}$

$c = \underline{\hspace{2cm}}$

$d = \underline{\hspace{2cm}}$



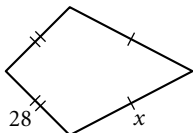
9. Construct an equiangular quadrilateral that is not regular.

Lesson 5.3 • Kite and Trapezoid Properties

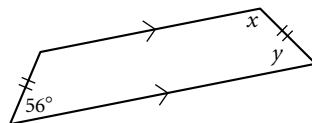
Name _____ Period _____ Date _____

In Exercises 1–4, find each lettered measure.

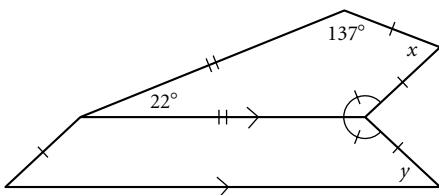
1. Perimeter = 116. $x =$ _____



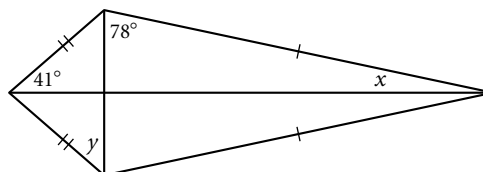
2. $x =$ _____, $y =$ _____



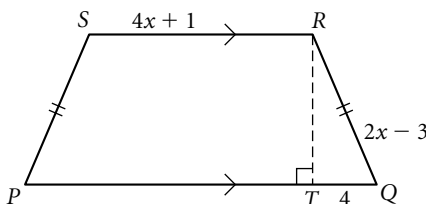
3. $x =$ _____, $y =$ _____



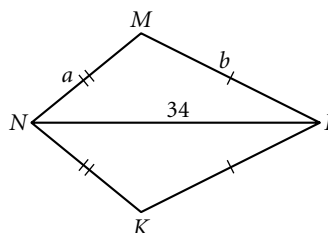
4. $x =$ _____, $y =$ _____



5. Perimeter $PQRS = 220$. $PS =$ _____

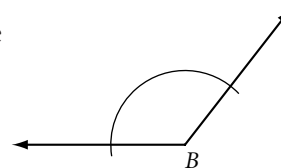
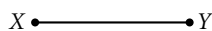
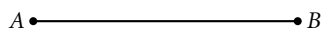


6. $b = 2a + 1$. $a >$ _____

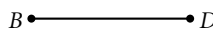
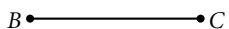
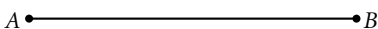


In Exercises 7 and 8, use the properties of kites and trapezoids to construct each figure. Use patty paper or a compass and a straightedge.

7. Construct an isosceles trapezoid given base \overline{AB} , $\angle B$, and distance between bases XY .



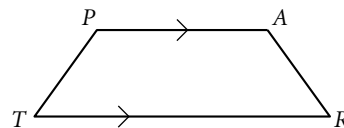
8. Construct kite $ABCD$ with \overline{AB} , \overline{BC} , and \overline{BD} .



9. Write a paragraph or flowchart proof of the Converse of the Isosceles Trapezoid Conjecture. *Hint:* Draw \overline{AE} parallel to \overline{TP} with E on \overline{TR} .

Given: Trapezoid $TRAP$ with $\angle T \cong \angle R$

Show: $\overline{TP} \cong \overline{RA}$



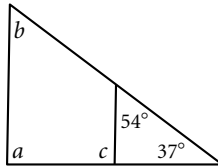
Lesson 5.4 • Properties of Midsegments

Name _____ Period _____ Date _____

In Exercises 1–3, each figure shows a midsegment.

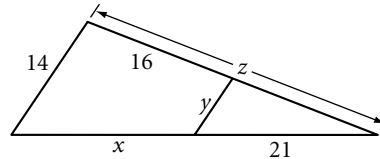
1. $a = \underline{\hspace{2cm}}$, $b = \underline{\hspace{2cm}}$,

$c = \underline{\hspace{2cm}}$



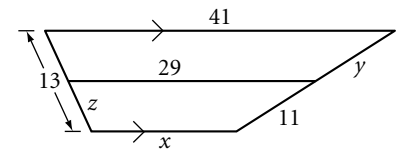
2. $x = \underline{\hspace{2cm}}$, $y = \underline{\hspace{2cm}}$,

$z = \underline{\hspace{2cm}}$



3. $x = \underline{\hspace{2cm}}$, $y = \underline{\hspace{2cm}}$,

$z = \underline{\hspace{2cm}}$

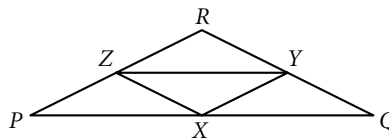


4. X , Y , and Z are midpoints. Perimeter $\triangle PQR = 132$, $RQ = 55$, and $PZ = 20$.

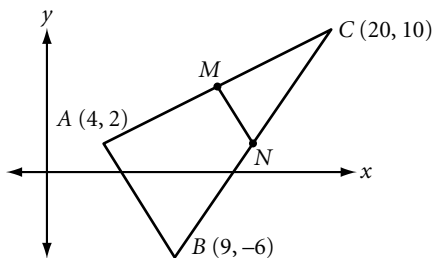
Perimeter $\triangle XYZ = \underline{\hspace{2cm}}$

$PQ = \underline{\hspace{2cm}}$

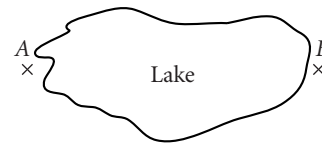
$ZX = \underline{\hspace{2cm}}$



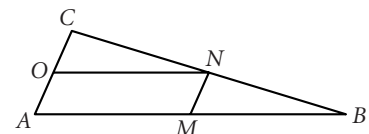
5. \overline{MN} is the midsegment. Find the coordinates of M and N . Find the slopes of \overline{AB} and \overline{MN} .



6. Explain how to find the width of the lake from A to B using a tape measure, but without using a boat or getting your feet wet.



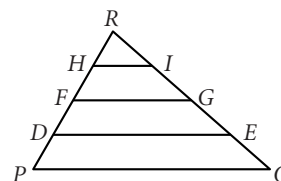
7. M , N , and O are midpoints. What type of quadrilateral is $AMNO$? How do you know? Give a flowchart proof showing that $\triangle ONC \cong \triangle MBN$.



8. Give a paragraph or flowchart proof.

Given: $\triangle PQR$ with $PD = DF = FH = HR$
and $QE = EG = GI = IR$

Show: $\overline{HI} \parallel \overline{FG} \parallel \overline{DE} \parallel \overline{PQ}$

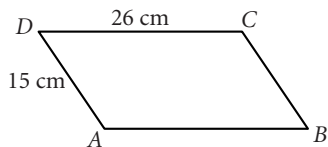


Lesson 5.5 • Properties of Parallelograms

Name _____ Period _____ Date _____

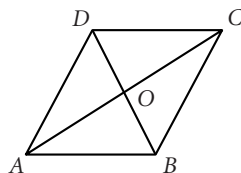
In Exercises 1–7, $ABCD$ is a parallelogram.

1. Perimeter $ABCD =$ _____



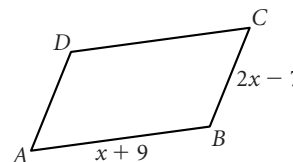
2. $AO = 11$, and $BO = 7$.

$AC =$ _____, $BD =$ _____



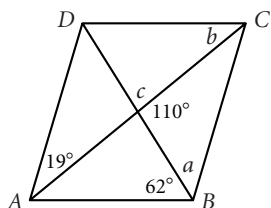
3. Perimeter $ABCD = 46$.

$AB =$ _____, $BC =$ _____



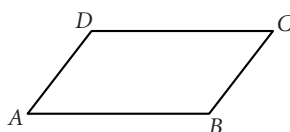
4. $a =$ _____, $b =$ _____,

$c =$ _____



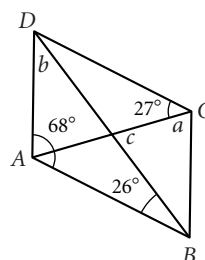
5. Perimeter $ABCD = 119$, and

$BC = 24$. $AB =$ _____

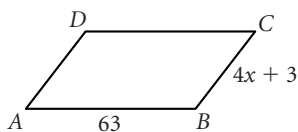


6. $a =$ _____, $b =$ _____,

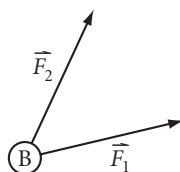
$c =$ _____



7. Perimeter $ABCD = 16x - 12$. $AD =$ _____



8. Ball B is struck at the same instant by two forces, \vec{F}_1 and \vec{F}_2 . Show the resultant force on the ball.



9. Find each lettered angle measure.

$a =$ _____ $g =$ _____

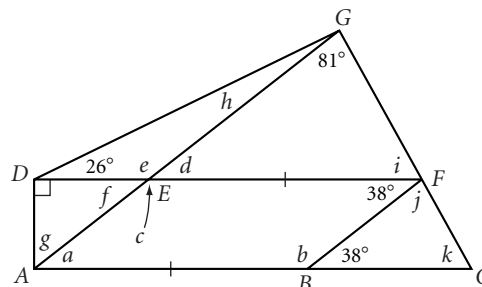
$b =$ _____ $h =$ _____

$c =$ _____ $i =$ _____

$d =$ _____ $j =$ _____

$e =$ _____ $k =$ _____

$f =$ _____



10. Construct a parallelogram with diagonals \overline{AC} and \overline{BD} .

Is your parallelogram unique? If not, construct a different (noncongruent) parallelogram.



Lesson 5.6 • Properties of Special Parallelograms

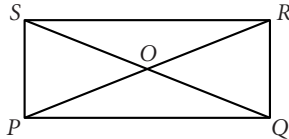
Name _____ Period _____ Date _____

1. $PQRS$ is a rectangle and $OS = 16$.

$OQ =$ _____

$m\angle QRS =$ _____

$PR =$ _____

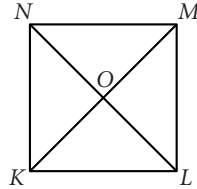


2. $KLMN$ is a square and $NM = 8$.

$m\angle OKL =$ _____

$m\angle MOL =$ _____

Perimeter $KLMN =$ _____

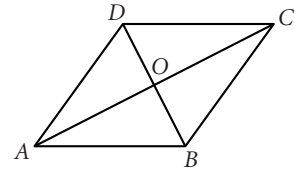


3. $ABCD$ is a rhombus, $AD = 11$, and $DO = 6$.

$OB =$ _____

$BC =$ _____

$m\angle AOD =$ _____



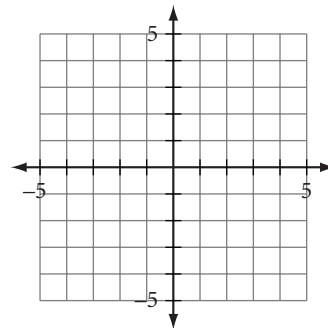
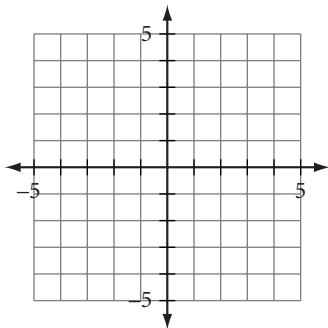
In Exercises 4–11, match each description with *all* the terms that fit it.

- | | | | |
|--|---|---|--|
| a. Trapezoid | b. Isosceles triangle | c. Parallelogram | d. Rhombus |
| e. Kite | f. Rectangle | g. Square | h. All quadrilaterals |
| 4. _____ Diagonals bisect each other. | 5. _____ Diagonals are perpendicular. | 6. _____ Diagonals are congruent. | 7. _____ Measures of interior angles sum to 360° . |
| 8. _____ Opposite sides are congruent. | 9. _____ Opposite angles are congruent. | 10. _____ Both diagonals bisect angles. | 11. _____ Diagonals are perpendicular bisectors of each other. |

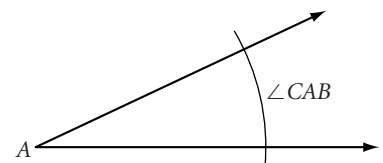
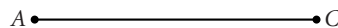
In Exercises 12 and 13, graph the points and determine whether $ABCD$ is a trapezoid, parallelogram, rectangle, or none of these.

12. $A(-4, -1)$, $B(0, -3)$, $C(4, 0)$, $D(-1, 5)$

13. $A(0, -3)$, $B(-1, 2)$, $C(-3, 4)$, $D(-2, -1)$



14. Construct rectangle $ABCD$ with diagonal \overline{AC} and $\angle CAB$.



Lesson 5.7 • Proving Quadrilateral Properties

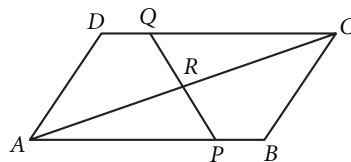
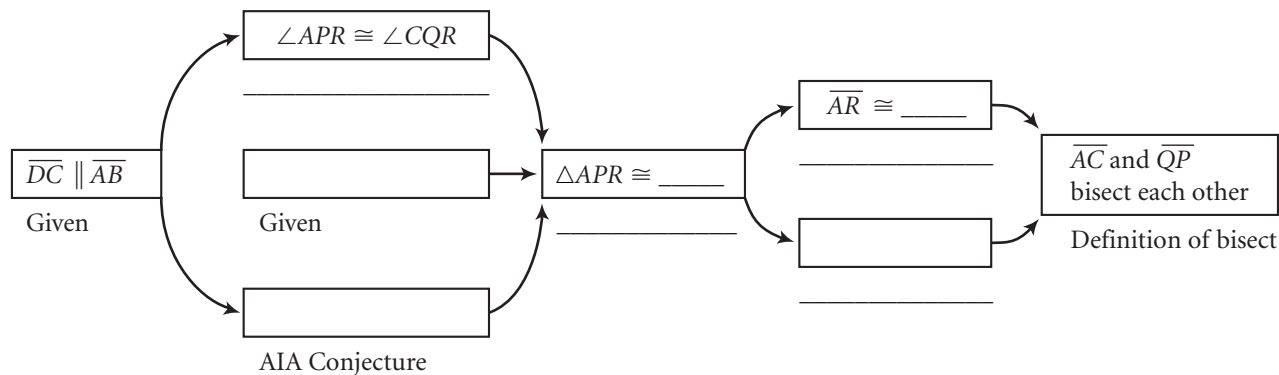
Name _____ Period _____ Date _____

Write or complete each flowchart proof.

1. **Given:** $ABCD$ is a parallelogram and $\overline{AP} \cong \overline{QC}$

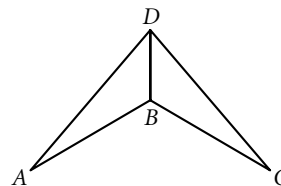
Show: \overline{AC} and \overline{PQ} bisect each other

Flowchart Proof



2. **Given:** Dart $ABCD$ with $\overline{AB} \cong \overline{BC}$ and $\overline{CD} \cong \overline{AD}$

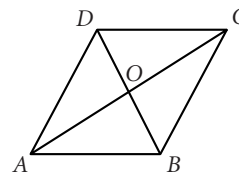
Show: $\angle A \cong \angle C$



3. Show that the diagonals of a rhombus divide the rhombus into four congruent triangles.

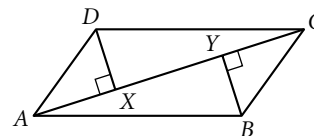
Given: Rhombus $ABCD$

Show: $\triangle ABO \cong \triangle CBO \cong \triangle CDO \cong \triangle ADO$



4. **Given:** Parallelogram $ABCD$, $\overline{BY} \perp \overline{AC}$, $\overline{DX} \perp \overline{AC}$

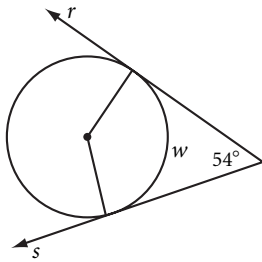
Show: $\overline{DX} \cong \overline{BY}$



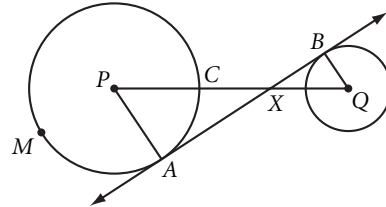
Lesson 6.1 • Tangent Properties

Name _____ Period _____ Date _____

1. Rays r and s are tangents. $w =$ _____

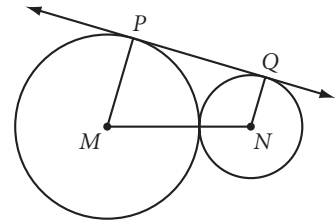


2. \overleftrightarrow{AB} is tangent to both circles and $m\widehat{AMC} = 295^\circ$. $m\angle BQX =$ _____

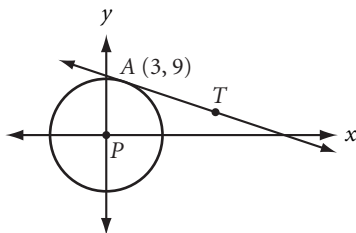


3. \overleftrightarrow{PQ} is tangent to two externally tangent noncongruent circles, M and N .

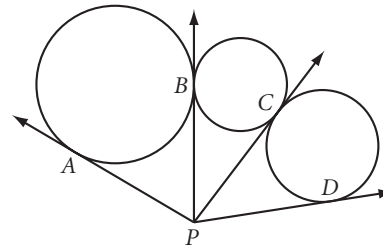
- a. $m\angle NQP =$ _____, $m\angle MPQ =$ _____
 b. What kind of quadrilateral is $MNQP$? Explain your reasoning.



4. \overleftrightarrow{AT} is tangent to circle P . Find the equation of \overleftrightarrow{AT} .



5. \overrightarrow{PA} , \overrightarrow{PB} , \overrightarrow{PC} , and \overrightarrow{PD} are tangents. Explain why $\overrightarrow{PA} \cong \overrightarrow{PD}$.



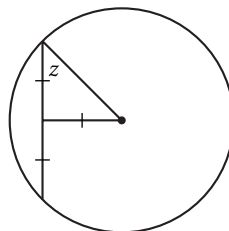
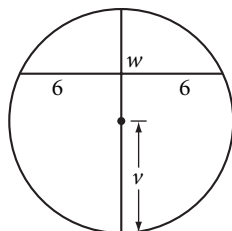
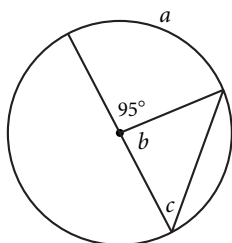
6. Circle A has diameter 16.4 cm. Circle B has diameter 6.7 cm.
 a. If A and B are internally tangent, what is the distance between their centers?
 b. If A and B are externally tangent, what is the distance between their centers?
7. Construct a circle, P . Pick a point, A , on the circle. Construct a tangent through A . Pick a point, T , on the tangent. Construct a second tangent to the circle through T .

Lesson 6.2 • Chord Properties

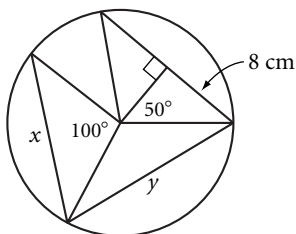
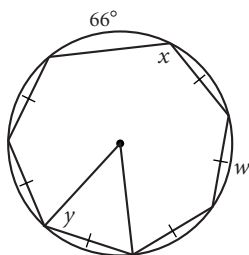
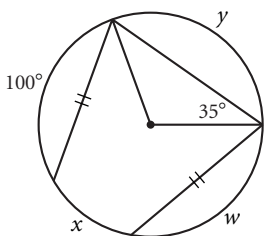
Name _____ Period _____ Date _____

In Exercises 1–6, find each unknown or write “cannot be determined.”

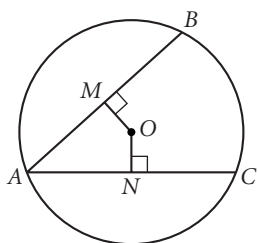
1. $a = \underline{\hspace{1cm}}$, $b = \underline{\hspace{1cm}}$, $c = \underline{\hspace{1cm}}$ 2. $w = \underline{\hspace{1cm}}$, $v = \underline{\hspace{1cm}}$ 3. $z = \underline{\hspace{1cm}}$



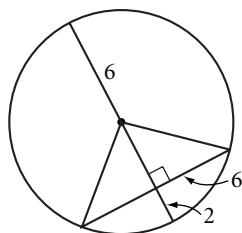
4. $w = \underline{\hspace{1cm}}$, $x = \underline{\hspace{1cm}}$, $y = \underline{\hspace{1cm}}$ 5. $w = \underline{\hspace{1cm}}$, $x = \underline{\hspace{1cm}}$, $y = \underline{\hspace{1cm}}$ 6. $x = \underline{\hspace{1cm}}$, $y = \underline{\hspace{1cm}}$



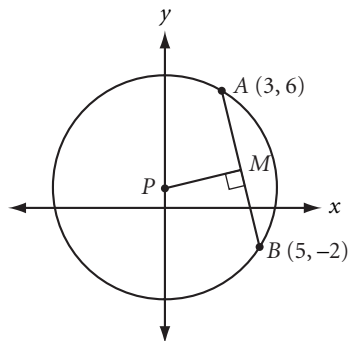
7. $\overline{AB} \cong \overline{AC}$. $\angle MON$ is a .
Justify your answer.



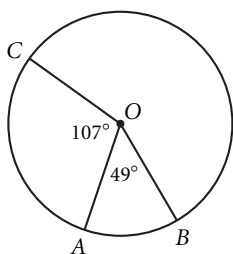
8. What's wrong with this picture?



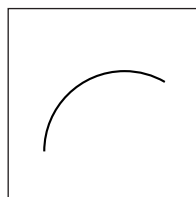
9. Find the coordinates of P and M .



10. $m\widehat{AB} = \underline{\hspace{1cm}}$
 $m\widehat{ABC} = \underline{\hspace{1cm}}$
 $m\widehat{BAC} = \underline{\hspace{1cm}}$
 $m\widehat{ACB} = \underline{\hspace{1cm}}$



11. Trace part of a circle onto patty paper. Fold to find the center. Explain your method.



Lesson 6.3 • Arcs and Angles

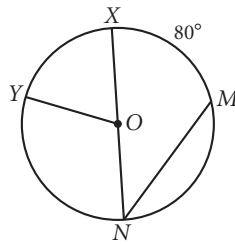
Name _____ Period _____ Date _____

1. $m\widehat{XM} = 80^\circ$

$m\angle XNM = \underline{\hspace{2cm}}$

$m\widehat{XN} = \underline{\hspace{2cm}}$

$m\widehat{MN} = \underline{\hspace{2cm}}$

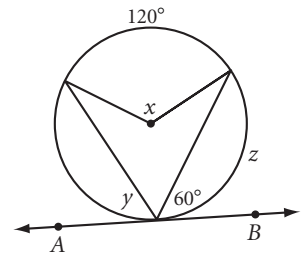


2. \overleftrightarrow{AB} is a tangent.

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$

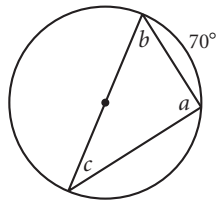
$z = \underline{\hspace{2cm}}$



3. $a = \underline{\hspace{2cm}}$

$b = \underline{\hspace{2cm}}$

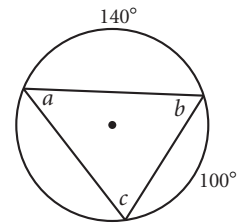
$c = \underline{\hspace{2cm}}$



4. $a = \underline{\hspace{2cm}}$

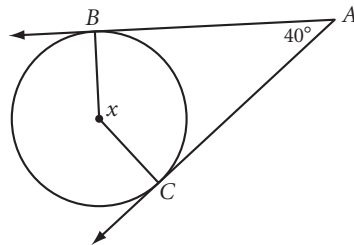
$b = \underline{\hspace{2cm}}$

$c = \underline{\hspace{2cm}}$



5. \overleftrightarrow{AB} and \overleftrightarrow{AC} are tangents.

$x = \underline{\hspace{2cm}}$



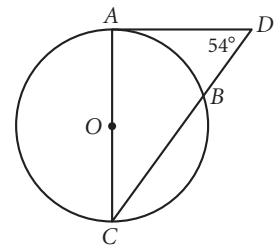
6. \overleftrightarrow{AD} is a tangent. \overleftrightarrow{AC} is a diameter.

$m\angle A = \underline{\hspace{2cm}}$

$m\widehat{AB} = \underline{\hspace{2cm}}$

$m\angle C = \underline{\hspace{2cm}}$

$m\widehat{CB} = \underline{\hspace{2cm}}$

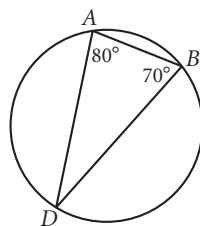


7. $m\widehat{AD} = \underline{\hspace{2cm}}$

$m\angle D = \underline{\hspace{2cm}}$

$m\widehat{AB} = \underline{\hspace{2cm}}$

$m\widehat{DAB} = \underline{\hspace{2cm}}$

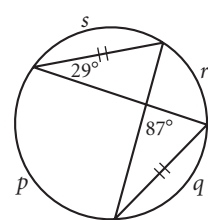


8. $p = \underline{\hspace{2cm}}$

$q = \underline{\hspace{2cm}}$

$r = \underline{\hspace{2cm}}$

$s = \underline{\hspace{2cm}}$



9. Find the lettered angle and arc measures. \overleftrightarrow{AT} and \overleftrightarrow{AZ} are tangents.

$a = \underline{\hspace{2cm}}$

$b = \underline{\hspace{2cm}}$

$c = \underline{\hspace{2cm}}$

$d = \underline{\hspace{2cm}}$

$e = \underline{\hspace{2cm}}$

$f = \underline{\hspace{2cm}}$

$g = \underline{\hspace{2cm}}$

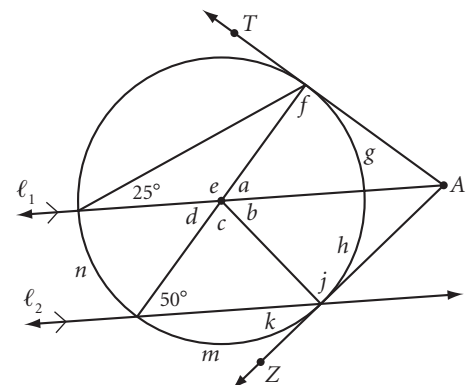
$h = \underline{\hspace{2cm}}$

$j = \underline{\hspace{2cm}}$

$k = \underline{\hspace{2cm}}$

$m = \underline{\hspace{2cm}}$

$n = \underline{\hspace{2cm}}$



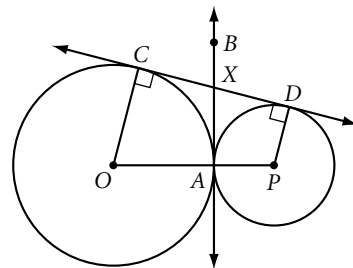
Lesson 6.4 • Proving Circle Conjectures

Name _____ Period _____ Date _____

In Exercises 1–4, complete each proof with a paragraph or a flowchart.

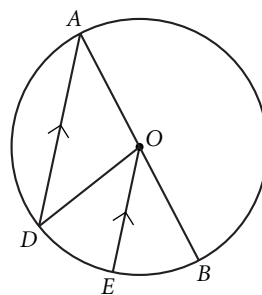
- 1. Given:** Circles O and P are externally tangent, with common tangents \overleftrightarrow{CD} and \overleftrightarrow{AB}

Show: \overleftrightarrow{AB} bisects \overleftrightarrow{CD} at X



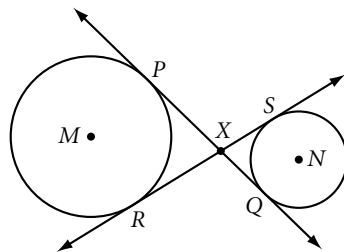
- 2. Given:** Circle O with diameter \overline{AB} and chord \overline{AD} . $\overline{OE} \parallel \overline{AD}$.

Show: $\widehat{DE} \cong \widehat{BE}$



- 3. Given:** \overleftrightarrow{PQ} and \overleftrightarrow{RS} are tangent to both circles.

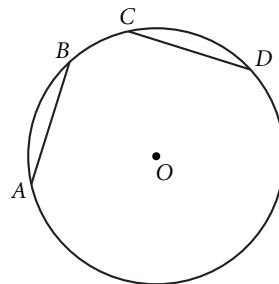
Show: $\overline{PQ} \cong \overline{RS}$.



- 4. Prove the converse of the Chord Arcs Conjecture:** If two arcs in a circle are congruent, then their chords are congruent. *Hint:* Draw radii.

Given: $\widehat{AB} \cong \widehat{CD}$

Show: $\overline{AB} \cong \overline{CD}$



Lesson 6.5 • The Circumference/Diameter Ratio

Name _____ Period _____ Date _____

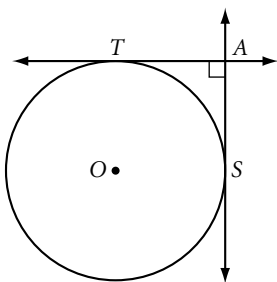
In Exercises 1–4, leave your answers in terms of π .

1. If $r = 10.5$ cm, find C .
2. If $C = 25\pi$ cm, find r .
3. What is the circumference of a circle whose radius is 30 cm?
4. What is the diameter of a circle whose circumference is 24π cm?

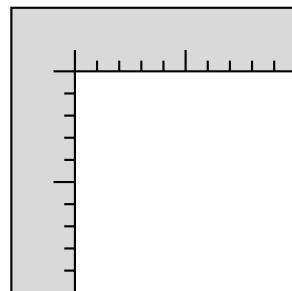
In Exercises 5–9, round your answer to the nearest 0.1 unit. Use the symbol \approx to show that your answer is an approximation.

5. If $d = 9.6$ cm, find C .
6. If $C = 132$ cm, find d and r .
7. A dinner plate fits snugly in a square box with perimeter 48 inches. What is the circumference of the plate?
8. Four saucers are part of the same set as the dinner plate in Exercise 7. Each has a circumference of 15.7 inches. Will they fit, side by side, in the same square box? If so, how many inches will there be between the saucers for padding?

9. \overleftrightarrow{AT} and \overleftrightarrow{AS} are tangents. $AT = 12$ cm. What is the circumference of circle O ?



10. How can you use a large carpenter's square to find the circumference of a tree?



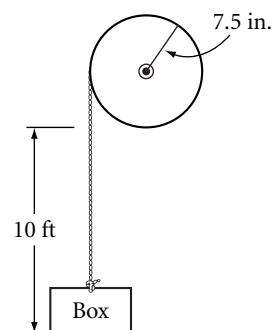
11. In order to increase the circumference of a circle from 16π cm to 20π cm, by how much must the diameter increase?

Lesson 6.6 • Around the World

Name _____ Period _____ Date _____

1. Alfonzo's Pizzeria bakes olive pieces in the outer crust of its 20-inch (diameter) pizza. There is at least one olive piece per inch of crust. How many olive pieces will you get in one slice of pizza? Assume the pizza is cut into eight slices.

2. To use the machine at right, you turn the crank, which turns the pulley wheel, which winds the rope and lifts the box. Through how many rotations must you turn the crank to lift the box 10 feet?



3. A satellite in *geostationary* orbit stays over the same spot on Earth. The satellite completes one orbit in the same time that Earth rotates once about its axis (23.93 hours). If the satellite's orbit has radius 4.23×10^7 m, calculate the satellite's orbital speed (tangential velocity) in meters per second.

4. You want to decorate the side of a cylindrical can by coloring a rectangular piece of paper and wrapping it around the can. The paper is 19 cm by 29 cm. Find the two possible diameters of the can to the nearest 0.01 cm. Assume the paper fits exactly.

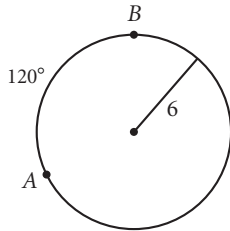
5. As you sit in your chair, you are whirling through space with Earth as it moves around the sun. If the average distance from Earth to the sun is 1.4957×10^{11} m and Earth completes one revolution every 364.25 days, what is your "sitting" speed in space relative to the sun? Give your answer in km/h, rounded to the nearest 100 km/h.

Lesson 6.7 • Arc Length

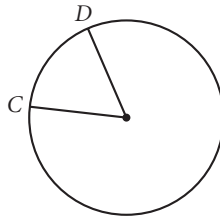
Name _____ Period _____ Date _____

In Exercises 1–10, leave your answers in terms of π .

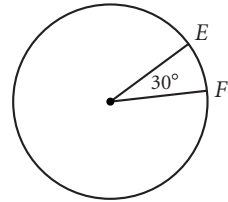
1. Length of \widehat{AB} = _____



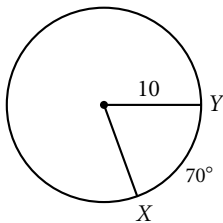
2. The circumference is 24π and $m\widehat{CD} = 60^\circ$. Length of \widehat{CD} = _____



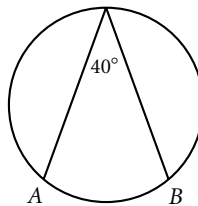
3. The length of \widehat{EF} is 5π . Radius = _____



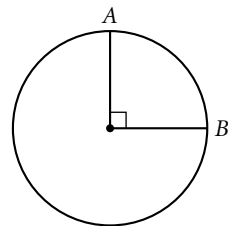
4. Length of \widehat{XY} = _____



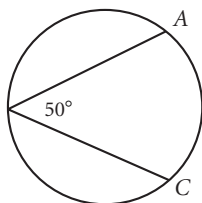
5. The radius is 20. Length of \widehat{AB} = _____



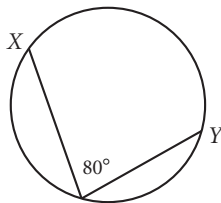
6. The circumference is 25π . Length of \widehat{AB} = _____



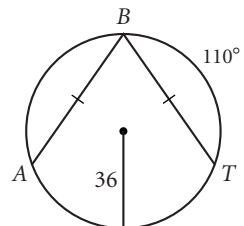
7. The diameter is 40. Length of \widehat{AC} = _____



8. The length of \widehat{XY} is 14π . Diameter = _____



9. Length of \widehat{AB} = _____

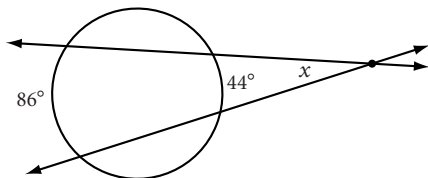


10. A circle has an arc with measure 80° and length 88π . What is the diameter of the circle?

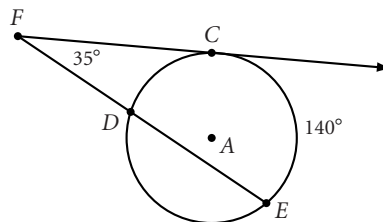
Exploration • Intersecting Secants, Tangents, and Chords

Name _____ Period _____ Date _____

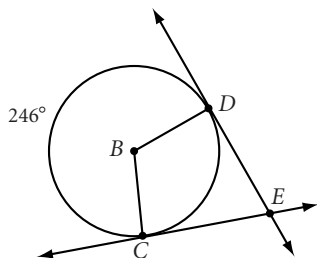
1. $x =$ _____



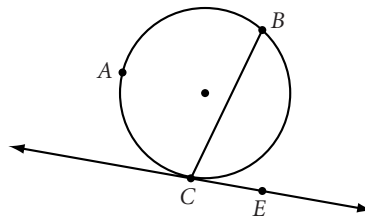
2. \overrightarrow{FC} is tangent to circle A at point C.
 $m\widehat{DC} =$ _____, $m\widehat{ED} =$ _____



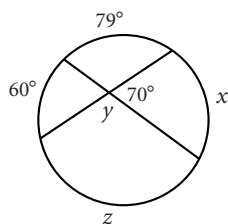
3. \overrightarrow{ED} and \overrightarrow{EC} are tangents.
 $m\widehat{DC} =$ _____, $m\angle DEC =$ _____



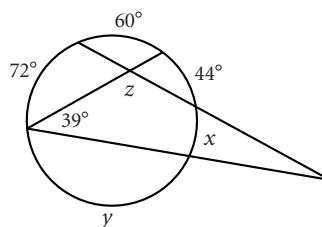
4. \overrightarrow{CE} is a tangent, $m\widehat{BC} = 150^\circ$
 $m\angle BCE =$ _____, $m\widehat{BAC} =$ _____



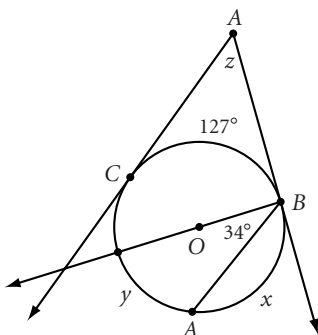
5. $x =$ _____, $y =$ _____, $z =$ _____



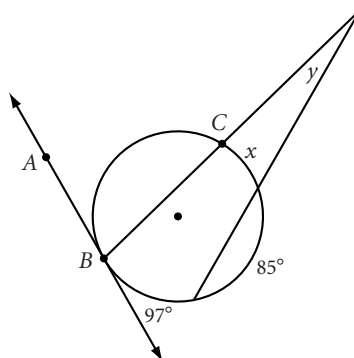
6. $x =$ _____, $y =$ _____, $z =$ _____



7. \overrightarrow{AB} and \overrightarrow{AC} are tangents.
 $x =$ _____, $y =$ _____, $z =$ _____



8. \overrightarrow{AB} is a tangent, $m\angle ABC = 75^\circ$
 $x =$ _____, $y =$ _____

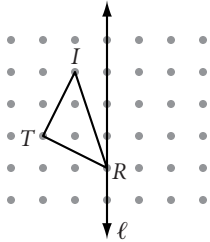


Lesson 7.1 • Transformations and Symmetry

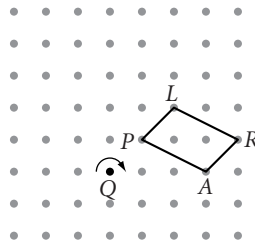
Name _____ Period _____ Date _____

In Exercises 1–3, perform each transformation.

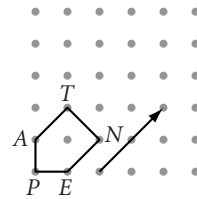
1. Reflect $\triangle TRI$ across line ℓ .



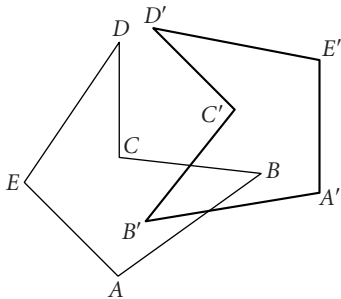
2. Rotate $PARL$ 270° clockwise about Q .



3. Translate $PENTA$ by the given vector.

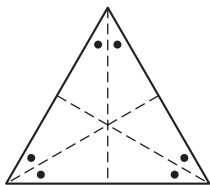


4. $ABCDE$ and its reflected image, $A'B'C'D'E'$, are shown below. Use construction tools to locate the line of reflection, ℓ . Explain your method.

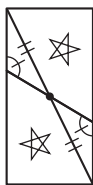


In Exercises 5–8, identify the type(s) of symmetry in each figure.

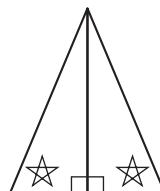
5. Equilateral triangle



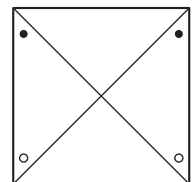
6. Rectangle



7. Isosceles triangle



8. Square



In Exercises 9–12, draw each polygon and identify the type(s) of symmetry in each. Draw all lines of reflection and mark centers of rotation.

9. Rhombus

10. Parallelogram

11. Isosceles trapezoid

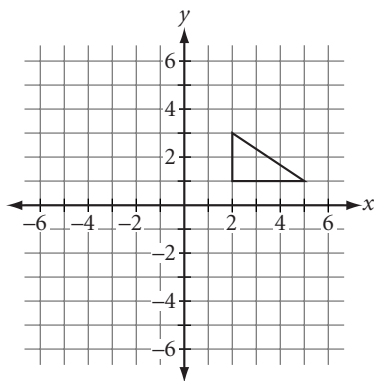
12. Square

Lesson 7.2 • Properties of Isometries

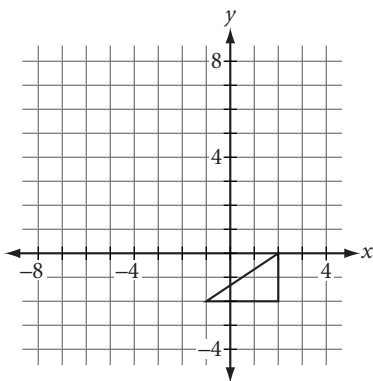
Name _____ Period _____ Date _____

In Exercises 1–3, draw the image according to the rule and identify the type of transformation.

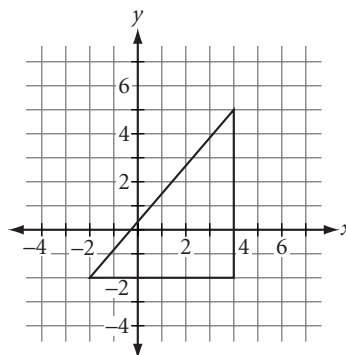
1. $(x, y) \rightarrow (-x, -y)$



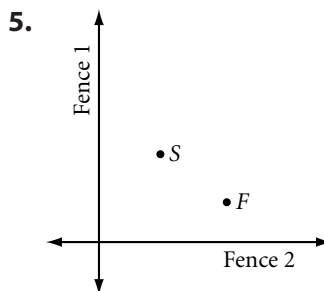
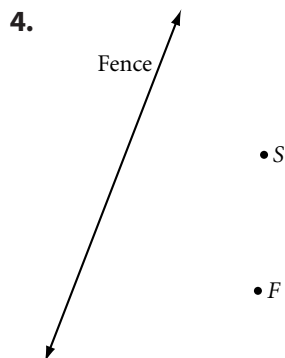
2. $(x, y) \rightarrow (x - 4, y + 6)$



3. $(x, y) \rightarrow (4 - x, y)$

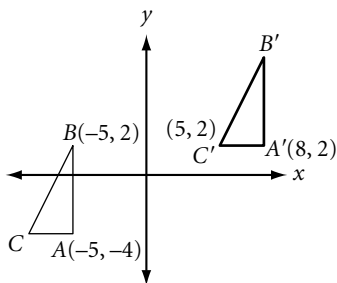


In Exercises 4 and 5, the Harbour High Geometry Class is holding a Fence Race. Contestants must touch each fence at some point as they run from S to F . Use your geometry tools to draw the shortest possible race path.

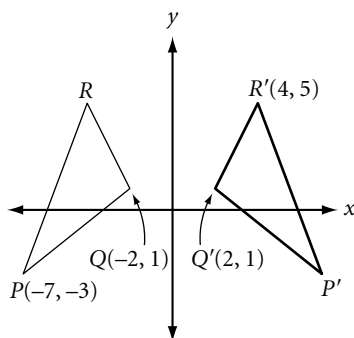


In Exercises 6–8, complete the ordered pair rule that transforms each triangle to its image. Identify the transformation. Find all missing coordinates.

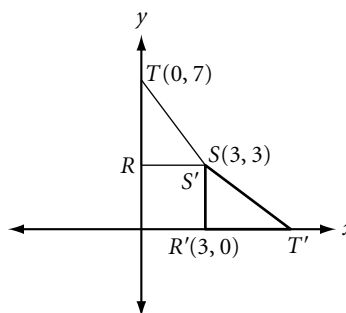
6. $(x, y) \rightarrow (\rule{1cm}{0.4pt}, \rule{1cm}{0.4pt})$



7. $(x, y) \rightarrow (\rule{1cm}{0.4pt}, \rule{1cm}{0.4pt})$



8. $(x, y) \rightarrow (\rule{1cm}{0.4pt}, \rule{1cm}{0.4pt})$



Lesson 7.3 • Compositions of Transformations

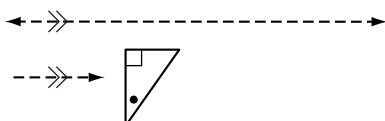
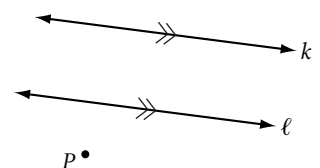
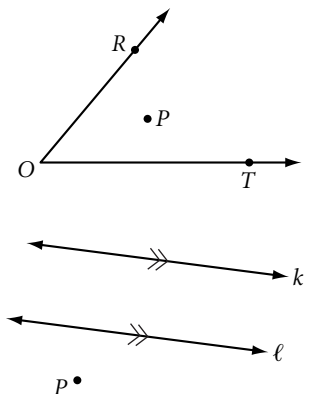
Name _____ Period _____ Date _____

In Exercises 1–8, name the single transformation that can replace the composition of each set of multiple transformations.

1. Translation by $\langle +4, +1 \rangle$, followed by $\langle +2, -3 \rangle$, followed by $\langle -8, +7 \rangle$
2. Rotation 60° clockwise, followed by 80° counterclockwise, followed by 25° counterclockwise all about the same center of rotation
3. Reflection across vertical line m , followed by reflection across vertical line n , where n is 8 units to the right of m
4. Reflection across vertical line p , followed by reflection across horizontal line q
5. Reflection across vertical line n , followed by reflection across vertical line m , where n is 8 units to the right of m
6. Reflection across horizontal line q , followed by reflection across vertical line p
7. Translation by $\langle +6, 0 \rangle$, followed by reflection across the y -axis
8. Reflection across the y -axis, followed by translation by $\langle +6, 0 \rangle$

In Exercises 9–11, copy the figure onto your paper and use your geometry tools to perform the given transformation.

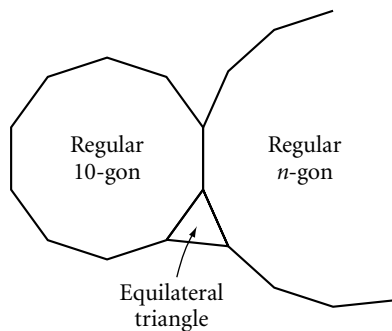
9. Locate P' , the reflected image across \overrightarrow{OR} , and P'' , the reflected image of P' across \overrightarrow{OT} . Find $m\angle ROT$ and give a single transformation that maps P to P'' .
10. Locate P' , the reflected image across k , and P'' , the reflected image of P' across ℓ . Find the distance between ℓ and k and give a single transformation that maps P to P'' .
11. Draw five glide-reflected images of the triangle.



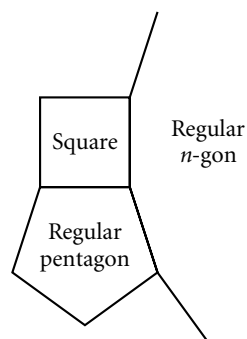
Lesson 7.4 • Tessellations with Regular Polygons

Name _____ Period _____ Date _____

1. Find n .



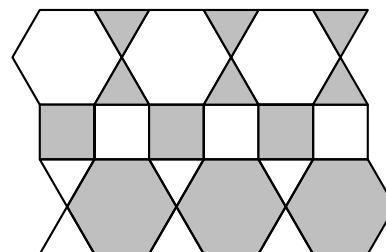
2. Find n .



3. What is a regular tessellation? Sketch an example to illustrate your explanation.

4. What is a 1-uniform tiling? Sketch an example of a 1-uniform tiling that is not a regular tessellation.

5. Give the numerical name for the tessellation at right.

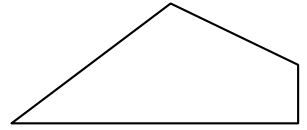


6. Use your geometry tools to draw the 4.8^2 tessellation.

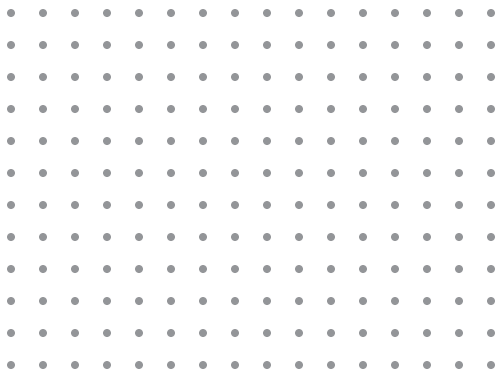
Lessons 7.5–7.8 • Tessellations

Name _____ Period _____ Date _____

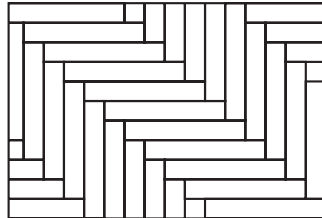
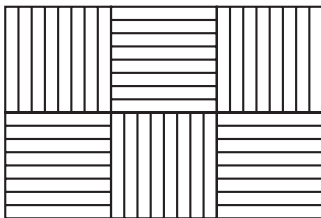
1. Trace the quadrilateral at right (or draw a similar one). Make the outline dark. Set another piece of paper on top of the quadrilateral and, by tracing, create a tessellation. (*Hint:* Trace vertices and use a straightedge to connect them.)



2. On dot paper, draw a small concave quadrilateral (vertices on dots). Allow no more than three dots inside the figure. Tessellate the entire paper with your quadrilateral. Color and shade your tessellation.



3. In *non-edge-to-edge tilings*, the vertices of the polygons do not have to coincide, as in these wooden deck patterns. Use graph paper to create your own non-edge-to-edge tiling.

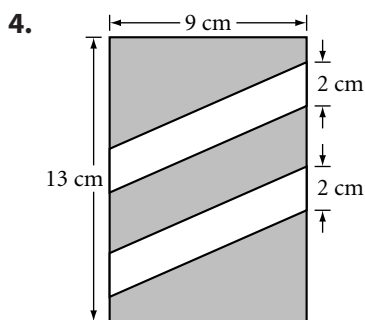
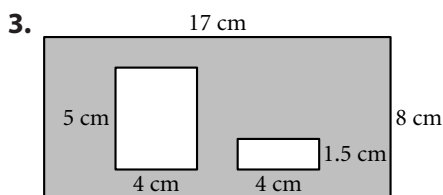
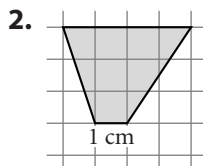
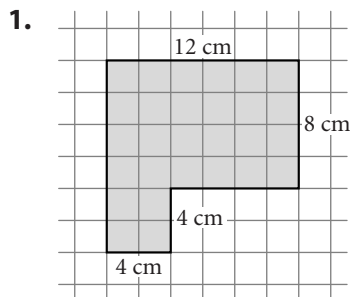


4. Use your geometry tools to draw a parallelogram. Draw squares on each side. Create a tessellation by duplicating your parallelogram and squares.

Lesson 8.1 • Areas of Rectangles and Parallelograms

Name _____ Period _____ Date _____

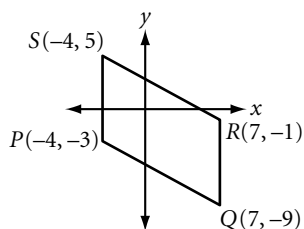
In Exercises 1–4, find the area of the shaded region.



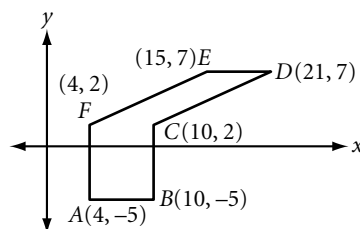
5. Rectangle $ABCD$ has area 2684 m^2 and width 44 m . Find its length.

6. Draw a parallelogram with area 85 cm^2 and an angle with measure 40° .
Is your parallelogram unique? If not, draw a different one.

7. Find the area of $PQRS$.



8. Find the area of $ABCDEF$.



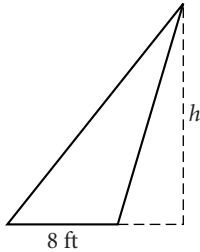
9. Dana buys a piece of carpet that measures 20 square yards. Will she be able to completely cover a rectangular floor that measures 12 ft 6 in. by 16 ft 6 in.? Explain why or why not.

Lesson 8.2 • Areas of Triangles, Trapezoids, and Kites

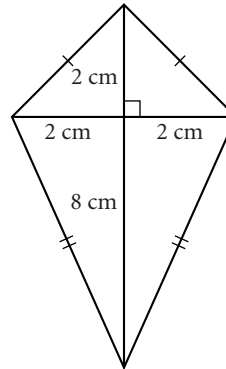
Name _____ Period _____ Date _____

In Exercises 1–4, solve for the unknown measures.

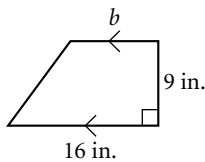
1. Area = 64 ft^2 , $h =$ _____.



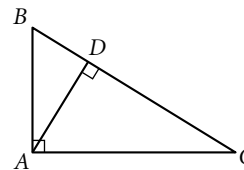
2. Area = _____.



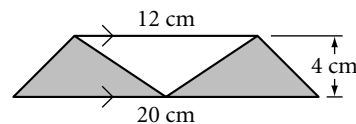
3. Area = 126 in^2
 $b =$ _____.



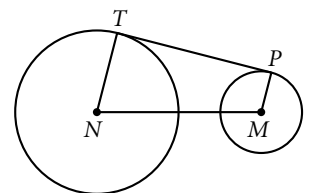
4. $AB = 6 \text{ cm}$, $AC = 8 \text{ cm}$, and $BC = 10 \text{ cm}$.
Find AD .



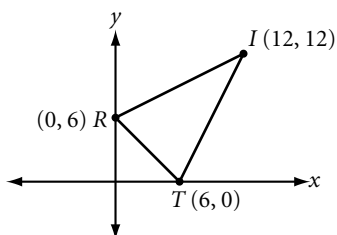
5. Find the area of the shaded region.



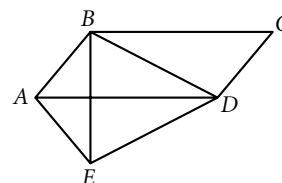
6. \overline{TP} is tangent to circles M and N . $TP = 16 \text{ cm}$. The radius of N is 7 cm and the radius of M is 4 cm. Find the area of $NMPT$.



7. Find the area of $\triangle TRI$.



8. $ABCD$ is a parallelogram, $ABDE$ is a kite, $AD = 18 \text{ cm}$, and $BE = 10 \text{ cm}$. Find the area of $ABCDE$.

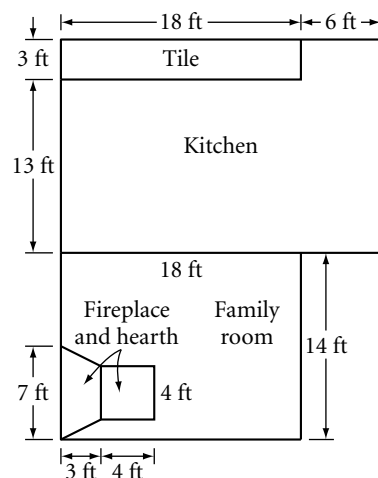


Lesson 8.3 • Area Problems

Name _____ Period _____ Date _____

1. A bundle of hardwood flooring contains $14\frac{1}{2}$ ft² and costs \$39.90.

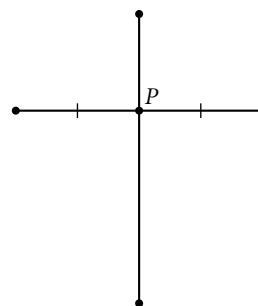
- How many square feet of flooring is needed to cover the kitchen and family room? Exclude the fireplace, hearth, and tiled area.
- You should buy 5% extra flooring to account for waste. How many bundles of flooring should you buy? What will be the cost?



2. Bert's Bigtime Bakery has baked the world's largest chocolate cake. It is a rectangular sheet cake that is 600 cm by 400 cm by 180 cm high. Bert wants to apply frosting to the four sides and the top. How many liters of frosting does he need if 1 liter of frosting covers about 1200 cm²?

3. For a World Peace Day celebration the students at Cabot Junior/Senior High School are making a 6 m-by-8 m flag. Each of the six grades will create a motif to honor the people of the six inhabited continents. Sketch three possible ways to divide the flag: one into six congruent triangles; one into six triangles with equal area but none congruent; and one into six congruent trapezoids. Give measurements or markings on your sketches so each class knows it has equal area.

4. Kit and Kat are building a kite for the big kite festival. Kit has already cut his sticks for the diagonals. He wants to position P so that he will have maximum kite area. He asks Kat for advice. What should Kat tell him?

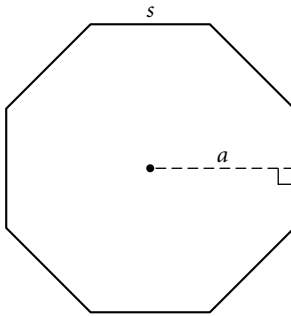


Lesson 8.4 • Areas of Regular Polygons

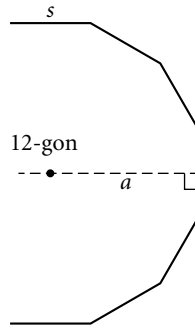
Name _____ Period _____ Date _____

In Exercises 1–3, the polygons are regular.

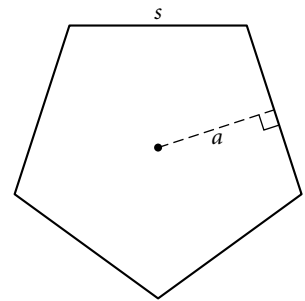
1. $s = 12$ cm
 $a \approx 14.5$ cm
 $A \approx$ _____



2. $s = 4.2$ cm
 $A \approx 197$ cm²
 $a \approx$ _____



3. $a = 6$ cm
 $A \approx 130.8$ cm²
 $p \approx$ _____

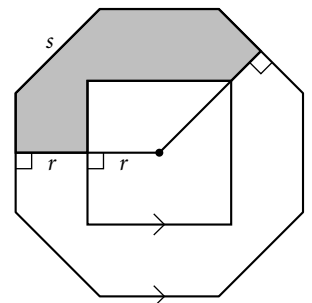


4. In a regular n -gon, $s = 4.8$ cm, $a \approx 7.4$ cm, and $A \approx 177.6$ cm². Find n .

5. Draw a regular pentagon so that it has perimeter 20 cm. Use the Regular Polygon Area Conjecture and a centimeter ruler to find its approximate area.

6. Use a compass and straightedge to construct a regular octagon and its apothem. Use a centimeter ruler to measure its side length and apothem, and use the Regular Polygon Area Conjecture to find its approximate area.

7. Find the area of the shaded region between the square and the regular octagon. $s \approx 5$ cm. $r = 3$ cm.



Lesson 8.5 • Areas of Circles

Name _____ Period _____ Date _____

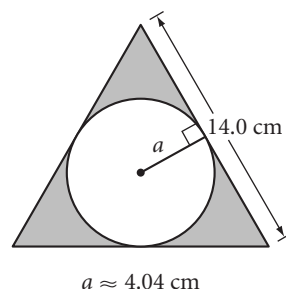
In Exercises 1–4, write your answers in terms of π .

1. If $r = 9$ cm, $A =$ _____.
2. If $d = 6.4$ cm, $A =$ _____.
3. If $A = 529\pi$ cm², $r =$ _____.
4. If $C = 36\pi$ cm, $A =$ _____.

In Exercises 5–8, round your answers to the nearest 0.01 unit.

5. If $r = 7.8$ cm, $A \approx$ _____.
6. If $A = 136.46$, $C \approx$ _____.
7. If $d = 3.12$, $A \approx$ _____.
8. If $C = 7.85$, $A \approx$ _____.

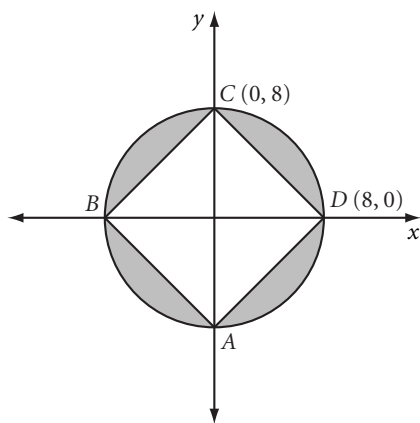
For Exercises 9 and 10, refer to the figure of a circle inscribed in an equilateral triangle. Round your answers to the nearest 0.1 unit.



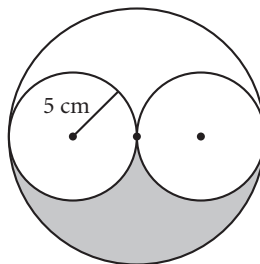
9. Find the area of the inscribed circle.
10. Find the area of the shaded region.

In Exercises 11 and 12, find the area of the shaded region. Write your answers in terms of π .

11. $ABCD$ is a square.



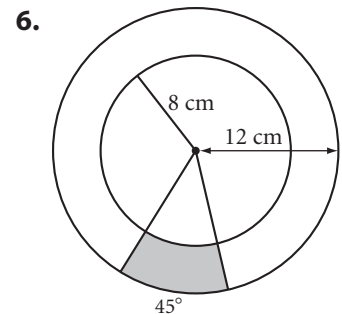
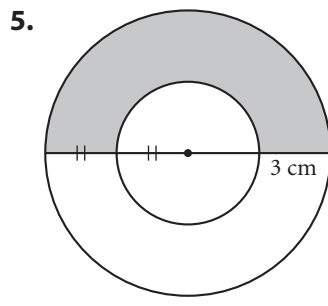
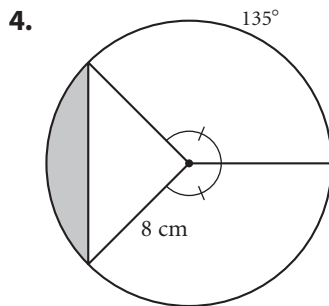
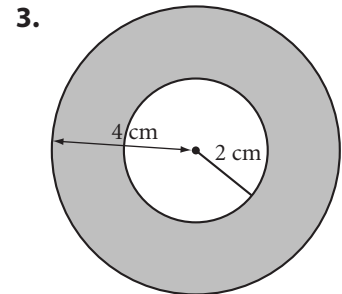
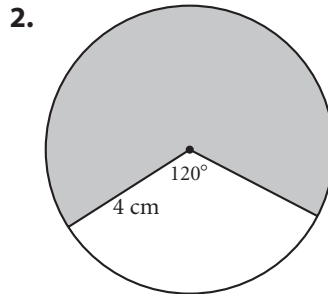
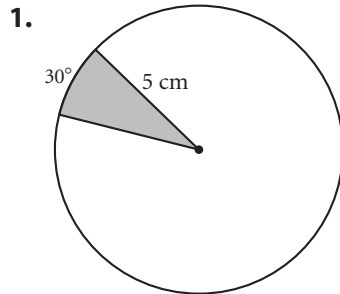
12. The three circles are tangent.



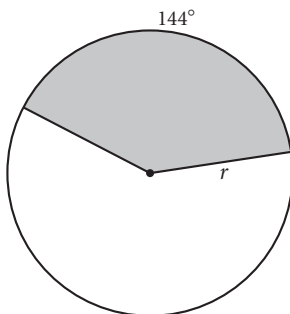
Lesson 8.6 • Any Way You Slice It

Name _____ Period _____ Date _____

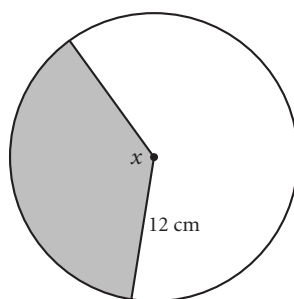
In Exercises 1–6, find the area of the shaded region. Write your answers in terms of π and rounded to the nearest 0.01 cm^2 .



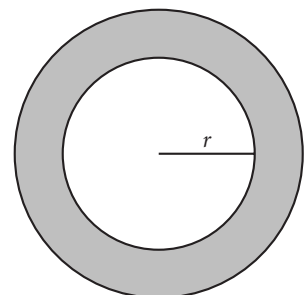
7. Shaded area is $40\pi \text{ cm}^2$.
Find r .



8. Shaded area is $54\pi \text{ cm}^2$.
Find x .



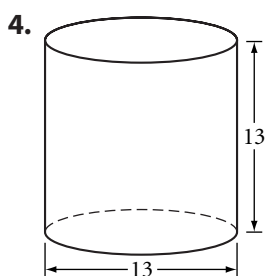
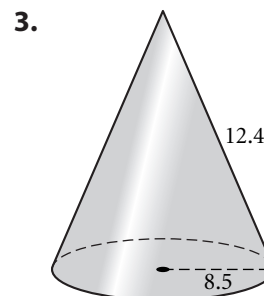
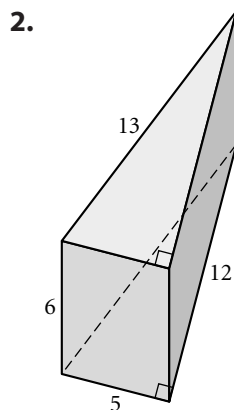
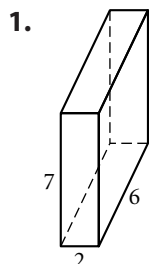
9. Shaded area is $51\pi \text{ cm}^2$.
The diameter of the larger circle is 20 cm. Find r .



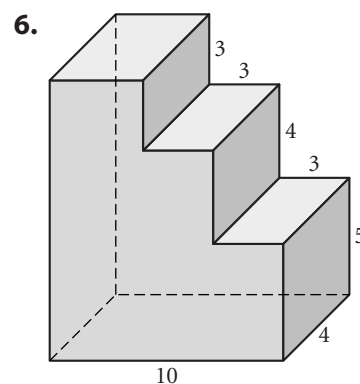
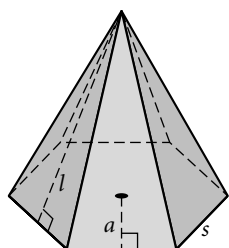
Lesson 8.7 • Surface Area

Name _____ Period _____ Date _____

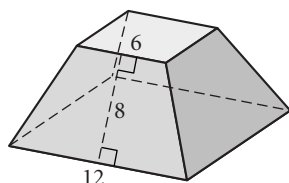
In Exercises 1–8, find the surface area of each solid. All quadrilaterals are rectangles, and all measurements are in centimeters. Round your answers to the nearest 0.1 cm^2 .



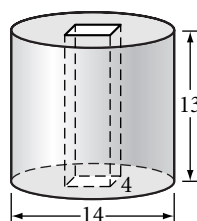
5. Base is a regular hexagon.
 $s = 6$, $a \approx 5.2$, and $l = 9$.



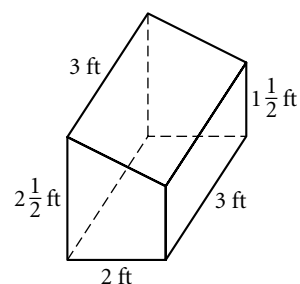
7. Both bases are squares.



8. A square hole in a round peg



9. Ilsa is building a museum display case. The sides and bottom will be plywood and the top will be glass. Plywood comes in 4 ft-by-8 ft sheets. How many sheets of plywood will she need to buy? Explain. Sketch a cutting pattern that will leave her with the largest single piece possible.

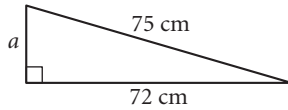


Lesson 9.1 • The Theorem of Pythagoras

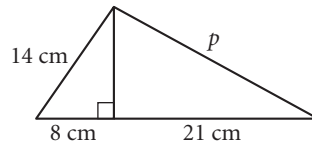
Name _____ Period _____ Date _____

Give all answers rounded to the nearest 0.1 unit.

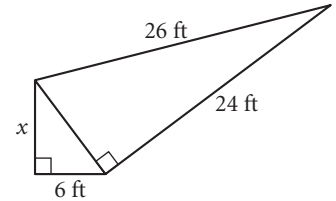
1. $a =$ _____



2. $p \approx$ _____

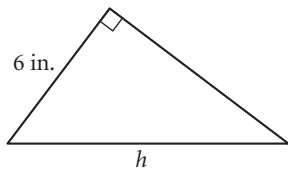


3. $x =$ _____

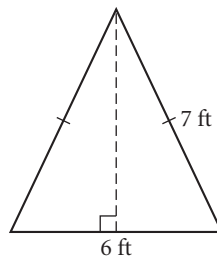


4. Area = 39 in^2

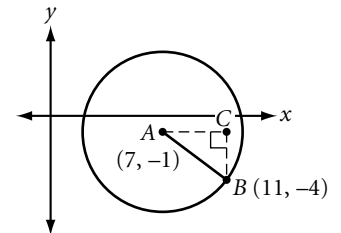
$h \approx$ _____



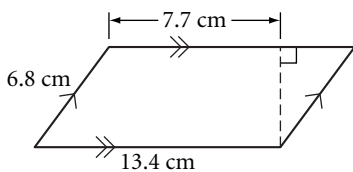
5. Find the area.



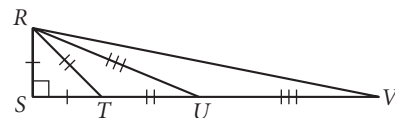
6. Find the coordinates of C and the radius of circle A.



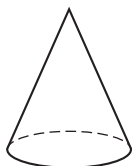
7. Find the area.



8. $RS = 3 \text{ cm}$. Find RV .



9. Base area = $16\pi \text{ cm}^2$ and slant height = 3 cm. What's wrong with this picture?



10. Given $\triangle PQR$, with $m\angle P = 90^\circ$, $PQ = 20 \text{ in.}$, and $PR = 15 \text{ in.}$, find the area of $\triangle PQR$, the length of the hypotenuse, and the altitude to the hypotenuse.

Lesson 9.2 • The Converse of the Pythagorean Theorem

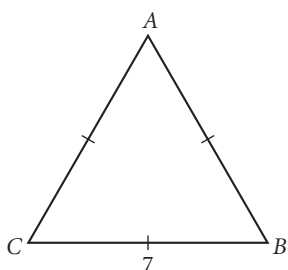
Name _____ Period _____ Date _____

All measurements are in centimeters. Give answers rounded to the nearest 0.01 cm.

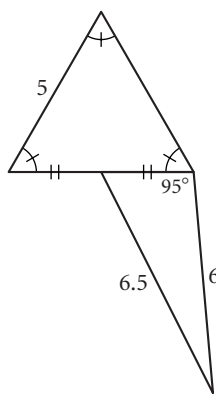
In Exercises 1–4, determine whether a triangle with the given side lengths is a right triangle.

1. 76, 120, 98 2. 221, 204, 85 3. 5.0, 1.4, 4.8 4. 80, 82, 18

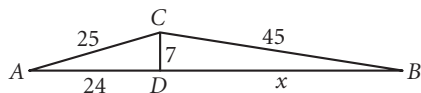
5. Find the area of $\triangle ABC$.



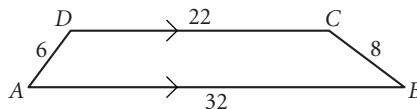
6. What's wrong with this picture?



7. Find x . Explain your method.

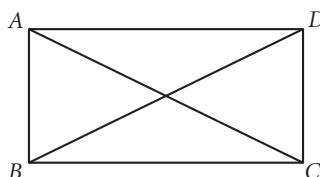


8. Find the area of $ABCD$.



In Exercises 9–11, determine whether $ABCD$ is a rectangle and justify your answer. If not enough information is given, write “cannot be determined.”

9. $AB = 3$, $BC = 4$, and $AC = 6$.



10. $AB = 3$, $BC = 4$, $DA = 4$, and $AC = 5$.

11. $AB = 3$, $BC = 4$, $CD = 3$, $DA = 4$, and $AC = BD$.

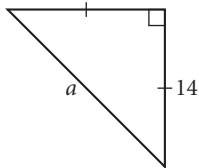
Lesson 9.3 • Two Special Right Triangles

Name _____ Period _____ Date _____

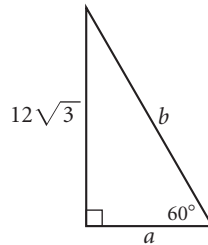
Give your answers in exact form unless otherwise indicated.
All measurements are in centimeters.

In Exercises 1–3, find the unknown lengths.

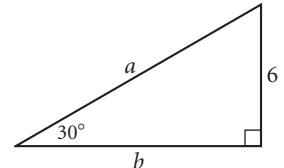
1. $a =$ _____



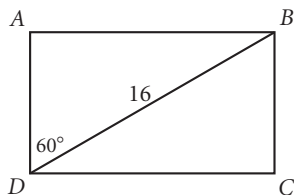
2. $a =$ _____, $b =$ _____



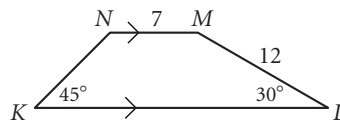
3. $a =$ _____, $b =$ _____



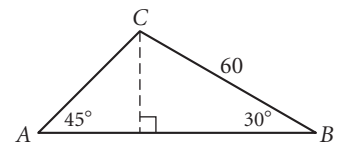
4. Find the area of rectangle $ABCD$.



5. Find the perimeter and area of $KLMN$.



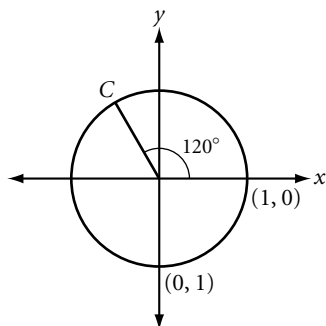
6. $AC =$ _____, $AB =$ _____,
and area $\triangle ABC =$ _____.



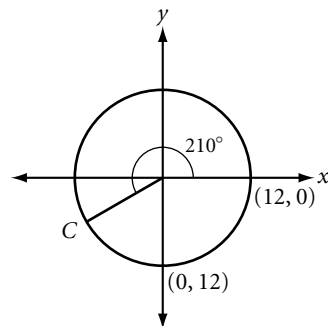
7. Find the area of an isosceles trapezoid if the bases have lengths 12 cm and 18 cm and the base angles have measure 60° .

In Exercises 8 and 9, find the coordinates of C .

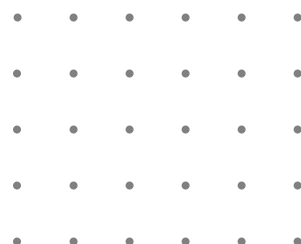
8.



9.



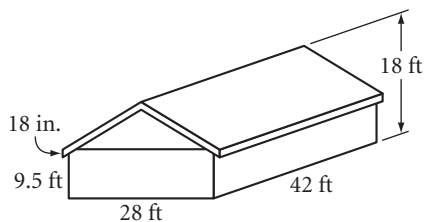
10. Sketch and label a figure to demonstrate that $\sqrt{18}$ is equivalent to $3\sqrt{2}$.



Lesson 9.4 • Story Problems

Name _____ Period _____ Date _____

1. A 20 ft ladder reaches a window 18 ft high. How far is the foot of the ladder from the base of the building? How far must the foot of the ladder be moved to lower the top of the ladder by 2 ft?
2. Robin and Dovey have four pet pigeons that they train to race. They release the birds at Robin's house and then drive to Dovey's to collect them. To drive from Robin's to Dovey's, because of one-way streets, they go 3.1 km north, turn right and go 1.7 km east, turn left and go 2.3 km north, turn right and go 0.9 km east, turn left and go 1.2 km north, turn left and go 4.1 km west, and finally turn left and go 0.4 km south. How far do the pigeons have to fly to go directly from Robin's house to Dovey's house?
3. Hans needs to paint the 18 in.-wide trim around the roof eaves and gable ends of his house with 2 coats of paint. A quart can of paint covers 175 ft^2 and costs \$9.75. A gallon can of paint costs \$27.95. How much paint should Hans buy? Explain.



4. What are the dimensions of the largest 30° - 60° - 90° triangle that will fit inside a 45° - 45° - 90° triangle with leg length 14 in.? Sketch your solution.

Lesson 9.5 • Distance in Coordinate Geometry

Name _____ Period _____ Date _____

In Exercises 1–3, find the distance between each pair of points.

1. $(-5, -5)$, $(1, 3)$ 2. $(-11, -5)$, $(5, 7)$ 3. $(8, -2)$, $(-7, 6)$

In Exercises 4 and 5, use the distance formula and the slope of segments to identify the type of quadrilateral. Explain your reasoning.

4. $A(-2, 1)$, $B(3, -2)$, $C(8, 1)$, $D(3, 4)$ 5. $T(-3, -3)$, $U(4, 4)$, $V(0, 6)$, $W(-5, 1)$

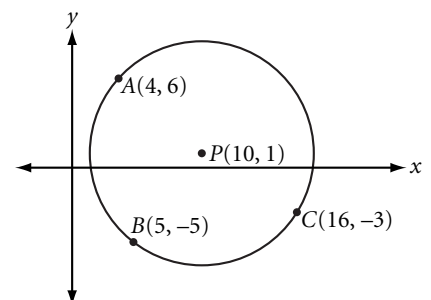
For Exercises 6 and 7, use $\triangle ABC$ with coordinates $A(4, 14)$, $B(10, 6)$, and $C(16, 14)$.

6. Determine whether $\triangle ABC$ is scalene, isosceles, or equilateral. Find the perimeter of the triangle.
7. Find the midpoints M and N of \overline{AB} and \overline{AC} , respectively. Find the slopes and lengths of \overline{MN} and \overline{BC} . How do the slopes compare? How do the lengths compare?

8. Find the equation of the circle with center $(-1, 5)$ and radius 2.

9. Find the center and radius of the circle whose equation is $x^2 + (y + 2)^2 = 25$.

10. P is the center of the circle. What's wrong with this picture?

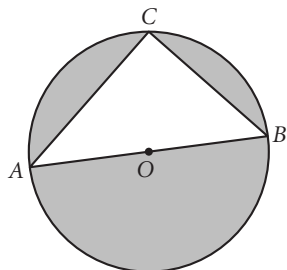


Lesson 9.6 • Circles and the Pythagorean Theorem

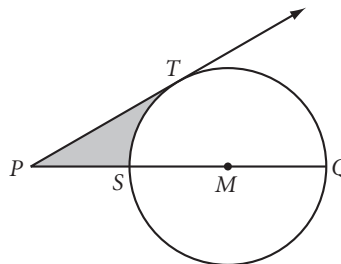
Name _____ Period _____ Date _____

In Exercises 1 and 2, find the area of the shaded region in each figure. All measurements are in centimeters. Write your answers in terms of π and rounded to the nearest 0.1 cm^2 .

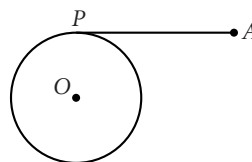
1. $AO = 5$. $AC = 8$.



2. Tangent \overrightarrow{PT} , $QM = 12$, $m\angle P = 30^\circ$



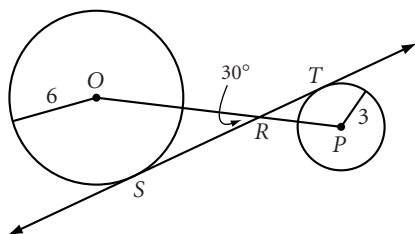
3. $AP = 63$ cm. Radius of circle $O = 37$ cm.
How far is A from the circumference of the circle?



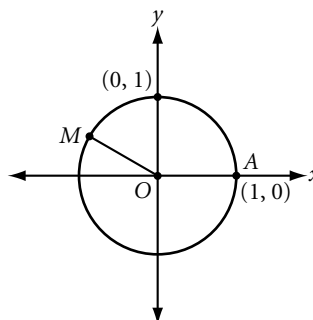
4. Two perpendicular chords with lengths 12.2 cm and 8.8 cm have a common endpoint. What is the area of the circle?

5. $ABCD$ is inscribed in a circle. \overline{AC} is a diameter. If $AB = 9.6$ cm, $BC = 5.7$ cm, and $CD = 3.1$ cm, find AD .

6. Find ST .



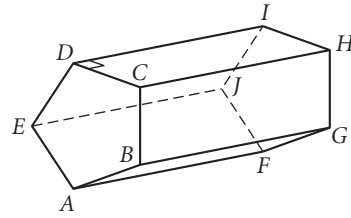
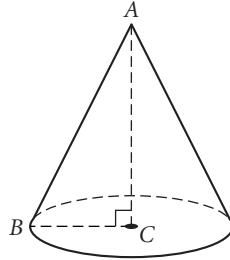
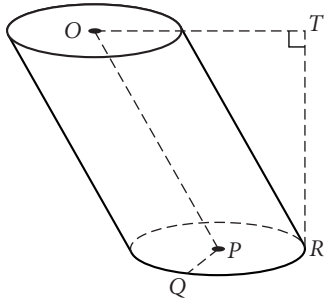
7. The coordinate of point M is $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$.
Find the measure of $\angle AOM$.



Lesson 10.1 • The Geometry of Solids

Name _____ Period _____ Date _____

For Exercises 1–14, refer to the figures below.



1. The cylinder is (oblique, right).
2. \overline{OP} is _____ of the cylinder.
3. \overline{TR} is _____ of the cylinder.
4. Circles O and P are _____ of the cylinder.
5. \overline{PQ} is _____ of the cylinder.
6. The cone is (oblique, right).
7. Name the base of the cone.
8. Name the vertex of the cone.
9. Name the altitude of the cone.
10. Name a radius of the cone.
11. Name the type of prism.
12. Name the bases of the prism.
13. Name all lateral edges of the prism.
14. Name an altitude of the prism.

In Exercises 15–17, tell whether each statement is true or false. If the statement is false, give a counterexample or explain why it is false.

15. The axis of a cylinder is perpendicular to the base.
16. A rectangular prism has four faces.
17. The bases of a trapezoidal prism are trapezoids.

For Exercises 18 and 19, draw and label each solid. Use dashed lines to show the hidden edges.

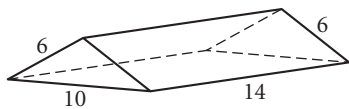
18. A right triangular prism with height equal to the hypotenuse
19. An oblique trapezoidal pyramid

Lesson 10.2 • Volume of Prisms and Cylinders

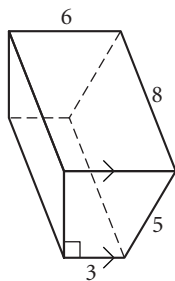
Name _____ Period _____ Date _____

In Exercises 1–3, find the volume of each prism or cylinder. All measurements are in centimeters. Round your answers to the nearest 0.01.

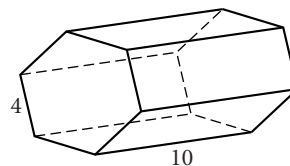
1. Right triangular prism



2. Right trapezoidal prism

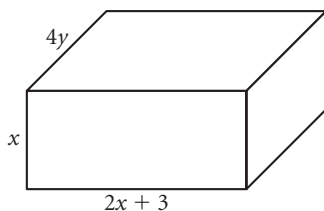


3. Regular hexagonal prism

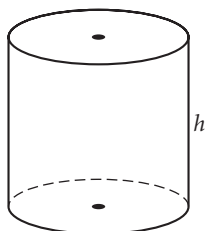


In Exercises 4–6, use algebra to express the volume of each solid.

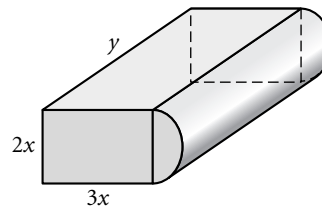
4. Right rectangular prism



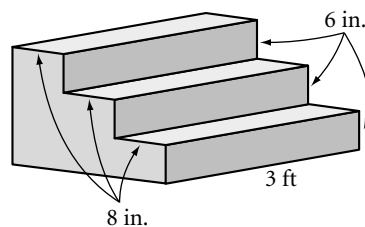
5. Right cylinder;
base circumference = $p\pi$



6. Right rectangular prism
and half of a cylinder



7. You need to build a set of solid cement steps for the entrance to your new house. How many cubic feet of cement do you need?

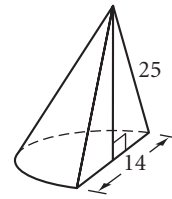
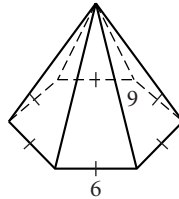
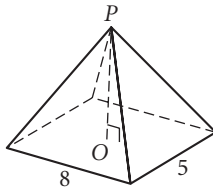


Lesson 10.3 • Volume of Pyramids and Cones

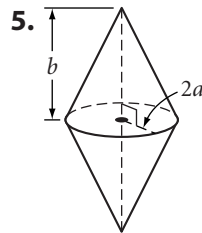
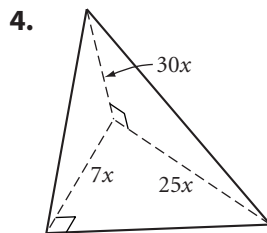
Name _____ Period _____ Date _____

In Exercises 1–3, find the volume of each solid. All measurements are in centimeters. Round your answers to two decimal places.

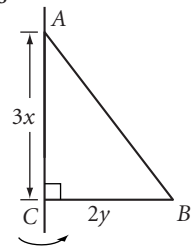
1. Rectangular pyramid; $OP = 6$ 2. Right hexagonal pyramid 3. Half of a right cone



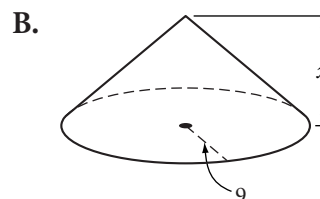
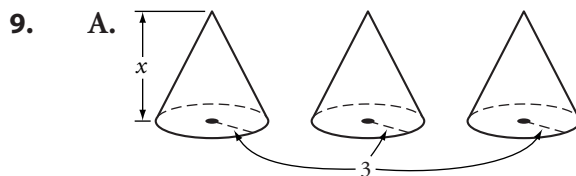
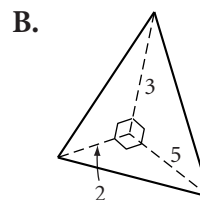
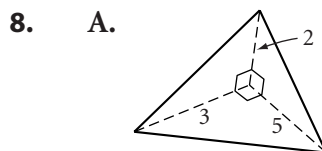
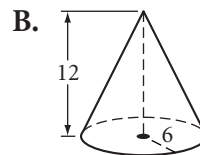
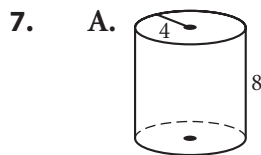
In Exercises 4–6, use algebra to express the volume of each solid.



6. The solid generated by spinning $\triangle ABC$ about the axis



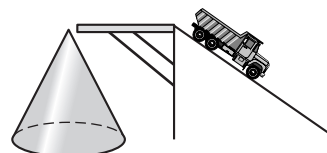
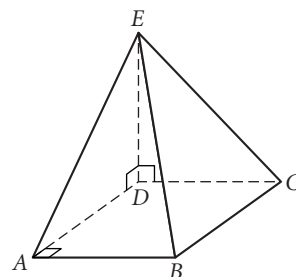
In Exercises 7–9, find the volume of each figure and tell which volume is larger.



Lesson 10.4 • Volume Problems

Name _____ Period _____ Date _____

1. A cone has volume 320 cm^3 and height 16 cm. Find the radius of the base. Round your answer to the nearest 0.1 cm.
2. How many cubic inches are there in one cubic foot? Use your answer to help you with Exercises 3 and 4.
3. Jerry is packing cylindrical cans with diameter 6 in. and height 10 in. tightly into a box that measures 3 ft by 2 ft by 1 ft. All rows must contain the same number of cans. The cans can touch each other. He then fills all the empty space in the box with packing foam. How many cans can Jerry pack in one box? Find the volume of packing foam he uses. What percentage of the box's volume is filled by the foam?
4. A king-size waterbed mattress measures 72 in. by 84 in. by 9 in. Water weighs 62.4 pounds per cubic foot. An empty mattress weighs 35 pounds. How much does a full mattress weigh?
5. Square pyramid $ABCDE$, shown at right, is cut out of a cube with base $ABCD$ and shared edge \overline{DE} . $AB = 2 \text{ cm}$. Find the volume and surface area of the pyramid.
6. In Dingwall the town engineers have contracted for a new water storage tank. The tank is cylindrical with a base 25 ft in diameter and a height of 30 ft. One cubic foot holds about 7.5 gallons of water. About how many gallons will the new storage tank hold?
7. The North County Sand and Gravel Company stockpiles sand to use on the icy roads in the northern rural counties of the state. Sand is brought in by tandem trailers that carry 12 m^3 each. The engineers know that when the pile of sand, which is in the shape of a cone, is 17 m across and 9 m high they will have enough for a normal winter. How many truckloads are needed to build the pile?



Lesson 10.5 • Displacement and Density

Name _____ Period _____ Date _____

1. A stone is placed in a 5 cm-diameter graduated cylinder, causing the water level in the cylinder to rise 2.7 cm. What is the volume of the stone?

2. A 141 g steel marble is submerged in a rectangular prism with base 5 cm by 6 cm. The water rises 0.6 cm. What is the density of the steel?

3. A solid wood toy boat with a mass of 325 g raises the water level of a 50 cm-by-40 cm aquarium 0.3 cm. What is the density of the wood?

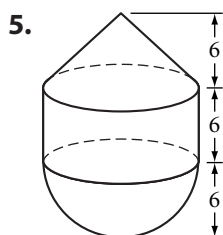
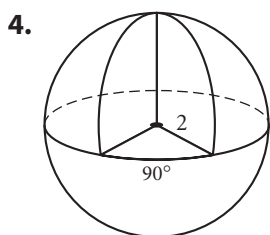
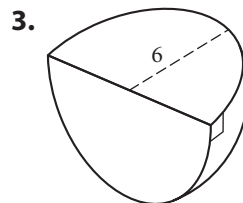
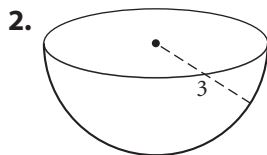
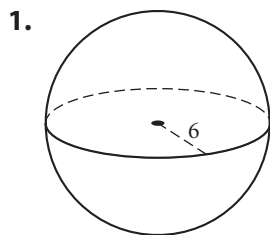
4. For Awards Night at Baddeck High School, the math club is designing small solid silver pyramids. The base of the pyramids will be a 2 in.-by-2 in. square. The pyramids should not weigh more than $2\frac{1}{2}$ pounds. One cubic foot of silver weighs 655 pounds. What is the maximum height of the pyramids?

5. While he hikes in the Gold Country of northern California, Sid dreams about the adventurers that walked the same trails years ago. He suddenly kicks a small bright yellowish nugget. Could it be gold? Sid quickly makes a balance scale using his walking stick and finds that the nugget has the same mass as the uneaten half of his 330 g nutrition bar. He then drops the stone into his water bottle, which has a 2.5 cm radius, and notes that the water level goes up 0.9 cm. Has Sid struck gold? Explain your reasoning. (Refer to the density chart in Lesson 10.5 in your book.)

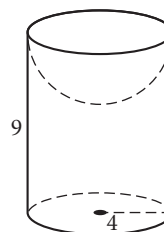
Lesson 10.6 • Volume of a Sphere

Name _____ Period _____ Date _____

In Exercises 1–6, find the volume of each solid. All measurements are in centimeters. Write your answers in exact form and rounded to the nearest 0.1 cm^3 .



6. Cylinder with hemisphere taken out of the top



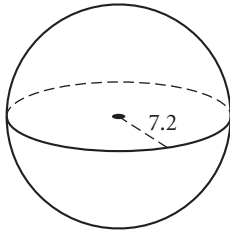
7. A sphere has volume $221\frac{5}{6}\pi \text{ cm}^3$. What is its diameter?
8. The area of the base of a hemisphere is $225\pi \text{ in}^2$. What is its volume?
9. Eight wooden spheres with radii 3 in. are packed snugly into a square box 12 in. on one side. The remaining space is filled with packing beads. What is the volume occupied by the packing beads? What percentage of the volume of the box is filled with beads?
10. The radius of Earth is about 6378 km, and the radius of Mercury is about 2440 km. About how many times greater is the volume of Earth than that of Mercury?

Lesson 10.7 • Surface Area of a Sphere

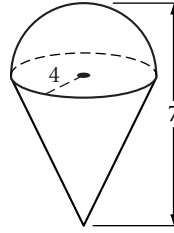
Name _____ Period _____ Date _____

In Exercises 1–4, find the volume and total surface area of each solid. All measurements are in centimeters. Round your answers to the nearest 0.1 cm.

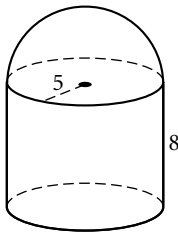
1.



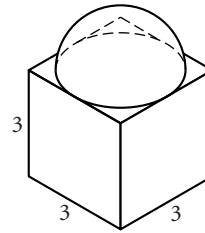
2.



3.



4.



5. If the surface area of a sphere is 48.3 cm^2 , find its diameter.
6. If the volume of a sphere is 635 cm^3 , find its surface area.
7. Lobster fishers in Maine often use spherical buoys to mark their lobster traps. Every year the buoys must be repainted. An average buoy has a 12 in. diameter, and an average fisher has about 500 buoys. A quart of marine paint covers 175 ft^2 . How many quarts of paint does an average fisher need each year?

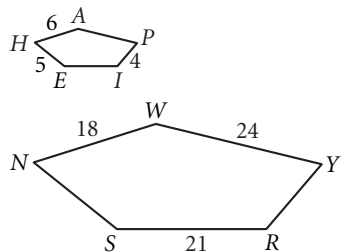
Lesson 11.1 • Similar Polygons

Name _____ Period _____ Date _____

All measurements are in centimeters.

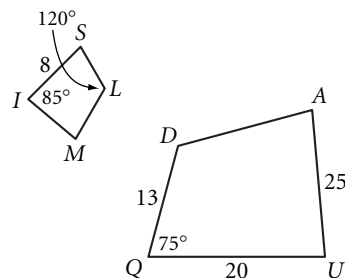
1. $HAPIE \sim NWYRS$

$AP =$ _____
 $EI =$ _____
 $SN =$ _____
 $YR =$ _____



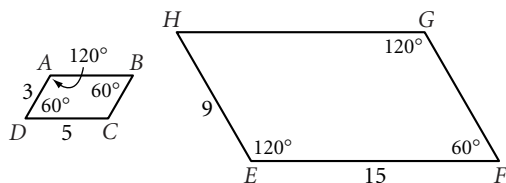
2. $QUAD \sim SIML$

$SL =$ _____
 $MI =$ _____
 $m\angle D =$ _____
 $m\angle U =$ _____
 $m\angle A =$ _____

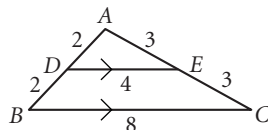


In Exercises 3–6, decide whether or not the figures are similar. Explain why or why not.

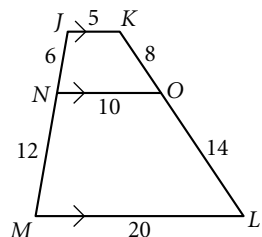
3. $ABCD$ and $EFGH$



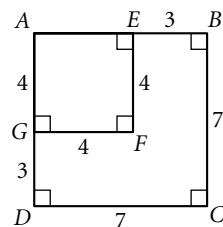
4. $\triangle ABC$ and $\triangle ADE$



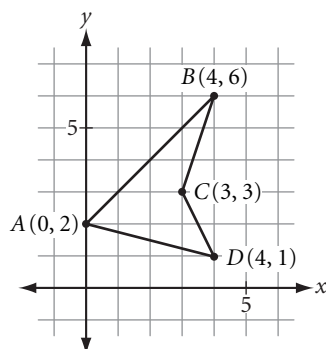
5. $JKON$ and $JKLM$



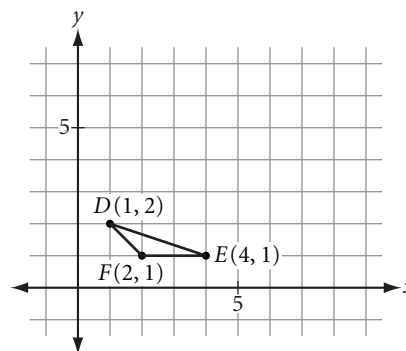
6. $ABCD$ and $AEFG$



7. Draw the dilation of $ABCD$ by a scale factor of $\frac{1}{2}$. What is the ratio of the perimeter of the dilated quadrilateral to the perimeter of the original quadrilateral?



8. Draw the dilation of $\triangle DEF$ by a scale factor of 2. What is the ratio of the area of the dilated triangle to the area of the original triangle?



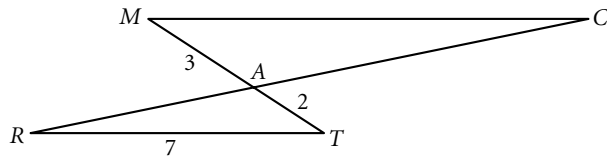
Lesson 11.2 • Similar Triangles

Name _____ Period _____ Date _____

All measurements are in centimeters.

1. $\triangle TAR \sim \triangle MAC$

$MC = \underline{\hspace{2cm}}$

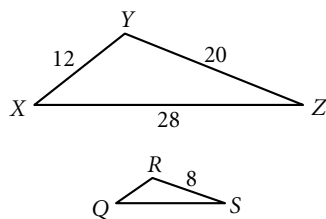


2. $\triangle XYZ \sim \triangle QRS$

$\angle Q \cong \underline{\hspace{2cm}}$

$QR = \underline{\hspace{2cm}}$

$QS = \underline{\hspace{2cm}}$

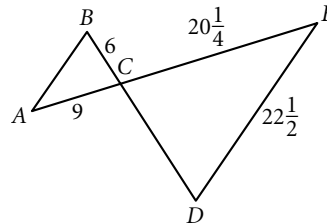


3. $\triangle ABC \sim \triangle EDC$

$\angle A \cong \underline{\hspace{2cm}}$

$CD = \underline{\hspace{2cm}}$

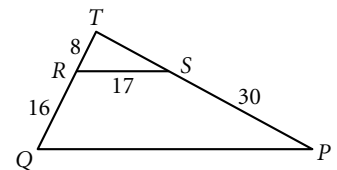
$AB = \underline{\hspace{2cm}}$



4. $\triangle TRS \sim \triangle TQP$

$TS = \underline{\hspace{2cm}}$

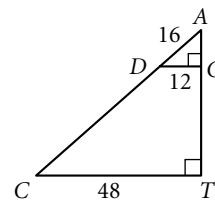
$QP = \underline{\hspace{2cm}}$



For Exercises 5 and 6, refer to the figure at right.

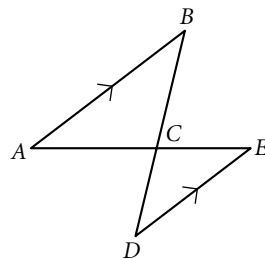
5. Explain why $\triangle CAT$ and $\triangle DAG$ are similar.

6. $CA = \underline{\hspace{2cm}}$

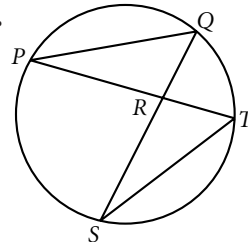


In Exercises 7–9, identify similar triangles and explain why they are similar.

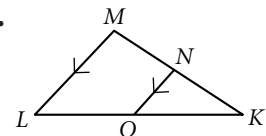
7.



8.



9.



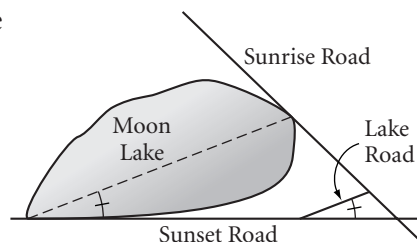
Lesson 11.3 • Indirect Measurement with Similar Triangles

Name _____ Period _____ Date _____

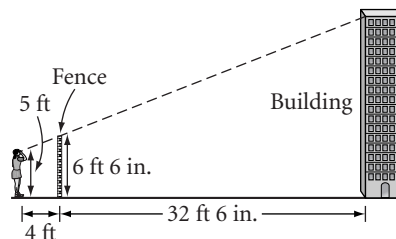
- At a certain time of day, a 6 ft man casts a 4 ft shadow. At the same time of day, how tall is a tree that casts an 18 ft shadow?

- Driving through the mountains, Dale has to go up and over a high mountain pass. The road has a constant incline for $7\frac{3}{4}$ miles to the top of the pass. Dale notices from a road sign that in the first mile he climbs 840 feet. How many feet does he climb in all?

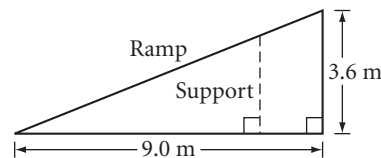
- Sunrise Road is 42 miles long between the edge of Moon Lake and Lake Road and 15 miles long between Lake Road and Sunset Road. Lake Road is 29 miles long. Find the length of Moon Lake.



- Marta is standing 4 ft behind a fence 6 ft 6 in. tall. When she looks over the fence, she can just see the top edge of a building. She knows that the building is 32 ft 6 in. behind the fence. Her eyes are 5 ft from the ground. How tall is the building? Give your answer to the nearest half foot.



- You need to add 5 supports under the ramp, in addition to the 3.6 m one, so that they are all equally spaced. How long should each support be? (One is drawn in for you.)

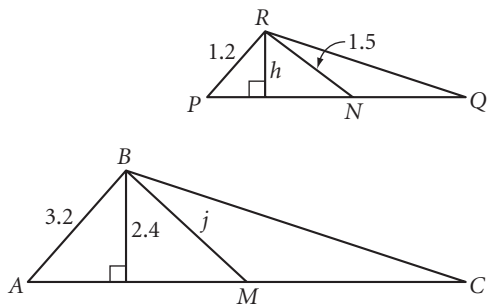


Lesson 11.4 • Corresponding Parts of Similar Triangles

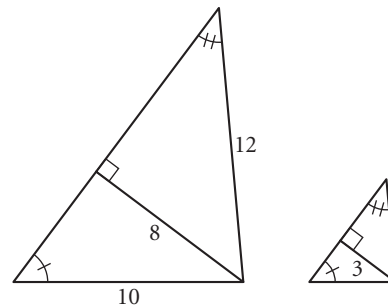
Name _____ Period _____ Date _____

All measurements are in centimeters.

1. $\triangle ABC \sim \triangle PRQ$. M and N are midpoints. Find h and j .

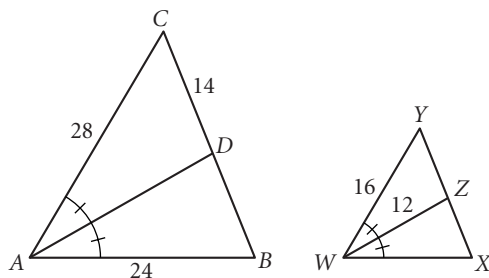


2. The triangles are similar.
Find the length of each side of the smaller triangle to the nearest 0.01.

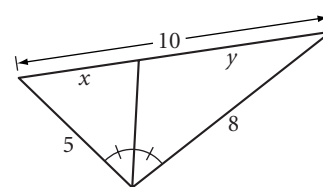


3. $\triangle ABC \sim \triangle WXY$

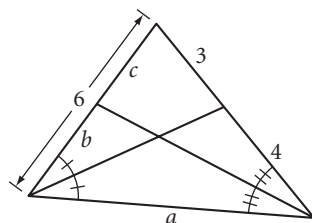
$WX = \underline{\hspace{2cm}}$ $AD = \underline{\hspace{2cm}}$
 $DB = \underline{\hspace{2cm}}$ $YZ = \underline{\hspace{2cm}}$
 $XZ = \underline{\hspace{2cm}}$



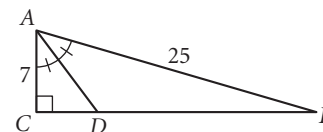
4. Find x and y .



5. Find a , b , and c .



6. Find CB , CD , and AD .

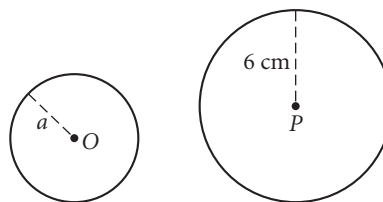
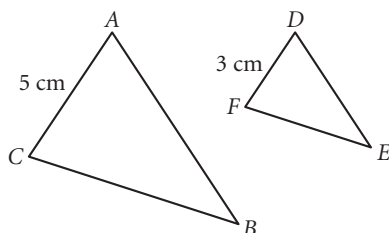


Lesson 11.5 • Proportions with Area

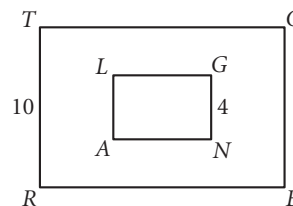
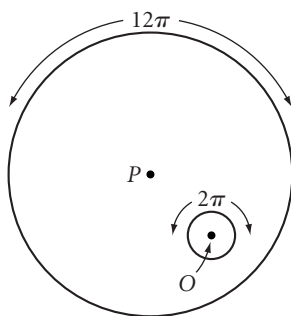
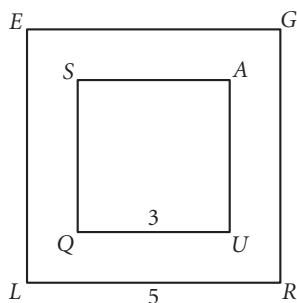
Name _____ Period _____ Date _____

All measurements are in centimeters unless otherwise indicated.

- $\triangle ABC \sim \triangle DEF$. Area of $\triangle ABC = 15 \text{ cm}^2$.
Area of $\triangle DEF =$ _____
- $\frac{\text{Area of circle } O}{\text{Area of circle } P} = \frac{4}{9}$.
 $a =$ _____



- $\frac{\text{Area of square } SQUA}{\text{Area of square } LRGE} =$ _____
- $\frac{\text{Area of circle } P}{\text{Area of circle } O} =$ _____
- $RECT \sim ANGL$.
 $\frac{\text{Area of } RECT}{\text{Area of } ANGL} =$ _____



- The ratio of the corresponding midsegments of two similar trapezoids is 4:5. What is the ratio of their areas?
- The ratio of the areas of two similar pentagons is 4:9. What is the ratio of their corresponding sides?
- If $ABCDE \sim FGHIJ$, $AC = 6 \text{ cm}$, $FH = 10 \text{ cm}$, and area of $ABCDE = 320 \text{ cm}^2$, then area of $FGHIJ =$ _____.
- Stefan is helping his mother retille the kitchen floor. The tiles are 4-by-4-inch squares. The kitchen is square, and the area of the floor is 144 square feet. Assuming the tiles fit snugly (don't worry about grout), how many tiles will be needed to cover the floor?

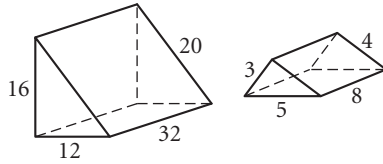
Lesson 11.6 • Proportions with Volume

Name _____ Period _____ Date _____

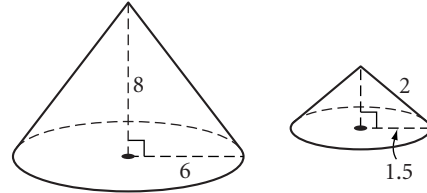
All measurements are in centimeters unless otherwise indicated.

In Exercises 1 and 2, decide whether or not the two solids are similar.

1.



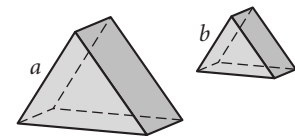
2.



3. The triangular prisms are similar and the ratio of a to b is $\frac{5}{2}$.

Volume of large prism = 250 cm^3

Volume of smaller prism = _____

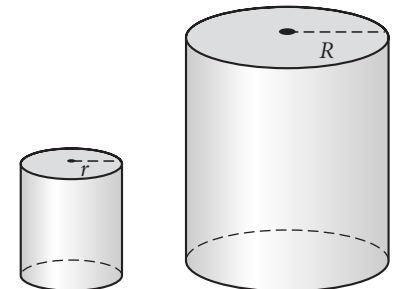


4. The right cylinders are similar and $r = 10 \text{ cm}$.

Volume of large cylinder = 64 cm^3

Volume of small cylinder = 8 cm^3

$R =$ _____



5. The corresponding heights of two similar cylinders is 2:5. What is the ratio of their volumes?

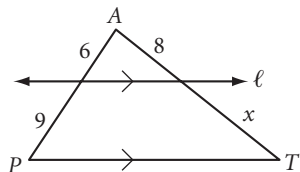
6. A rectangular prism aquarium holds 64 gallons of water. A similarly shaped aquarium holds 8 gallons of water. If a 1.5 ft^2 cover fits on the smaller tank, what is the area of a cover that will fit on the larger tank?

Lesson 11.7 • Proportional Segments Between Parallel Lines

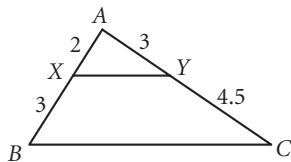
Name _____ Period _____ Date _____

All measurements are in centimeters.

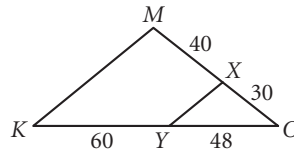
1. $x =$ _____



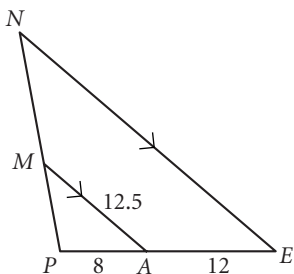
2. Is $\overline{XY} \parallel \overline{BC}$?



3. Is $\overline{XY} \parallel \overline{MK}$?



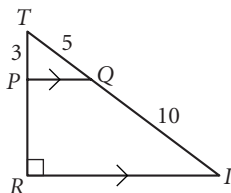
4. $NE =$ _____



5. $PR =$ _____

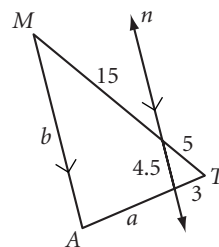
$PQ =$ _____

$RI =$ _____



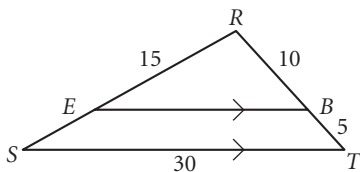
6. $a =$ _____

$b =$ _____



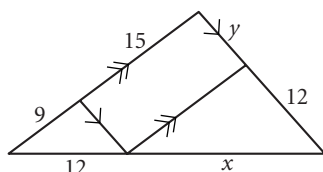
7. $RS =$ _____

$EB =$ _____



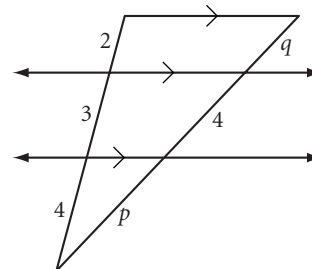
8. $x =$ _____

$y =$ _____



9. $p =$ _____

$q =$ _____



Lesson 12.1 • Trigonometric Ratios

Name _____ Period _____ Date _____

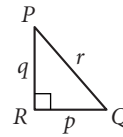
In Exercises 1–4, give each answer as a fraction in terms of p , q , and r .

1. $\sin P =$ _____

2. $\cos P =$ _____

3. $\tan P =$ _____

4. $\sin Q =$ _____



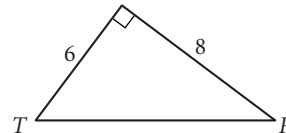
In Exercises 5–8, give each answer as a decimal accurate to the nearest 0.001.

5. $\sin T =$ _____

6. $\cos T =$ _____

7. $\tan T =$ _____

8. $\sin R =$ _____



For Exercises 9–11, solve for x . Express each answer accurate to the nearest 0.01.

9. $\cos 64^\circ = \frac{x}{28}$

10. $\sin 24^\circ = \frac{12.1}{x}$

11. $\tan 51^\circ = \frac{x}{14.8}$

For Exercises 12–14, find the measure of each angle to the nearest degree.

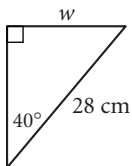
12. $\sin A = 0.9455$

13. $\tan B = \frac{4}{3}$

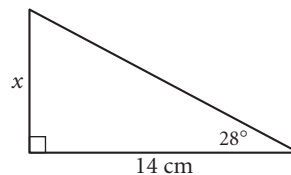
14. $\cos C = 0.8660$

For Exercises 15–17, write a trigonometric equation you can use to solve for the unknown value. Then find the value to the nearest 0.1.

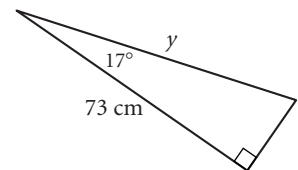
15. $w \approx$ _____



16. $x \approx$ _____

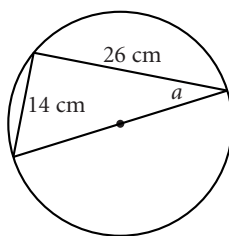


17. $y \approx$ _____

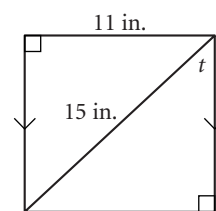


For Exercises 18–20, find the value of each unknown to the nearest degree.

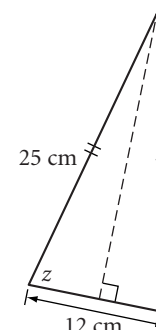
18. $a \approx$ _____



19. $t \approx$ _____



20. $z \approx$ _____

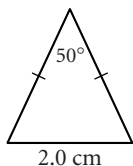


Lesson 12.2 • Problem Solving with Right Triangles

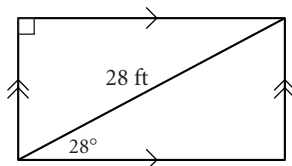
Name _____ Period _____ Date _____

For Exercises 1–3, find the area of each figure to the nearest square unit.

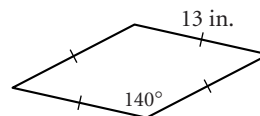
1. Area \approx _____



2. Area \approx _____



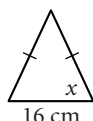
3. Area \approx _____



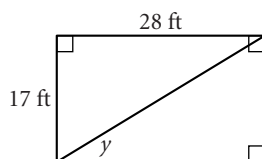
For Exercises 4–9, find each unknown to the nearest tenth of a unit.

4. Area = 88 cm²

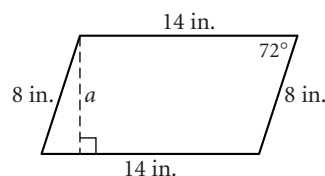
$x \approx$ _____



5. $y \approx$ _____

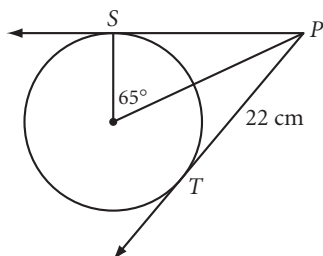


6. $a \approx$ _____



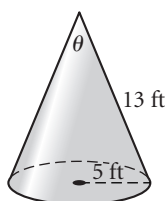
7. \overrightarrow{PS} and \overrightarrow{PT} are tangents.

Diameter \approx _____



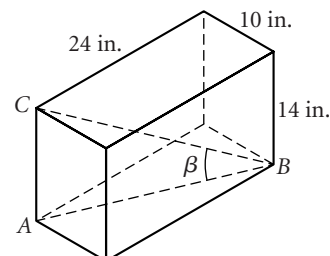
8. Right cone

$\theta \approx$ _____



9. Right rectangular prism

$m\angle ABC = \beta \approx$ _____



In Exercises 10–12, give each answer to the nearest tenth of a unit.

10. A ladder 7 m long stands on level ground and makes a 73° angle with the ground as it rests against a wall. How far from the wall is the base of the ladder?

11. To see the top of a building 1000 feet away, you look up 24° from the horizontal. What is the height of the building?

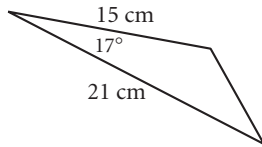
12. A guy wire is anchored 12 feet from the base of a pole. The wire makes a 58° angle with the ground. How long is the wire?

Lesson 12.3 • The Law of Sines

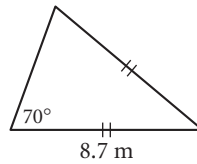
Name _____ Period _____ Date _____

In Exercises 1–3, find the area of each figure to the nearest square unit.

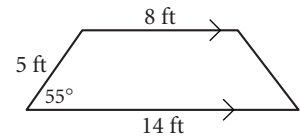
1. Area \approx _____



2. Area \approx _____

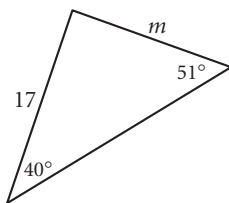


3. Area \approx _____

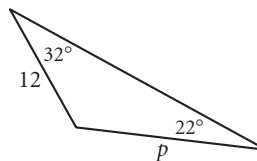


In Exercises 4–6, find each length to the nearest centimeter. All lengths are in centimeters.

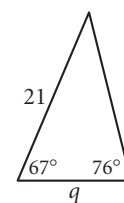
4. $m \approx$ _____



5. $p \approx$ _____



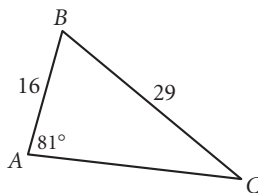
6. $q \approx$ _____



In Exercises 7–9, find the measure of each angle to the nearest degree.

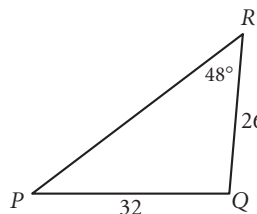
7. $m\angle B \approx$ _____

$m\angle C \approx$ _____



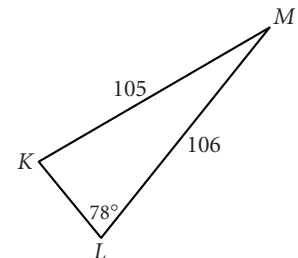
8. $m\angle P \approx$ _____

$m\angle Q \approx$ _____



9. $m\angle K \approx$ _____

$m\angle M \approx$ _____



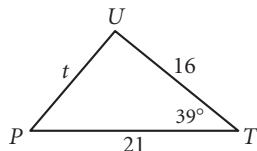
10. A large helium balloon is tethered to the ground by two taut lines. One line is 100 feet long and makes an 80° angle with the ground. The second line makes a 40° angle with the ground. How long is the second line, to the nearest foot? How far apart are the tethers?

Lesson 12.4 • The Law of Cosines

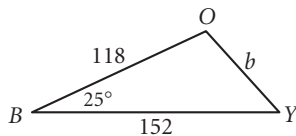
Name _____ Period _____ Date _____

In Exercises 1–3, find each length to the nearest centimeter. All lengths are in centimeters.

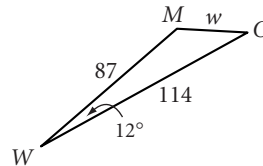
1. $t \approx$ _____



2. $b \approx$ _____



3. $w \approx$ _____

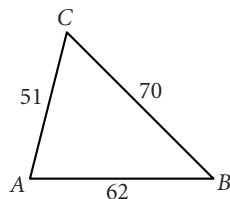


In Exercises 4–6, find each angle measure to the nearest degree.

4. $m\angle A \approx$ _____

$m\angle B \approx$ _____

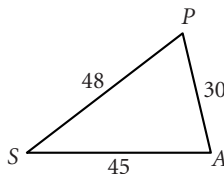
$m\angle C \approx$ _____



5. $m\angle A \approx$ _____

$m\angle P \approx$ _____

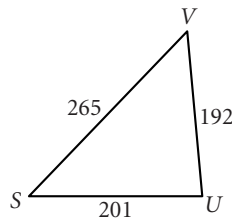
$m\angle S \approx$ _____



6. $m\angle S \approx$ _____

$m\angle U \approx$ _____

$m\angle V \approx$ _____



7. A circle with radius 12 in. has radii drawn to the endpoints of a 5 in. chord. What is the measure of the central angle?

8. A parallelogram has side lengths 22.5 cm and 47.8 cm. One angle measures 116° . What is the length of the shorter diagonal?

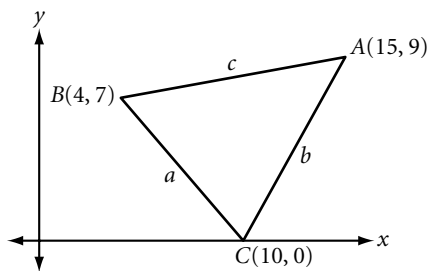
9. The diagonals of a parallelogram are 60 in. and 70 in. and intersect at an angle measuring 64° . Find the length of the shorter side of the parallelogram.

Lesson 12.5 • Problem Solving with Trigonometry

Name _____ Period _____ Date _____

1. While floating down a river with a 2.75 mi/h current, Alicia decides to swim directly toward the river bank. She can swim 0.75 mi/h in still water. What is the actual speed at which she moves toward the bank? At what angle will she approach the bank, measured with respect to the bank?

2. Find the measure of each angle to the nearest hundredth of a degree.



3. Two fire watchtowers 8.4 km apart spot a fire at the same time. Tower 1 reports the fire at a 36° angle measure from its line of sight to Tower 2. Tower 2 reports a 68° angle measure between the fire and Tower 1. How far is the fire from each tower?
4. Two airplanes leave O'Hare Airport in Chicago at the same time. One plane flies 280 mi/h at bearing 55° . The other plane flies 350 mi/h at bearing 128° . How far apart are the two planes after 2 hours 15 minutes?
5. Carla needs to fence her triangular plot of land. The angle between the two shorter sides measures 83° . The shortest side is 122 ft and the longest is 215 ft. How much fencing does Carla need? What is the area of her plot of land?

Lesson 13.1 • The Premises of Geometry

Name _____ Period _____ Date _____

1. Provide the missing property of equality or arithmetic as a reason for each step to solve the equation.

Solve for x : $5(x - 4) = 2x + 17$

Solution: $5(x - 4) = 2x + 17$ **a.** _____

$$5x - 20 = 2x + 17 \quad \mathbf{b.} \underline{\hspace{2cm}}$$
 $3x - 20 = 17$ c. _____ $3x = 37$ d. _____
$$x = \frac{37}{3} \quad \text{e. } \underline{\hspace{2cm}}$$

In Exercises 2–4, identify each statement as true or false. If the statement is true, tell which definition, property, or postulate supports your answer. If the statement is false, give a counterexample.

- 2.** If $AM = BM$, then M is the midpoint of \overline{AB} .

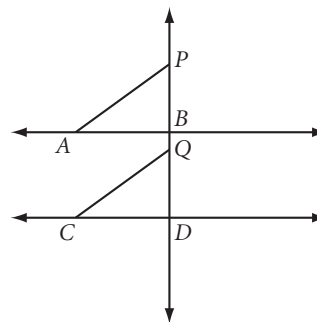
- 3.** If P is on \overleftrightarrow{AB} and D is not, then $m\angle APD + m\angle BPD = 180^\circ$.

- 4.** If $\overline{PQ} \cong \overline{ST}$ and $\overline{PQ} \cong \overline{KL}$, then $ST = KL$.

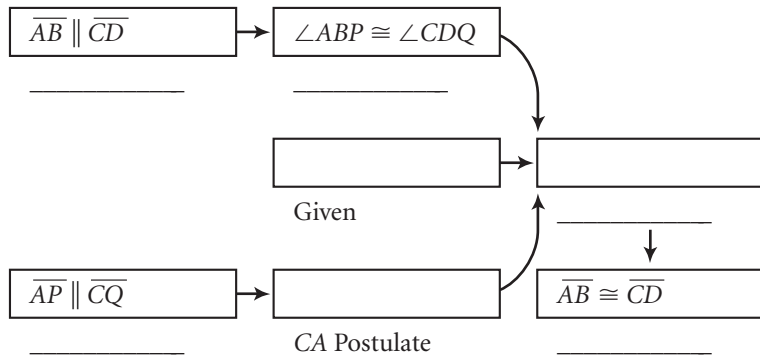
- 5.** Complete the flowchart proof.

Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AP} \parallel \overline{CQ}$, $\overline{PB} \cong \overline{QD}$

Show: $\overline{AB} \cong \overline{CD}$



Flowchart Proof



Lesson 13.2 • Planning a Geometry Proof

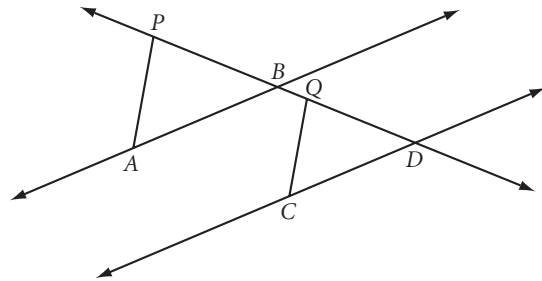
Name _____ Period _____ Date _____

For these exercises, you may use theorems added to your theorem list through the end of Lesson 13.2.

In Exercises 1–3, write a paragraph proof or a flowchart proof for each situation.

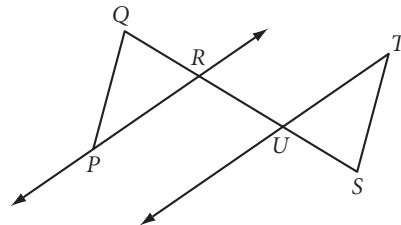
1. Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AP} \parallel \overline{CQ}$

Show: $\angle PAB \cong \angle QCD$



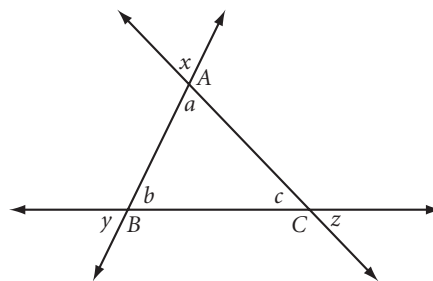
2. Given: $\overline{PQ} \parallel \overline{ST}$, $\angle QPR \cong \angle STU$

Show: $\overline{PR} \parallel \overline{UT}$



3. Given: Noncongruent, nonparallel segments AB , BC , and AC

Show: $x + y + z = 180^\circ$



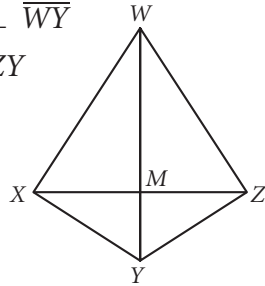
Lesson 13.3 • Triangle Proofs

Name _____ Period _____ Date _____

Write a proof for each situation. You may use theorems added to your theorem list through the end of Lesson 13.3.

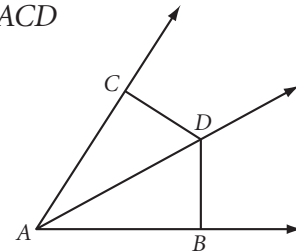
1. **Given:** $\overline{XY} \cong \overline{ZY}$, $\overline{XZ} \perp \overline{WY}$

Show: $\triangle WXY \cong \triangle WZY$



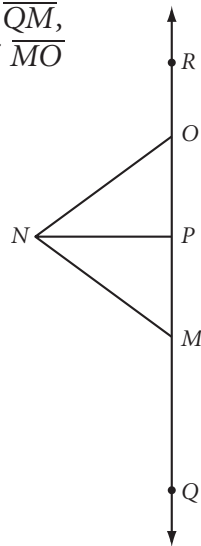
2. **Given:** $\overline{CD} \perp \overline{AC}$, $\overline{BD} \perp \overline{AB}$, $\overline{CD} \cong \overline{BD}$

Show: $\triangle ABD \cong \triangle ACD$



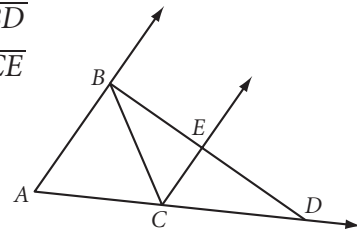
3. **Given:** $\overline{MN} \cong \overline{QM}$, $\overline{NO} \cong \overline{QO}$,
P is the midpoint of \overline{MO}

Show: $\angle QMN \cong \angle QON$



4. **Given:** $\overline{AB} \cong \overline{BC}$, $\angle ACB \cong \angle ECD$,
 $\overline{AB} \perp \overline{BD}$

Show: $\overline{BD} \perp \overline{CE}$



Lesson 13.4 • Quadrilateral Proofs

Name _____ Period _____ Date _____

In Exercises 1–6, write a proof of each conjecture on a separate piece of paper. You may use theorems added to your theorem list through the end of Lesson 13.4.

1. The diagonals of a parallelogram bisect each other. (Parallelogram Diagonals Theorem)
2. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (Converse of the Parallelogram Diagonals Theorem)
3. The diagonals of a rhombus bisect each other and are perpendicular. (Rhombus Diagonals Theorem)
4. If the diagonals of a quadrilateral bisect each other and are perpendicular, then the quadrilateral is a rhombus. (Converse of the Rhombus Diagonals Theorem)
5. If the base angles on one base of a trapezoid are congruent, then the trapezoid is isosceles. (Converse of the Isosceles Trapezoid Theorem)
6. If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles. (Converse of the Isosceles Trapezoid Diagonals Theorem)

In Exercises 7–9, decide if the statement is true or false. If it is true, prove it. If it is false, give a counterexample.

7. A quadrilateral with one pair of parallel sides and one pair of congruent angles is a parallelogram.
8. A quadrilateral with one pair of congruent opposite sides and one pair of parallel sides is a parallelogram.
9. A quadrilateral with one pair of parallel sides and one pair of congruent opposite angles is a parallelogram.

Lesson 13.5 • Indirect Proof

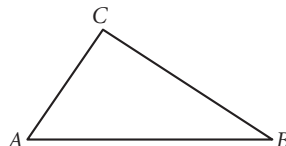
Name _____ Period _____ Date _____

- Complete the indirect proof of the conjecture: In a triangle the side opposite the larger of two angles has a greater measure.

Given: $\triangle ABC$ with $m\angle A > m\angle B$

Show: $BC > AC$

Proof: Assume _____



Case 1: $BC = AC$

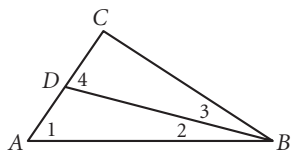
If $BC = AC$, then $\triangle ABC$ is _____ by _____.

By _____, $\angle A \cong \angle B$, which contradicts _____.

So, $BC \neq AC$.

Case 2: $BC < AC$

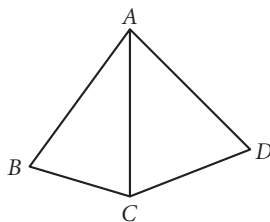
If $BC < AC$, then it is possible to construct point D on \overline{CA} such that $\overline{CD} \cong \overline{CB}$, by the Segment Duplication Postulate. Construct \overline{DB} , by the Line Postulate. $\triangle DBC$ is _____. Complete the proof.



In Exercises 2–5, write an indirect proof of each conjecture.

- Given:** $\overline{AD} \cong \overline{AB}$, $\overline{DC} \not\cong \overline{BC}$

Show: $\angle DAC \not\cong \angle BAC$



- If two sides of a triangle are not congruent, then the angles opposite them are not congruent.
- If two lines are parallel and a third line in the same plane intersects one of them, then it also intersects the other.

Lesson 13.6 • Circle Proofs

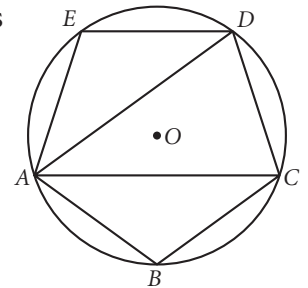
Name _____ Period _____ Date _____

Write a proof for each conjecture or situation. You may use theorems added to your theorem list through the end of Lesson 13.6.

1. If two chords in a circle are congruent, then their arcs are congruent.

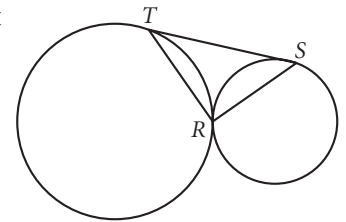
2. **Given:** Regular pentagon $ABCDE$ inscribed in circle O , with diagonals \overline{AC} and \overline{AD}

Show: \overline{AC} and \overline{AD} trisect $\angle BAE$



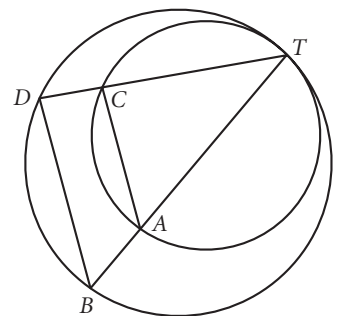
3. **Given:** Two circles externally tangent at R , common external tangent segment \overline{TS}

Show: $\angle TRS$ is a right angle



4. **Given:** Two circles internally tangent at T with chords \overline{TD} and \overline{TB} of the larger circle intersecting the smaller circle at C and A

Show: $\overline{AC} \parallel \overline{BD}$



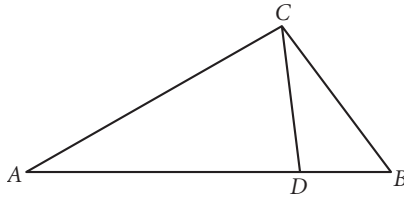
Lesson 13.7 • Similarity Proofs

Name _____ Period _____ Date _____

Write a proof for each situation. You may use theorems added to your theorem list through the end of Lesson 13.7.

1. **Given:** $\triangle ABC$ with $\angle A \cong \angle BCD$

Show: $BC^2 = AB \cdot BD$



2. The diagonals of a trapezoid divide each other into segments with lengths in the same ratio as the lengths of the bases.
3. In a right triangle the product of the lengths of the two legs equals the product of the lengths of the hypotenuse and the altitude to the hypotenuse.
4. If a quadrilateral has one pair of opposite right angles and one pair of opposite congruent sides, then the quadrilateral is a rectangle.